SIDEBAND INSTABILITY OF MODULATED TRAVELING WAVE CONVECTION

M. de la Torre Juárez, W. Zimmermann, and I.Rehberg
Physikalisches Institut, Universität Bayreuth, (FRG)

INTRODUCTION

It has been shown theoretically (Riecke et al. 1988, Walgraef 1988), that in a system exhibiting a supercritical Hopf bifurcation a temporal modulation of the driving force with a modulation frequency ω_m of about double of the Hopf frequency can stabilize standing waves (SW). An experimental verification was presented (Rehberg et al. 1988) for the electro-hydrodynamic convection of liquid crystals, where the driving ac voltage is modulated as $V(t)= V_c\cdot\cos(\omega\cdot t)\cdot[1+\varepsilon+b\cdot\cos(\omega_m\cdot t)]$, with b being the modulation and ε the reduced driving amplitude. The theoretical model is very similar to the one describing parametrically excited waves which are known to exhibit Benjamin-Feir turbulence that is characterized by a transfer of energy from the fundamental Fourier mode to the side bands (Craik 1985). When increasing ε for a constant modulation amplitude b the simplified theoretical model predicts a supercritical bifurcation from SW to modulated traveling waves (TW). In the experiment SW become unstable via a different mechanism which we clarify here. The scenario includes the appearance of the sideband instability, defects and stable undulated rolls of a very short wavelength.

TRANSITION FROM STANDING TO TRAVELING WAVES

The solution describing a system of traveling waves can be written as

$$u(x,t)= A_1(X,T)\cdot\exp(i\cdot(q_c\cdot x+\omega_c\cdot t)) + A_2(X,T)\cdot\exp(i\cdot(q_c\cdot x-\omega_c\cdot t)) + c.c.$$

where $u(x,t)$ describes one of the observable quantities and A_1 and A_2 are the slowly varying amplitudes of the left and right TW, respectively. The fast variable $u(x,t)$ is translational invariant $(x \to x + d)$ and has reflectional symmetry $(x \to -x)$. Using these symmetries the normal form valid for small amplitudes near a supercritical Hopf-bifurcation is (Ioss 1987):

$$\partial_t A_1 = [(\mu+i\nu)-(1+i\beta)|A_1|^2-(\delta+i\gamma)|A_2|^2]A_1 + O(A_1^5)$$
$$\partial_t A_2 = [(\mu-i\nu)-(1-i\beta)|A_2|^2-(\delta-i\gamma)|A_1|^2]A_2 + O(A_1^5)$$

As long as $\delta > 0$ left TW or right TW are stable with respect to SW.

When the driving frequency is time modulated with the frequency $\omega_m \simeq 2\cdot\omega_c$ the system acquires a new invariance under the transformation $t \to t + 2\pi/\omega_m$ and leads to a linear coupling between A_1 and A_2 proportional to the modulation amplitude b:

Nonlinear Evolution of Spatio-Temporal Structures in Dissipative Continuous Systems
Edited by F.H. Busse and L. Kramer

$$\partial_t A_1 = [(\mu+i\nu)-(1+i\beta)|A_1|^2-(\delta+i\gamma)|A_2|^2]A_1 + \lambda A_2 + O(A_1^5)$$
$$\partial_t A_2 = [(\mu-i\nu)-(1-i\beta)|A_2|^2-(\delta-i\gamma)|A_1|^2]A_2 + \lambda A_1 + O(A_1^5) \qquad (I)$$

where the phases have been chosen adequately to make $\lambda \propto b$ real. For $\lambda \neq 0$ the simple solutions of this equation are SW or modulated TW instead the TW solutions.

In large aspect ratio systems long wavelength perturbations can destabilize the coherent pattern. Spatial derivatives have to be added to (I) in order to describe this (Fauve 1987):

$$(\partial_t+s\partial_x)A_1 = [(\mu+i\nu)+(\varphi+i\eta)\partial_{xx}-(1+i\beta)|A_1|^2-(\delta+i\gamma)|A_2|^2]A_1+\lambda A_2+O(A_1^5)$$
$$(\partial_t-s\partial_x)A_2 = [(\mu-i\nu)+(\varphi-i\eta)\partial_{xx}-(1-i\beta)|A_2|^2-(\delta-i\gamma)|A_1|^2]A_2+\lambda A_1+O(A_1^5) \qquad (II)$$

The linear stability analysis of (I) around the trivial state $A_1=A_2=0$ shows a Hopf bifurcation at $\mu=0$, $\lambda^2<\nu^2$ leading to modulated TW; and a steady one at $\mu^2+\nu^2=\lambda^2$, $\lambda^2>\nu^2$ that gives rise to SW. One can also see that at an $\varepsilon>0$ for $\lambda^2>\nu^2$ the SW become unstable against modulated TW along a threshold line of cubic order in ε. This transition could not be seen in the experiment. This is understandable when a stability analysis based on the more realistic model (II) is done for long wavelength perturbations. It turns out that the SW become unstable against spatial modulations before the transition to temporally modulated TW occurs. This kind of instability is called sideband instability since it destabilizes the Fourier modes close to the critical one. One example of sideband instability for a steady bifurcation is the Eckhaus instability in a dissipative system like liquid crystals (Lowe and Gollub 1985). Experimentally it was first described in the case of an oscillatory bifurcation in a conservative system, namely trains of water waves propagating downstream from a wavemaker (Benjamin and Feir 1966), thus the name Benjamin-Feir instability (BF) is also used for this instability.

An example for this situation is shown in the phase diagram (Fig. 1, from Rehberg et al. 1988). Here along the closed circles the SW become unstable against a solution with alternating SW and TW even for $\varepsilon<0$ (Fig. 2). This line can be explained by taking into account the effect of spatial modulations. A good fit of equation (II) to this line was possible with values of the coefficients of the order of magnitude of the ones measured by de la Torre Juárez and Rehberg (1989) and in addition the line obtained for the saddle node bifurcation predicted for (I) fitted very well with the threshold shift measured for $\varepsilon<0$ and $\lambda^2<\nu^2$.

EXPERIMENTAL SETUP

The sample studied consisted of one cell of 15μm thickness filled with Merck Phase V. This is a nematic liquid crystal mixture whose parameter values are mostly unknown. However most of the ones appearing in the context of the simplified model (Riecke et al. 1988) could be estimated and reported elsewhere (de la Torre Juárez and Rehberg 1989). The measurements presented here were done in a different cell than those by Rehberg et al. (1988) and de la Torre and Rehberg (1989), but all three cells were filled with Phase V and had 15μm thickness, so their behaviour is expected to be very similar.

The cell has been treated to impose an initial homogeneously planar orientation on the director field. It was kept inside a thermalized box to achieve a temperature stabilization of about 0.005 K. It was also hermetically sealed in order to avoid impurities to come from outside into the fluid. This stabilized the values of the bifurcation diagrams over month.

The measurements were done using a shadowgraph technique (Rasenat et al. 1989) by illuminating the sample along the direction of the electric field. The data were taken with a CCD camera mounted on a microscope and digitized by a computer to be stored and treated. The basic treatment consisted in taking a whole image with a resolution of 512x512 pixels or measuring along a line parallel to the initial orientation of the director of the sample (x-direction).

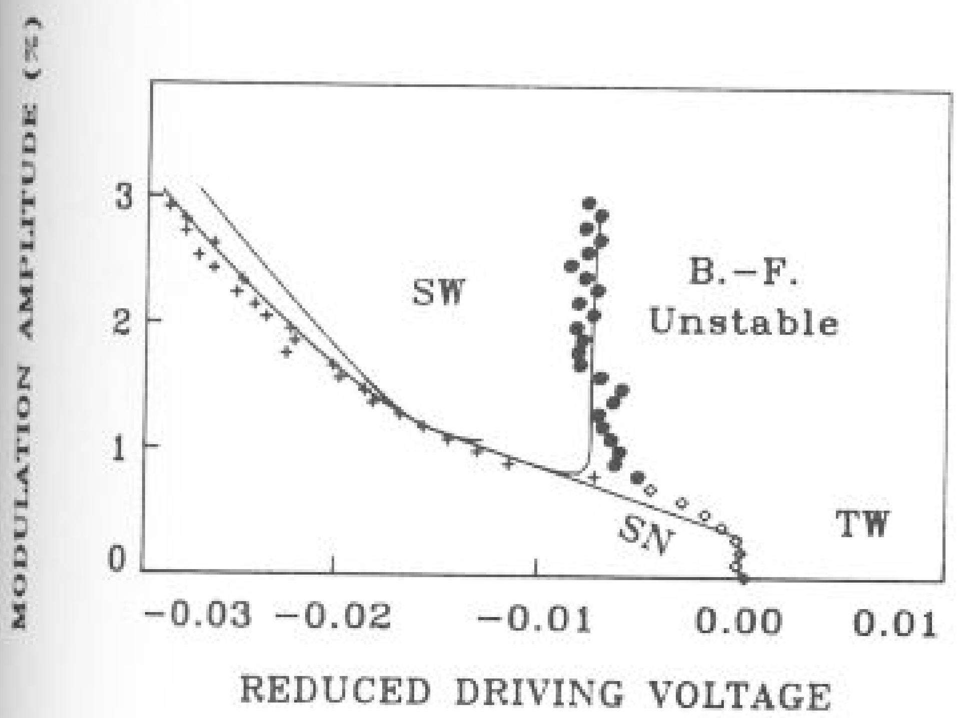

Fig. 1. Phase diagram and lines obtained by fitting the theory to the stability threshold. Open dimonds: transition to TW. Crosses: transition to SW. Solid circles: transition to the unstable regime.

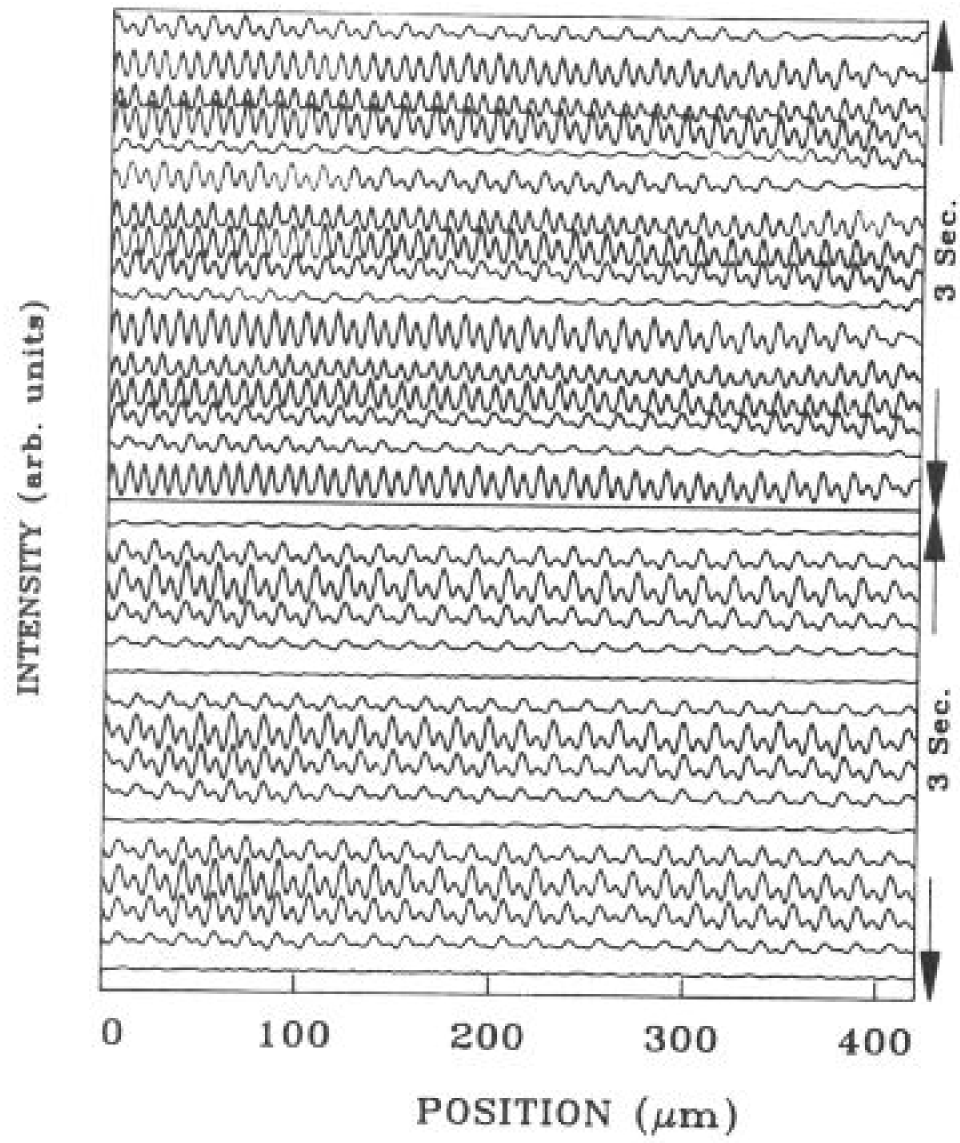

Fig. 2. Time evolution of stable SW (below) and the unstable solution.

EXPERIMENTAL RESULTS

The SW state on the left of the instability line shows a well defined wavelength stable in time as shown in Fig. 3. Here we have averaged the Fourier spectra of the structure along the x direction over 24 periods of the modulation frequency after having waited for the transient response to pass. The SW spectrum (down) was made for $\varepsilon=-0.005$. The narrow wave number peak indicates that the structure stays stable in time. At a higher value of $\varepsilon=0.055$ the peak becomes wider and shifted to another wave number. This broadening appears because now there is no stable wave number and defects appear continuously in the structure.

To give proof that this instability is the predicted one, we imposed a periodic state of the type $u(t)\cdot\exp(iq_c x)$ as shown in Fig. 3a for $\varepsilon=-0.005$ and jumped into the unstable region, where the spatial perturbations are supposed to grow. In Fig. 4 the temporal evolution of the fundamental mode

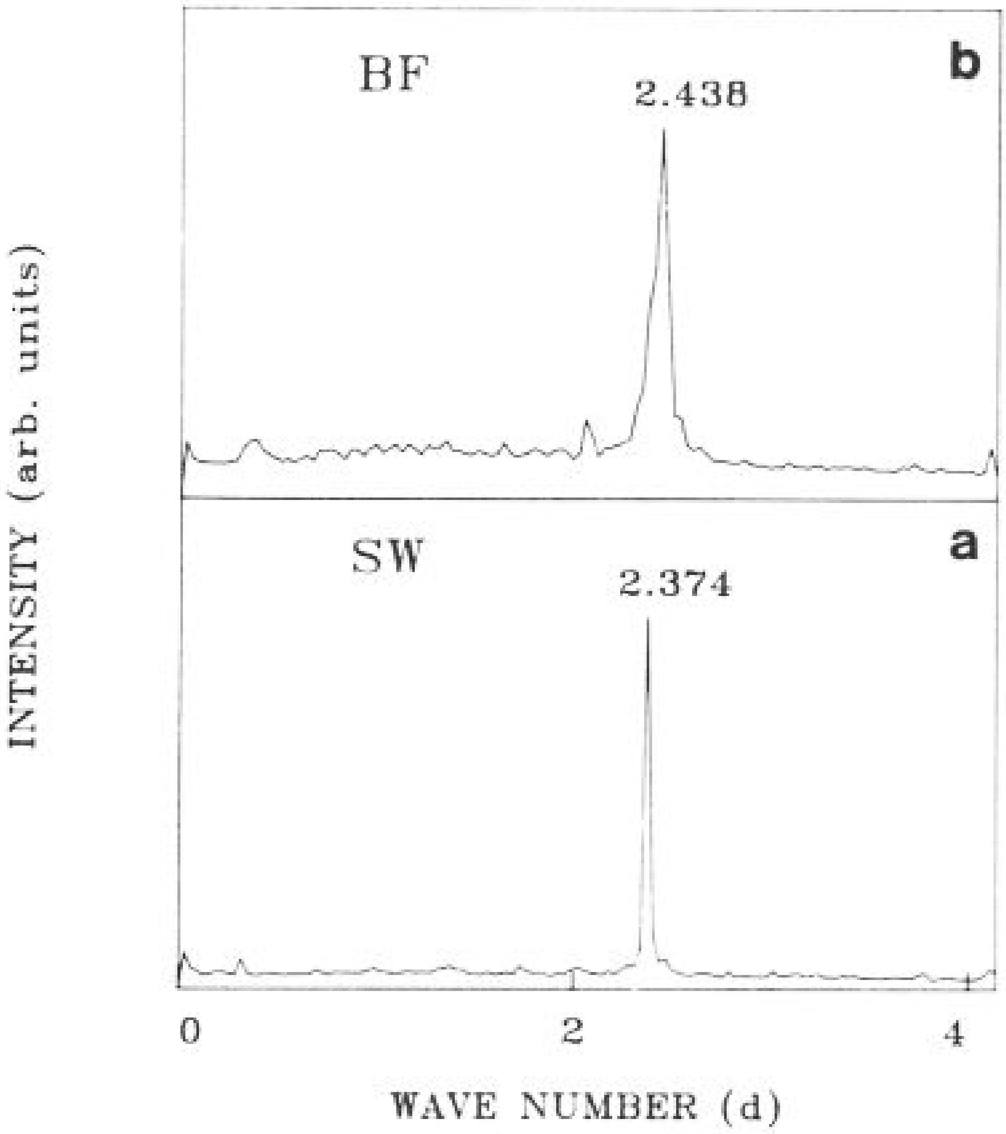

Fig. 3. Time average over 24 modulation periods of the Fourier spectra of the pattern in a) the stable SW state at ε=0.005; b) the unstable state at ε=0.055

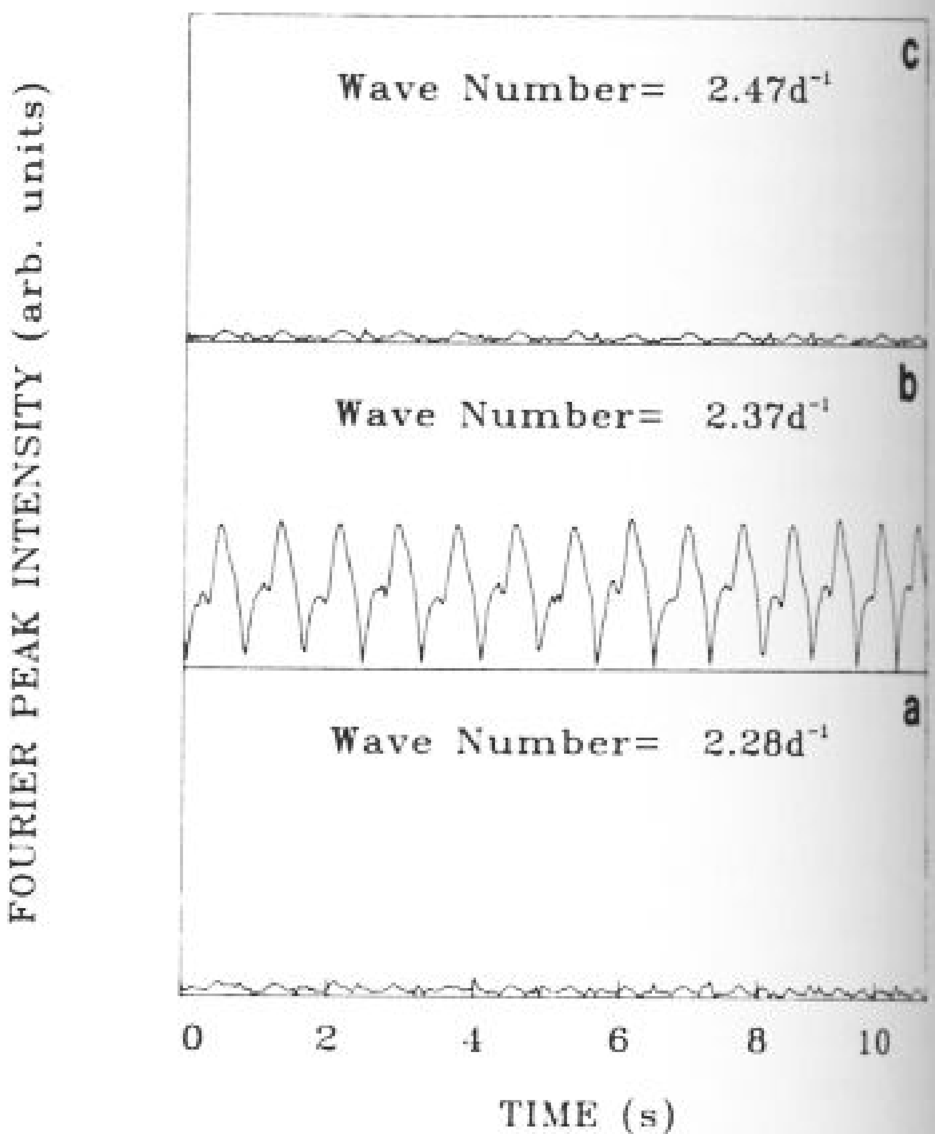

Fig. 4. Time evolution in the stable state (ε=-0.005) of the Fourier modes: a) $2.28d^{-1}$; b) $2.37d^{-1}$ (fundamental mode); c) $2.47d^{-1}$.

is plotted in the middle and compared to two neighboring modes. The resolution of our spectra is $\Delta q=0.03d^{-1}$. We are showing as an example the third next mode in our discrete serie. The time evolution of the same three modes q_n during the transient decay from the periodic structure is shown in Fig. 5. One can see that while the fundamental mode decreases its intensity, the modes on the right and on the left of it show a positive growth factor. This experimental behaviour corresponds clearly to the definition of a sideband instability.

We measured the structure function (Rehberg et al. 1989) of the final state resulting after this transients to quantify the properties of the unstable regime. We made a time average over 512 modulation periods for constant modulation b=5% and slowly increasing values of ε. It shows a decay in both directions (Fig. 6) and a very interesting additional property: In the x-direction (perpendicular to the roll axis) above ε=0.20 a slight wavelength change. This wavelength shift in the x-direction is correlated with the appearance of finite periodicity in the y-direction. For ε=0.02 and 0.03 the wavelength parallel to the roll axis changes from about 36d to 30d, and above ε=0.04 makes a big jump into values of the order of magnitude of 8d. This corresponds to a spatial structure of Zig-Zag rolls of a very short wavelength. The spatial structure of this solution is shown in Fig. 7 where 7a corresponds to SW with normal straight rolls, 7b corresponds to the instability where the defects start to appear, and a short wavelength sets in in the y-direction. In Fig. 7c one can see the stable structure corresponding to the high wave number Zig-Zags. Further measurements demonstrating the relationship between these two instabilities are in progress.

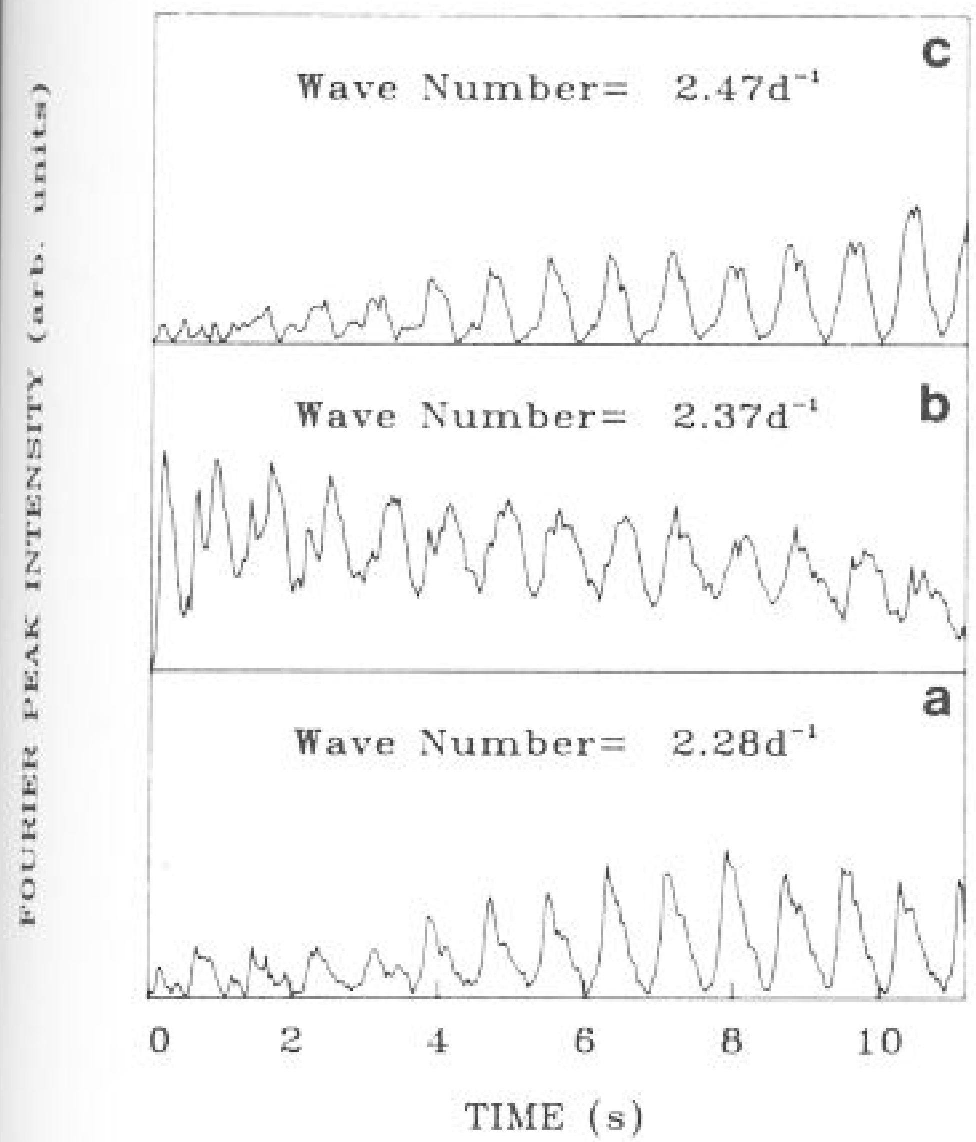

Fig. 5. Time evolution during the jump from SW into the unstable state at ε=0.04 of the Fourier modes: a) $2.28d^{-1}$; b) $2.37d^{-1}$ (the fundamental mode); c) $2.47d^{-1}$.

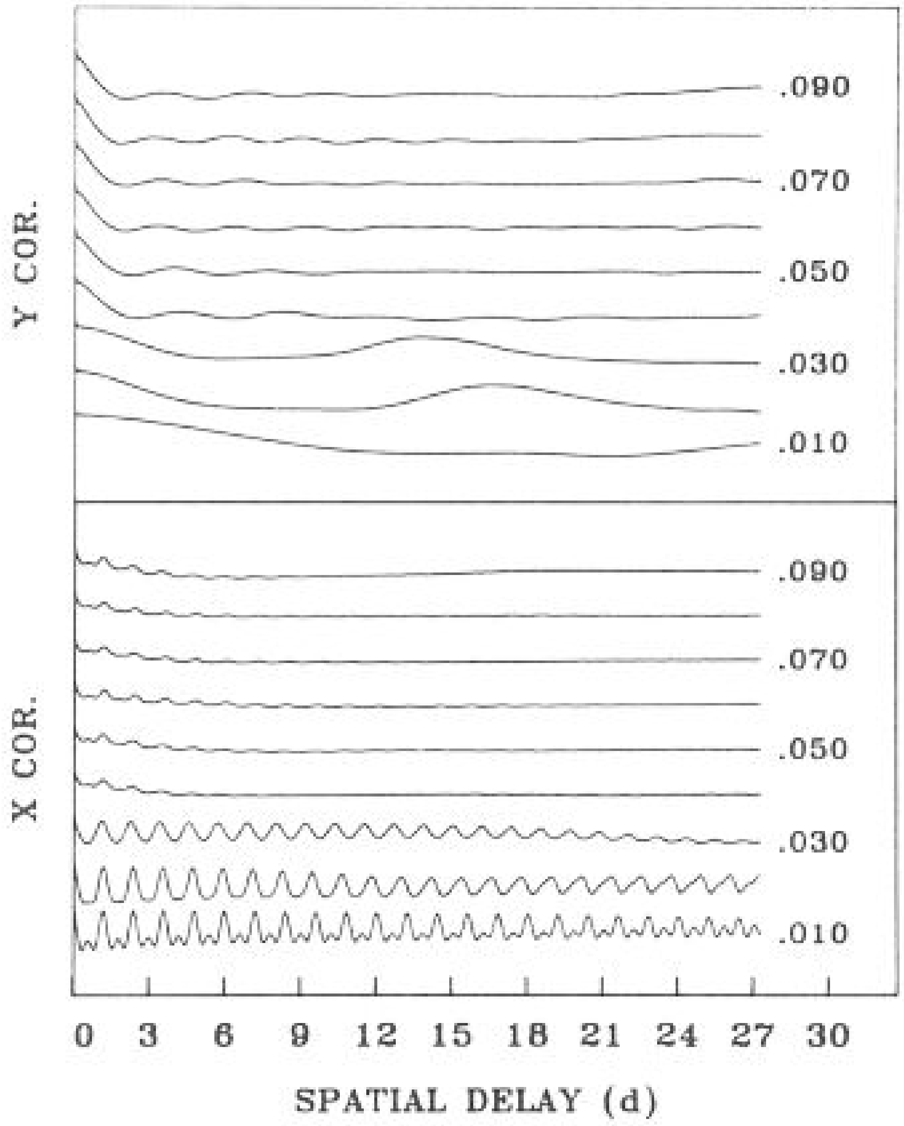

Fig. 6. Spatial correlations of the structure in x- and y-direction averaged over 512 modulation periods for a constant modulation frequency of 1.3 Hz as a function of ε for b=7%.

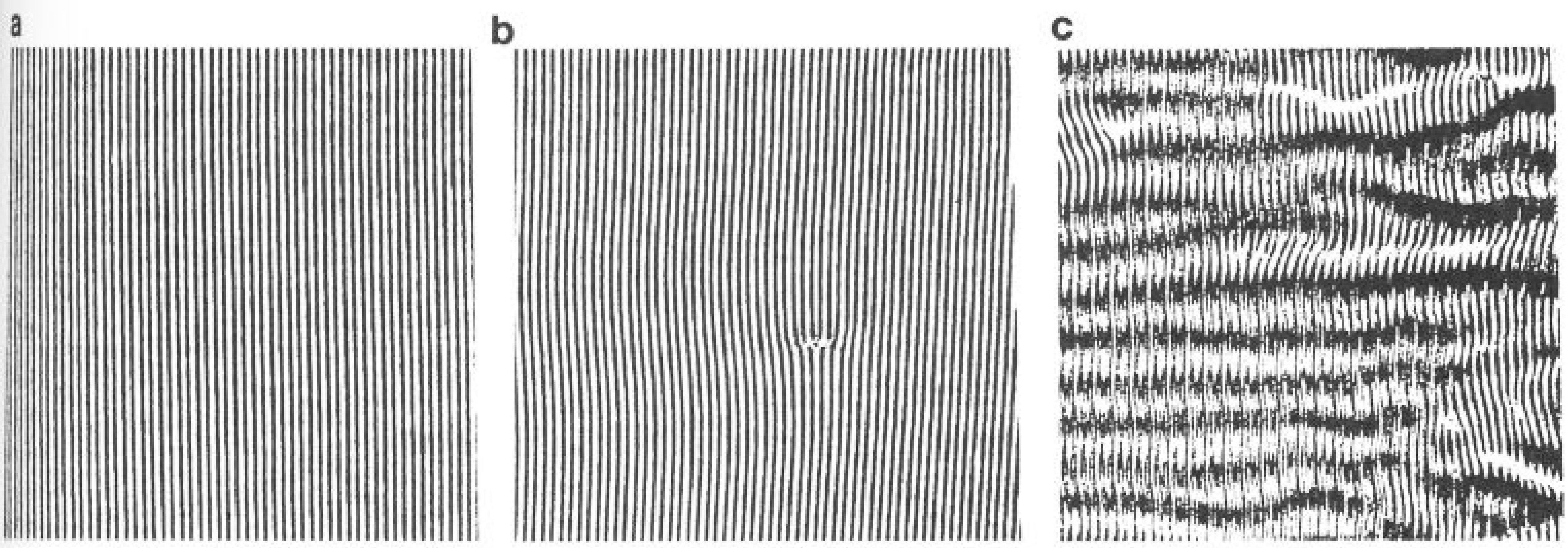

Fig. 7. Spatial structures observed at a) ε=-0.005 b) ε=0.04 c) ε=0.08).

ACKNOWLEDGMENT

The measurements were supported by Deutsche Forschungsgemeinschaft. M.T.J. acknowledges financial support by the Spanish Ministerio de Educación y Ciencia. Its a pleasure to thank F.H. Busse for very useful comments and suggestions.

REFERENCES

Benjamin, T.B and Feir, J.E., 1967; The disintegration of wave trains on deep water, J. Fluid Mech. 27:417.

Craik, A.D.D., 1985; "Wave interactions and fluid flows" (and references therein), Cambridge University Press.

Fauve, S. 1987; in "Instabilities and Nonequilibrium Structures", D. Tirapegui, ed., D.Reidel Publishing Company.

Ioss, E., 1987; in "Instabilities and Nonequilibrium Structures", D. Tirapegui, ed., D.Reidel Publishing Company.

Lowe, M. and Gollub, J.P. 1985, Pattern Selection near the Onset of Convection: The Eckhaus Instability, Phys. Rev. Lett. 55:2575.

Rasenat, S., Winkler, B.L., Hartung, G., Rehberg, I., 1989; The shadograph method in convection experiments, Experiments in fluids 7:412.

Rehberg, I., Rasenat, S. Fineberg, J., de la Torre Juárez, M. Steinberg, V., 1988, Temporal Modulation of Travelling Waves, Phys. Rev. Lett., 61:2449.

Rehberg, I., Winkler, B.L., de la Torre Juárez, M., Rasenat, S., Schöpf, W., 1989, Pattern Formation in a Liquid Crystal, in "Festkörperprobleme/Advances in Solid State Physics", U. Rössler, ed., Vieweg.

Riecke, H., Crawford, J.D., Knobloch, E., 1988, Time-Modulated Oscillatory Convection , Phys. Rev. Lett., 61:1942;

de la Torre Juárez, M. ,Rehberg, I., 1989, Experiments with travelling waves in electrohydrodynamic convection, in "New Trends in Nonlinear Dynamics: the Geometry of Nonequilibrium", P.Huerre and P.Coullet, ed., NATO-ASI Series, Plenum Press.

Walgraef, D., 1988, External Forcing of Spatio-Temporal Patterns, Europhysics Lett. 7, 485.