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The shadowgraph method at the Fréedericksz transition

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Abstract

A theoretical treatment of the light deflection of the extraordinary light beam in a nematic liquid crystal in the neighborhood of the Fréedericksz transition is presented. The analysis explains recent experimental observations of director fluctuations near the electrically driven splay Fréedericksz transition. As an application of this method, we propose the measurement of material parameters of a nematic liquid crystal via a spatio-temporal analysis of the measured light intensity.

1 Introduction

In a planarly aligned nematic liquid crystal, spatial variations of the director field lead to spatial variations of the the light intensity observed behind the cell, provided that the illuminating light is polarized parallel to the director. A quantitative analysis of this effect allows the determination of the orientation fluctuations of the director, as had been described in Ref. [1]. Because of a certain similarity of this measurement procedure to the one used in isotropic (i.e. not showing birefringence) fluids to measure spatial variations of the index of refraction[1], it is called shadowgraph method as well. For small orientation fluctuations of the director the analysis presented in Ref. [1] leads to a simple and robust measurement procedure. It is based on the fact that the light intensity variations observed behind the cell are proportional to these fluctuations [2]. That method had been used to analyze the thermally driven director fluctuations in the neighborhood of a pattern forming instability, namely the onset of electroconvection in a nematic liquid crystal [3]. A spatio-temporal analysis of the observed intensity fluctuations allows for the determination of the characteristic decay times of those fluctuations at different wavelengths [4]. While in Refs. [1], [2], [4] only a planarly aligned nematic liquid crystal was considered, it is the goal of this paper to extend the analysis presented in Ref. [1] to the director deformation above the splay Fréedericksz transition. It is motivated by a recent experimental study of that transition using the shadowgraph method [5]. These studies yielded fluctuating structures both above and below the Fréedericksz transition. One of the striking features of the measurement presented there was an unexplained increase of the fluctuation intensity slightly above the transition voltage V_c. The following calculations will show that this can be understood as a purely optical effect, and that it is thus not indicative of any critical behavior of the director fluctuations in the neighborhood of the transition.

2 Calculation of the light deflection

In the treatment of Ref. [1], deviations of the director field from the trivial planarly aligned configuration were considered. Above the Fréedericksz transition, the director field is more complicated, and the distribution of the deformation angle $\theta_F(x)$ over the cell thickness d can be calculated numerically by solving an integro-differential equation describing the director deformation as done in Ref. [5]. We will calculate the path of light inside the cell for small distortions $\theta_D(x,y)$ of that field $\theta_F(x)$.

The index of refraction of an anisotropic liquid crystal for a beam of light polarized in the x-y plane is given by

$$n(x,y,y') = \frac{n_o n_e}{\left[n_o^2 \cos^2(\beta) + n_e^2 \sin^2(\beta)\right]^{1/2}} . \tag{2.1}$$

with $\beta = -\theta(x, y) + \arctan(y')$, and n_o, n_e are the indices of refraction. We will use the 5CB values $n_o = 1.54$, $n_e = 1.72$ in the numerical calculations below.

Applying Fermat's principle, that $\int n ds =$ minimum, we obtain the Euler-Lagrange differential equation

$$\frac{\partial f}{\partial y} - \frac{\partial^2 f}{\partial y' \partial x} - \frac{\partial^2 f}{\partial y' \partial y} \cdot y' - \frac{\partial^2 f}{\partial y'^2} \cdot y'' = 0 \tag{2.2}$$

with $f(x, y, y') = n(x, y, y') \cdot (1 + y'^2)^{1/2}$.

We discuss here a director field of the form

$$\theta(x, y) = \theta_F(x) + \theta_D(x, y) \tag{2.3}$$

with

$$\theta_D(x, y) = \theta_0 \sin(\frac{\pi}{d} x) \cos(ky) \ . \tag{2.4}$$

Here d is the cell thickness, $\theta_F(x)$ represents the deformation of the director field above the Fréedericksz transition, which in general has to be obtained numerically [5], while $\theta_D(x, y)$ represents a spatial Fourier component of the distortion of this director field, which might be caused by the thermal fluctuations in the case of the Fréedericksz transition, or by electroconvection in the case of the transition to Williams rolls. The light intensity at a distance x_1 from the top of the cell is given by

$$I(x_1, y) = \frac{I_0}{\frac{\partial y(d)}{\partial y(0)} + x_1 \frac{\partial y'(d)}{\partial y(0)}} \ . \tag{2.5}$$

The normalized light intensity signal I/I_0 measured at a fixed x_1 has a periodicity with the wavelength $\lambda = 2\pi/k$ of the director modulation and can thus be decomposed in a Fourier series. We define the modulation amplitude A as the modulus of the fundamental mode. In the absence of a Fréedericksz effect, it had been shown that this light intensity modulation A is proportional to a small director perturbation θ_0, i.e.

$$A \approx s \cdot \theta_0 \tag{2.6}$$

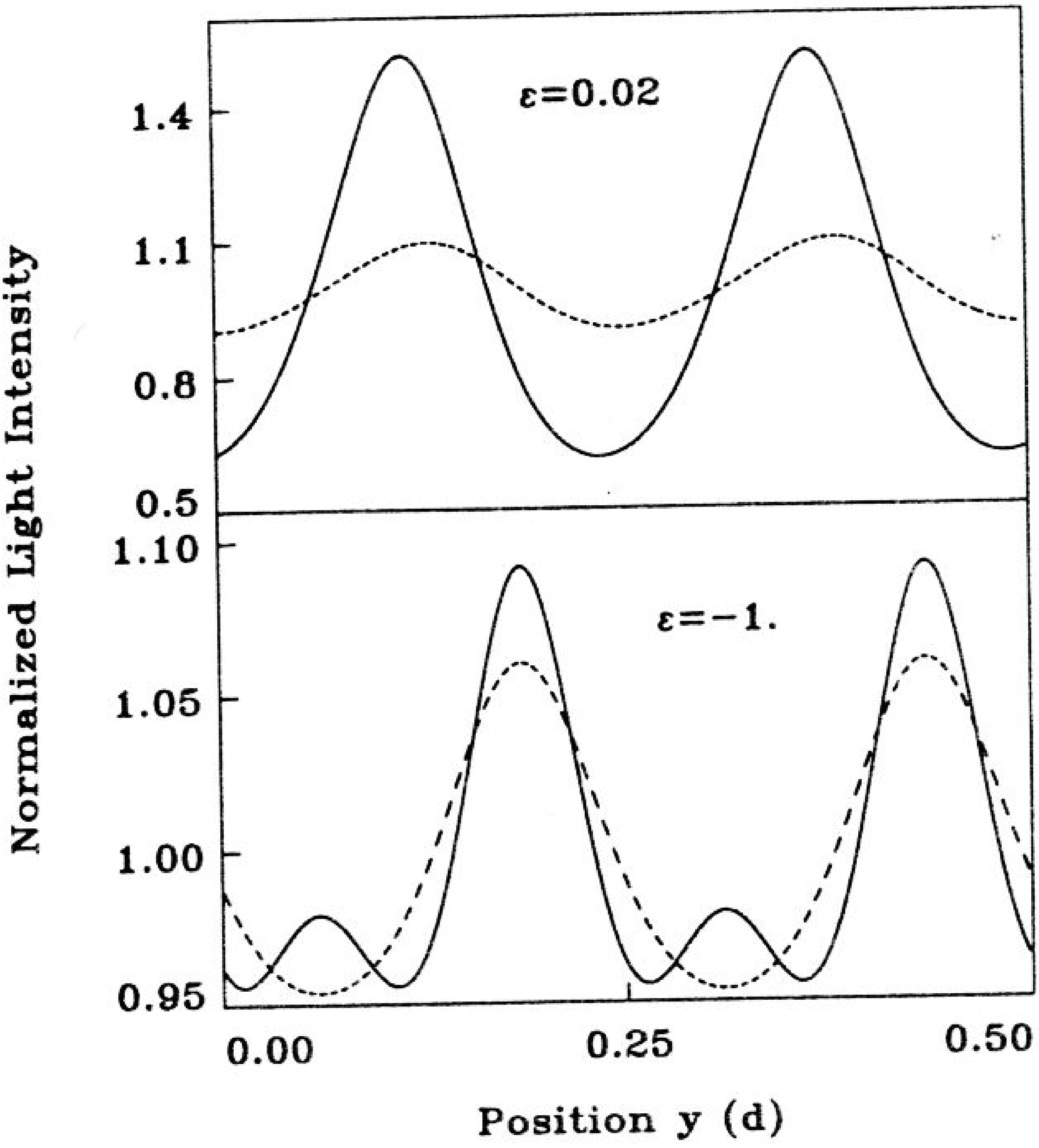

Figure 1: The light intensity modulation as calculated numerically for a distortion with amplitude $\theta_0 = 0.01$ and wavelength $\lambda = d/4$. The dashed lines are obtained for $x_1 = 0$, and the solid lines for $x_1 = 2d$.

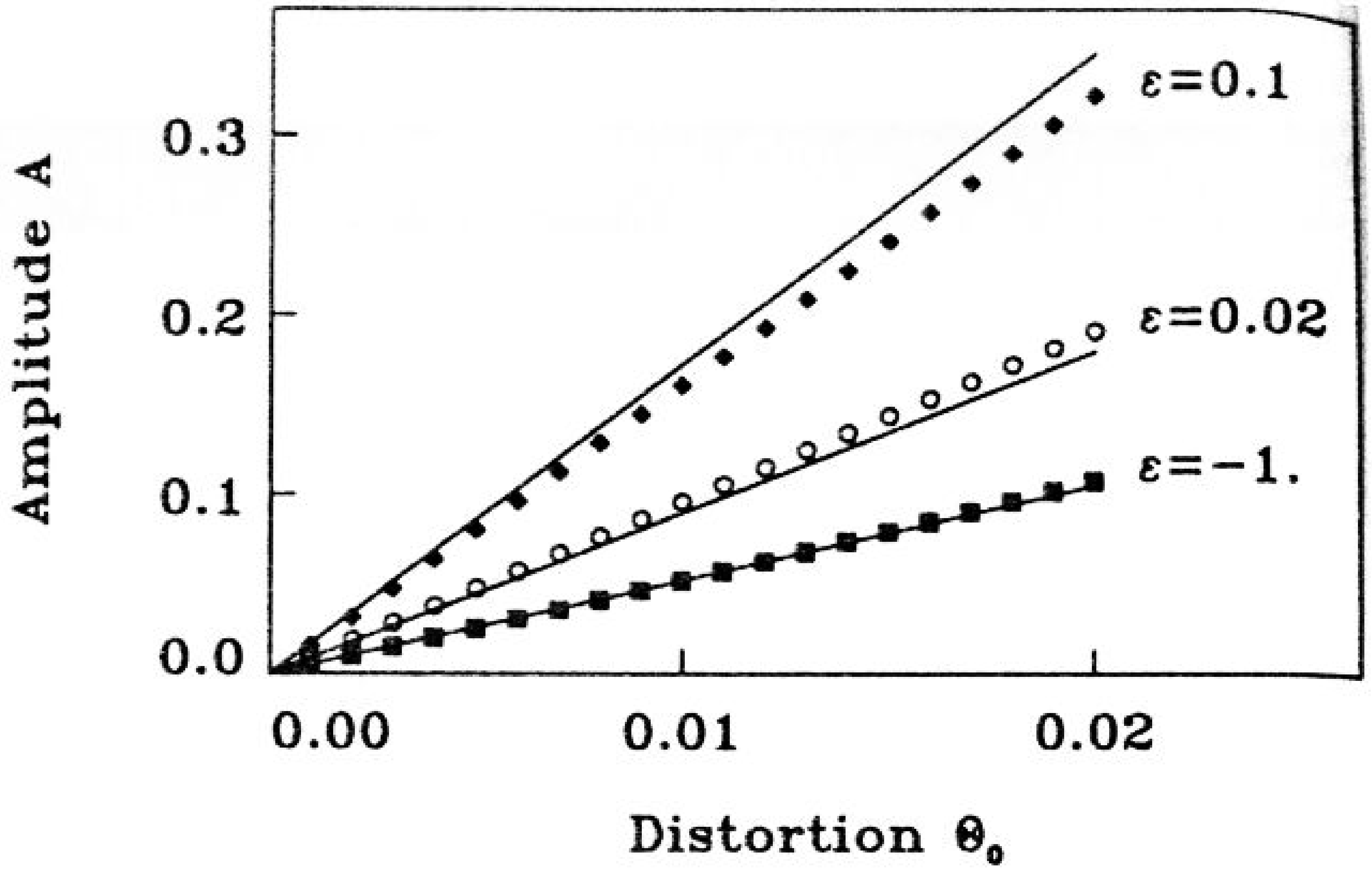

Figure 2: The symbols represent the numerically obtained light modulation amplitude below (solid squares), slighly above (open circles) and well above (solid diamonds) the Fréedericksz transition, for $x_1 = 0$. The straight lines represent the analytical approximation.

and the sensitivity s had been shown to be [2]:

$$s = 2\frac{\hat{n}}{\hat{n}+1}\frac{d}{\pi}k\ . \tag{2.7}$$

This fact allows for a robust measurement procedure because A is almost independent of x_1 (which experimentally might be hard to determine precisely), i.e. it is only a correction of second order in θ_0. In the case of a finite θ_F, the sensitivity s is enhanced as demonstrated in Fig. 1. Here, Eq. (2.2) is solved numerically, both for a value of $\theta_F = 0$, which is the case below the Fréedericksz transition, and for a finite value of θ_F. θ_F has been calculated numerically for a positive value of the control parameter $\epsilon = V^2/V_c^2 - 1$, V_c being the threshold voltage for the Fréedericksz transition. The most striking feature caused by the Fréedericksz effect seems the enhancement of the fundamental mode, so that the higher harmonics are less pronounced in the light intensity signal shown in the upper part of the figure. The dashed lines indicate the intensity signal measured directly on the upper boundary of the cell ($x_1 = 0$), while the solid lines show this signal at a distance of $x_1 = 2d$

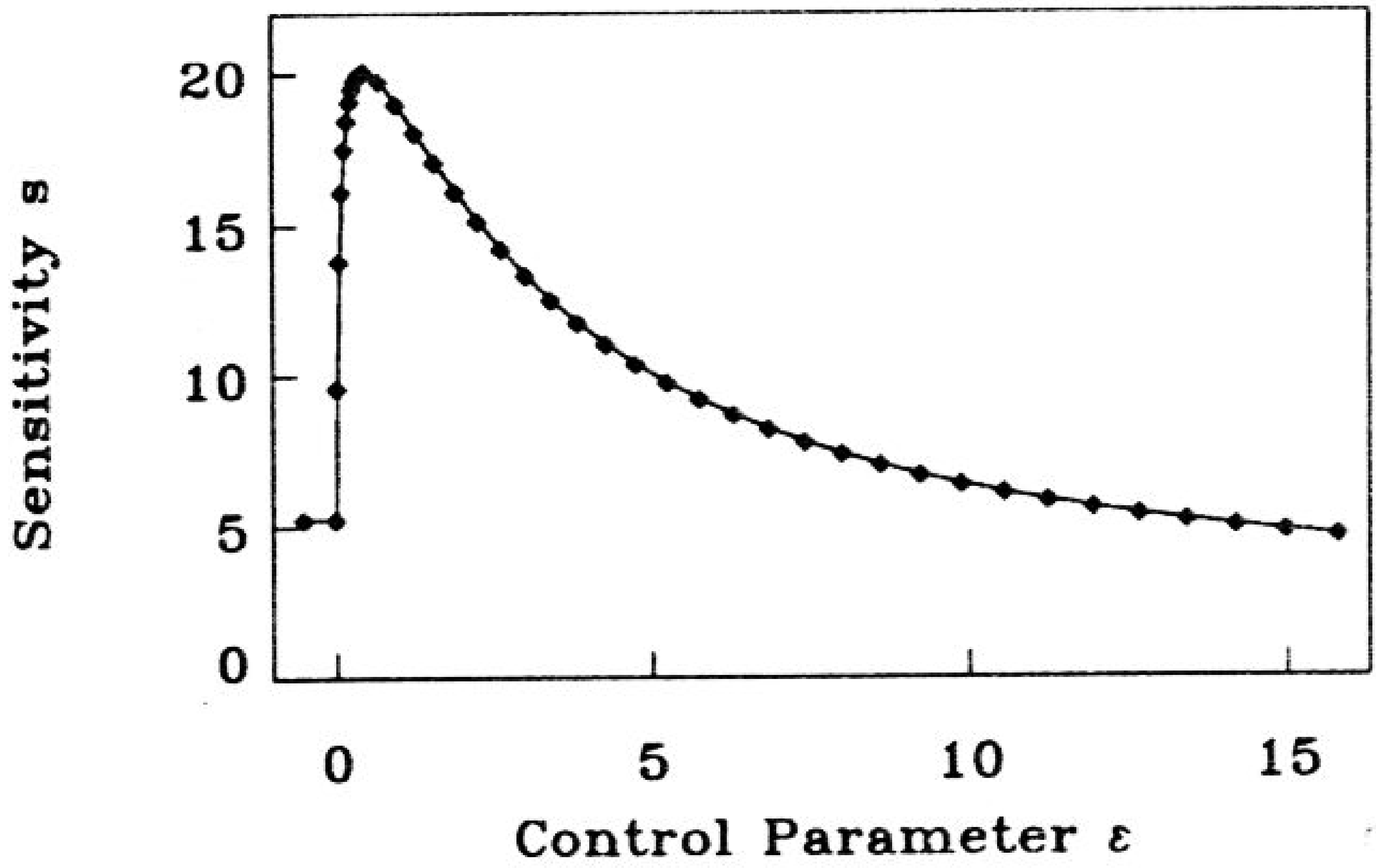

Figure 3: The symbols denote the numerically obtained sensitivity s. The line is a guide for the eye.

behind the cell.

In Fig. 2 the amplitude of the light intensity modulation for three different values of the control parameter $\epsilon = V^2/V_c^2 - 1$ are plotted versus θ_0. The symbols were obtained via a numerical calculation. It can be seen that the amplitude A is proportional to θ_0 for small values of θ_0, and the slope of this line serves to define the sensitivity s.

In order to get an analytical expression for the sensitivity we follow Ref. [1] and expand f in a Taylor series up to second order in θ, $y - y_0$ and their derivatives. Inserting this expression in eq. (2.2) and omitting all terms higher than second order implies

$$\hat{n}\,\theta\,\frac{\partial\theta}{\partial y} + \hat{n}\,\frac{\partial\theta}{\partial x} - (\hat{n}+1)\,y'' = 0 \tag{2.8}$$

with $\hat{n} = \dfrac{n_o^2 - n_e^2}{n_o^2}$.

Furthermore $\theta_F(x)$ is substituded by $\theta_m\ \sin(\frac{\pi}{d}x)$, which is correct for small ϵ.

The influence of the fluctuations and the Fréedericksz effect can be seen in an expansion up to order $\theta_0\,\theta_m$ of the path of light $y(x)$ at $x = d$ and its first derivative $y'(d)$ $(N = \dfrac{\hat{n}}{\hat{n}+1})$:

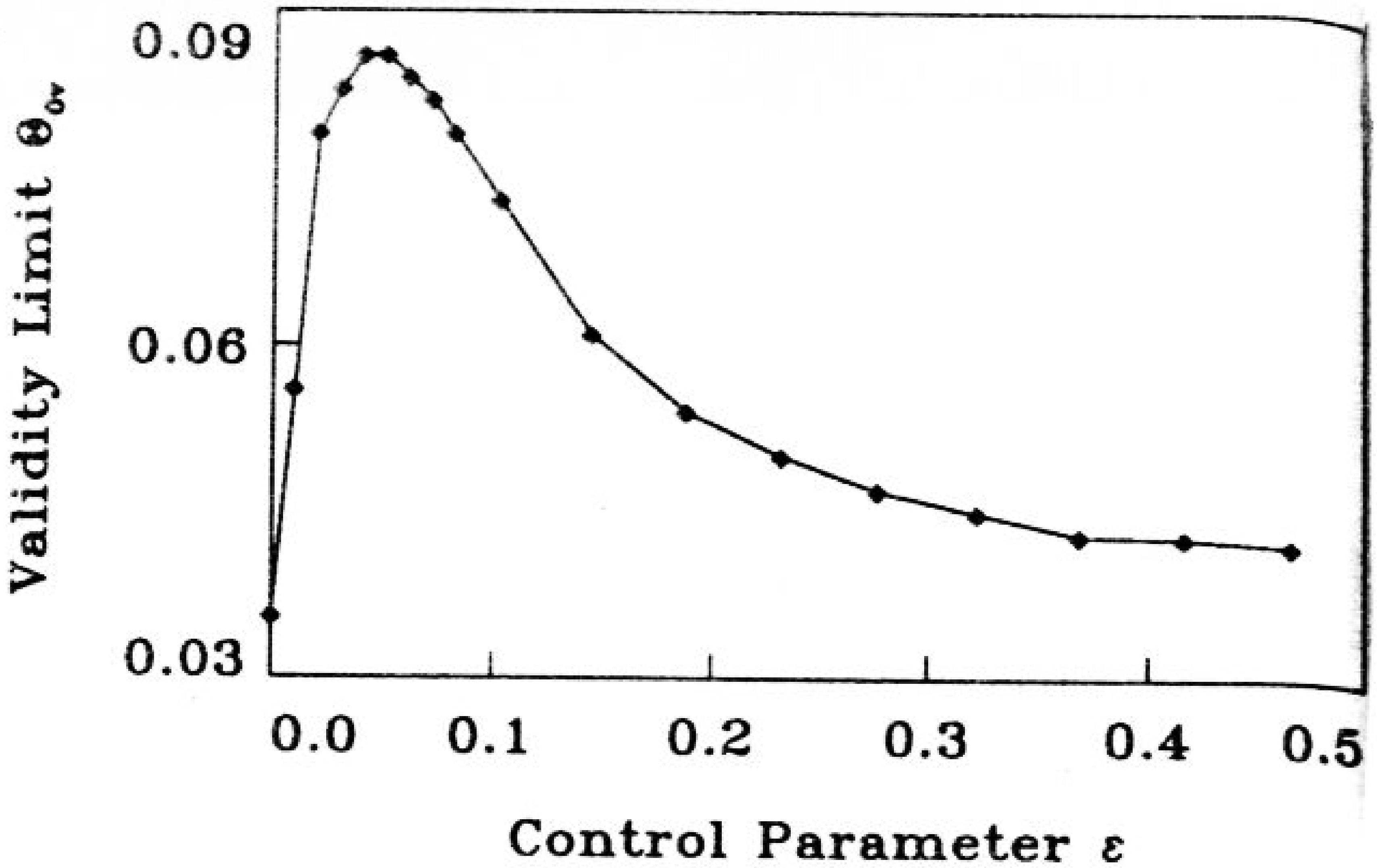

Figure 4: The validity range of the linear approximation is calculated for $x_1 = 0$ and $\lambda = d/4$. The line is a guide for the eye.

$$
\begin{aligned}
y(d) - y_0 = \theta_m \, 2 \, \frac{d}{\pi} \, N & \\
+ \theta_0 \, 2 \, N \, \frac{d}{\pi} \left[\cos(k y_0) + \theta_m \, k \sin(k y_0) \, \frac{d}{\pi} \, \left(-\frac{1}{8}\pi^2 + N(\frac{1}{8}\pi^2 - 1) \right) \right] &
\end{aligned}
\tag{2.9}
$$

$$
\begin{aligned}
y'(d) &= \theta_0 \theta_m N(-1 + N) \frac{1}{2} \, dk \sin(k y_0) \\
& + \theta_0^2 N \frac{1}{2} \, dk \left[-\frac{1}{\hat{n} + 1} \frac{1}{2} \sin(2 k y_0) - \theta_m N \frac{dk}{\pi} \frac{1}{2} (1 - \cos(2 k y_0)) \right]
\end{aligned}
\tag{2.10}
$$

The straight lines in Fig. 2 present the linear approximation in θ_0 given by eq. (2.10). The bad accordance of the two curves for higher ϵ is due to the relative large values of θ_m (for $\epsilon = 0.1025$, $\theta_m \approx 0.3$). Thus, in the following we will calculate the sensitivity s numerically.

The sensitivity s of the method is a nontrivial function of the director field $\theta_F(x)$, as indicated in Fig. 3. Here $\theta_F(x)$ is calculated numerically for various values of ϵ, and the amplitude of the light intensity modulation obtained for a fixed value of the wavelength $\lambda = d/4$ is shown. We would

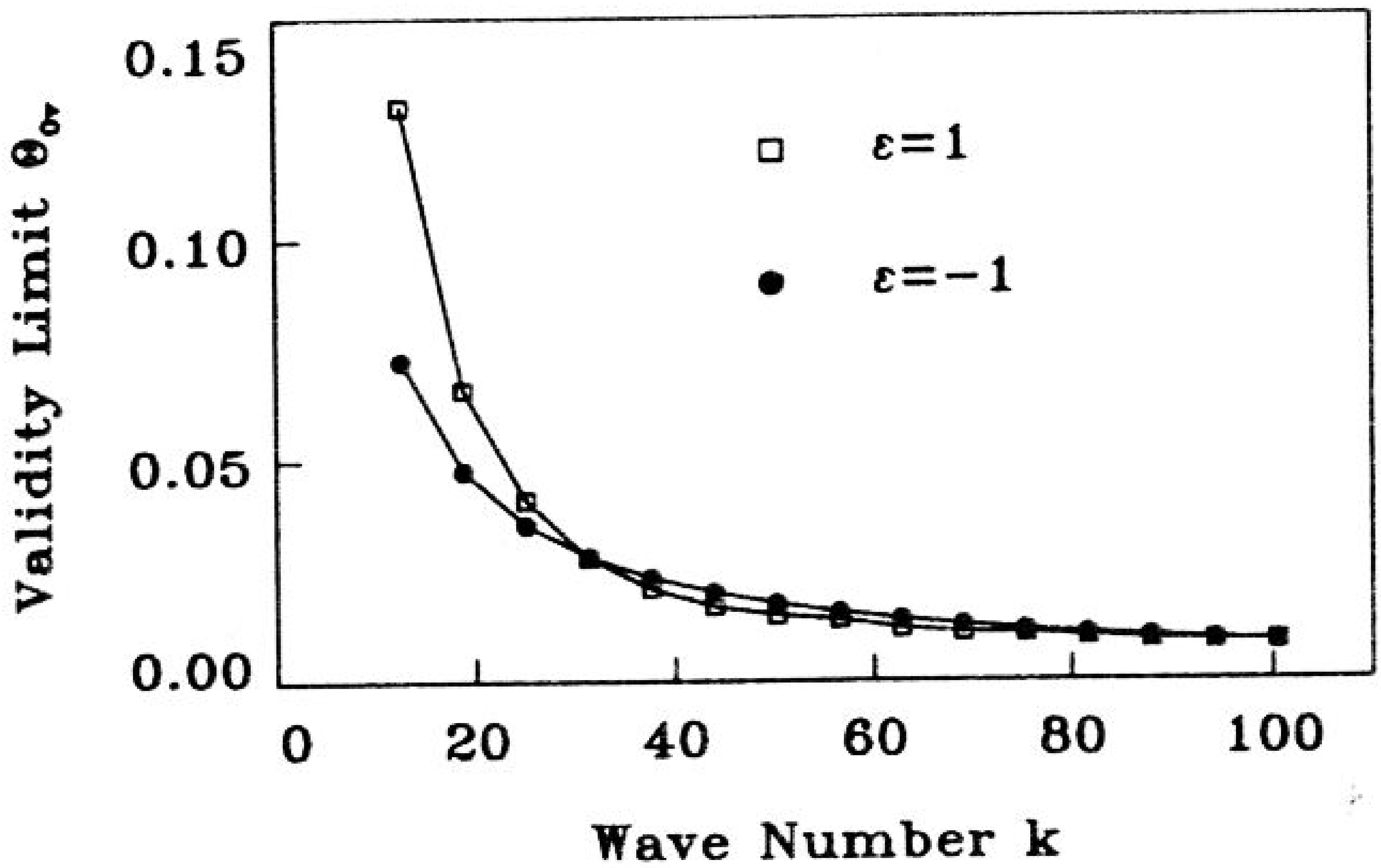

Figure 5: The validity range below (solid circles) and above (open squares) the Fréedericksz transition. The lines are guides for the eye.

like to point out that there is a striking similarity of this curve with the measured one presented in Fig. 4 of Ref. [5].

The qualitative feature of the curve presented in Fig. 3 to have a maximum at a finite voltage seems understandable. Both cases $V = 0$ and V being very large correspond to director configurations (parallel or perpendicular to y) where due to symmetry considerations small perturbations of the director field cause only a second order effect in the index of refraction. Thus the sensitivity s has to become small in those limiting cases. If this interpretation is correct, one would expect an increase in the observed light intensity fluctuations once the polarized light enters the cell at an oblique angle. In order to test this prediction we have tilted the cell inside the microscope with respect to the optical axis, and indeed found an increase of the light intensity fluctuations.

Of fundamental interest for the measurement is the question of the linearity of the shadowgraph method. By inspection of Fig. 2 it is clear that eq. (2.6) is valid only for small values of θ_0. To get a measure for the deviation from the linearity we define the quantity

$$Q_v = \frac{\mid A - s\theta_0 \mid}{s\theta_0} . \qquad (2.11)$$

The value of θ_0 where Q_v first exceeds 10 percent is named θ_{0v}. This validity range is a function of the focal height x_1 of the microscope. In addition, it is a function of both the control parameter ϵ (Fig. 4) and the wave number k (Fig. 5) of the distortion.

3 Conclusion

We have considered the optical effect of periodic distortions of the director field along the axis of the director. In the case of thermally induced fluctuations of the director field, distortions perpendicular to this axis (i.e. a twist) cannot be excluded. It has been shown, however, that the influence of these twist deformations is only second order in θ_0, thus it might be justified to neglect this effect for such small values of θ_0 as they are provided by the thermal fluctuations.

The comparision of our Fig. 3 with the measurement presented in Fig. 4 of Ref. [5] can only be of a qualitative nature, because in that paper the root mean sqare value of the intensity fluctuations was presented, rather than the amplitude of one mode with wavelength λ. It should be no problem, however, to extract that amplitude via a spatial Fourier decomposition of the measured signal, as it has been done for the case of a pattern forming instability in Ref. [4]. That procedure would not only yield the intensity of the fluctuations, but also the characteristic decay time of a specific director deformation. That idea is identical to the one used for measuring material parameters like the elastic constants and the viscosity via dynamic light scattering methods. The shadowgraph method might have some merits compared to dynamical light scattering, however. It works well at long wavelengths comparable to the cell thickness, and it seems to be simpler from a mechanical point of view. Test measurements of material parameters of 5CB using this method are in progress.

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