PATTERN FORMATION BY HORIZONTAL VIBRATION OF GRANULAR MATERIAL

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Experiments on the spontaneous formation of patterns in a horizontally vibrated sand layer are presented. We developed a cellular automaton in a first attempt to model the essential parts of the mechanism leading to this patterns.

1 Introduction

Although the fascinating well-known patterns like beach cusps, sand ripples and dunes have been admired by our ancestors and inspired several painters, nevertheless no physicist or engineer is able to give a satisfying answer to his children asking simply "Why ?"[1,2,3]. Besides this deplorable state of affairs there is a reasonable interest on the part of engineers who have to handle granular media in several technical applications. Ore mining, recycling, pharmaceutic and chemical industries are just a few catchwords to be named here[4]. There is a limited amount of hope that the more principally oriented studies of physicists may throw light upon the understanding of this practical questions. A large amount of work has been devoted to the formation of patterns under vertical vibration, which is obviously considered as a paradigmatic system. Recent examples of these investigations are presented in Refs.[5,6,7]. In contrast we present here investigations on horizontal vibration in granular media. Our experiments are accompanied by studies of a cellular automaton modelling horizontal vibration.

2 Experiment

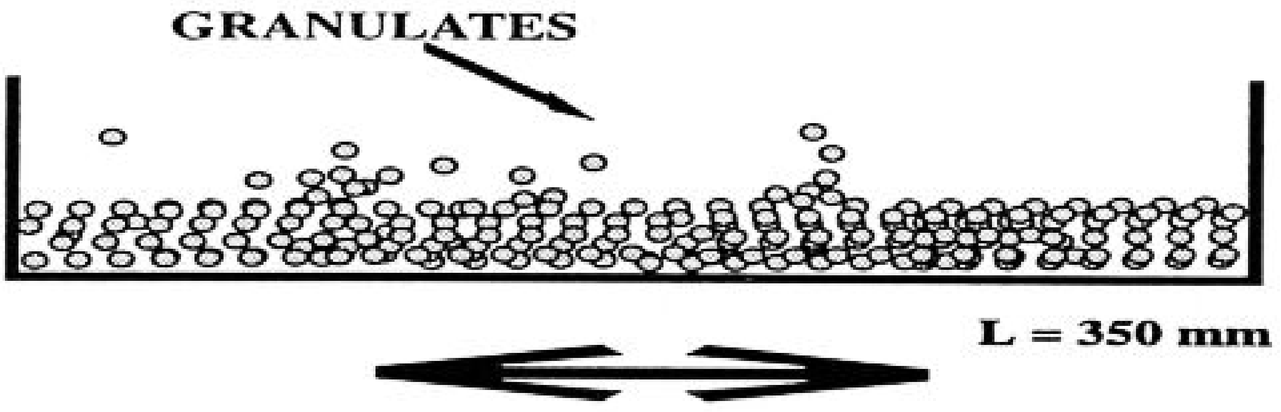

Figure 1: *Experimental setup*

As shown in Fig. 1 the experimental setup consists of a box $(350 \cdot 50 \cdot 20 mm)$ which is filled with sand and oscillated horizontally. The container is made of aluminium and has a surface roughness of $7\mu m$. Filling the box with a certain amount of sand leads to a dumping height H. H is varied from one to three monolayers. As sand we use glass beads with a density of $2.5 g/cm^3$ and different diameters D ($150 - 160\mu m, 210 - 250\mu m$ and $420 - 600\mu m$). The frequency f of the oscillatory motion can be varied in the following range: $0 \leq f \leq 5Hz$. The amplitude of the motion A is fixed at $10mm$. A CCD-camera allows visualization of the pattern from above. We use grazing incidence of light along the direction of the oscillatory motion to visualize the modulation of the surface.

First we fluidize the granular material by using a sufficiently high frequency of motion (about $5.0Hz$). Decreasing the frequency results in ripple-like patterns at a critical frequency. These patterns depend on the frequency of motion, grain size and initial dumping height. In addition the roughening of the surface of the channel seems to play an important role for the pattern formation.
In Fig. 2 some images of patterns for different parameters are shown. In Fig. 2a the grain size D ranges from $150\mu m - 160\mu m$, the initial height H consists of two monolayers, the driving frequency is $f = 4.3Hz$. In Fig. 2b $210\mu m \leq D \leq 250\mu m$, $H = 3$ monolayers, $f = 4.0Hz$. In Fig. 2c $420\mu m \leq D \leq 600\mu m$, $H = 1$ monolayer, $f = 4.0Hz$.

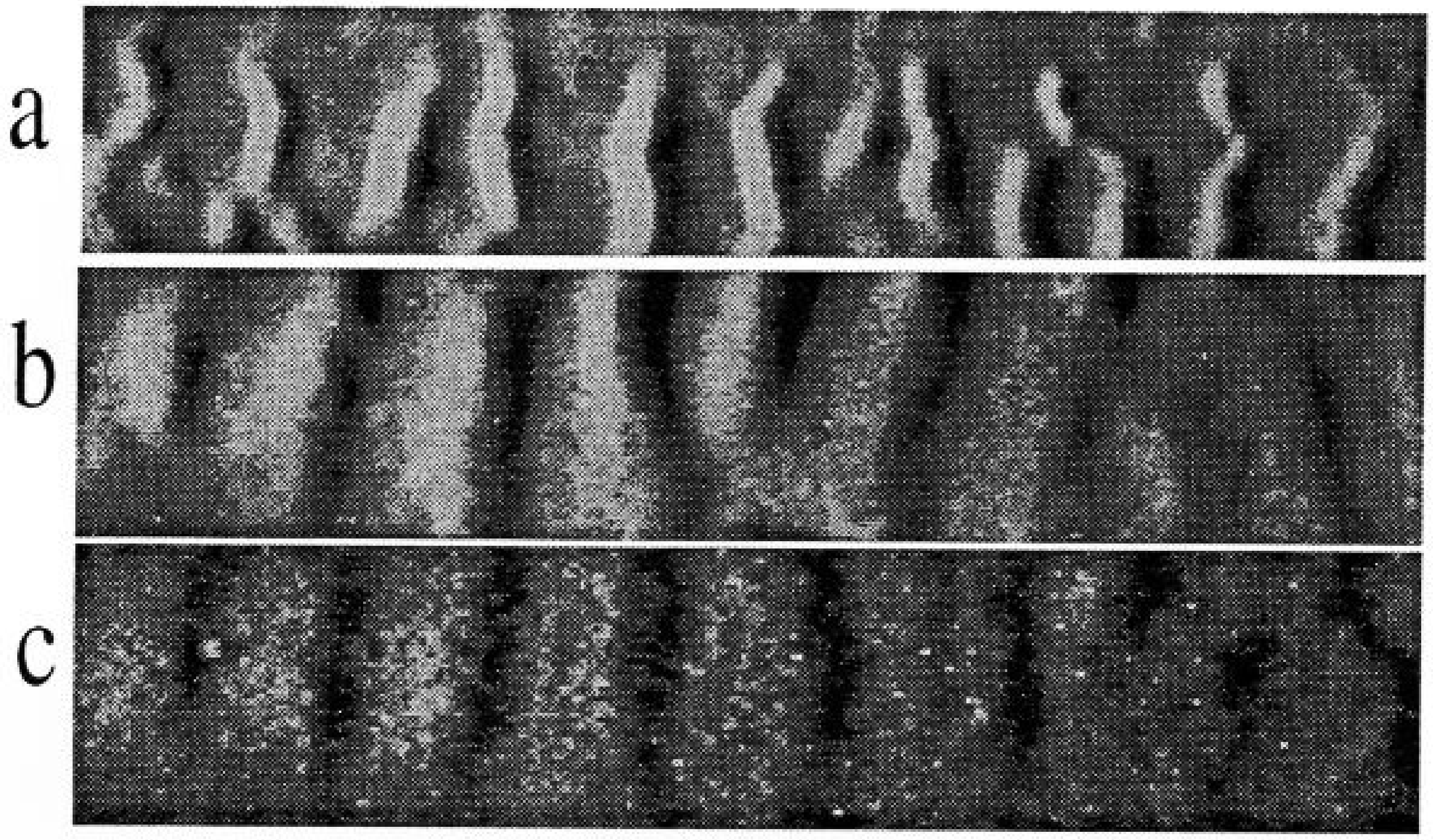

Figure 2: Several Images of Patterns

3 Modelling by a Cellular Automaton

3.1 Rules

In the following we introduce a cellular automaton which we have developed to model the dynamics of the horizontal vibration in granular media.

The indices of the elements of the underlying matrix of the cellular-automaton represent the location of the grains of sand in our model. The *value* of the elements is an indicator for the height above the initially plain surface. There are two rules applied to the top layer of the sand which transform the configuration at the moment T_n to the configuration at T_{n+1}:

1. *Simultaneously* every grain of the top layer flies alternatively to the left or to the right on parabolic trajectories until it bumps against another grain (Fig. 3).
2. Particles fall until they come to rest on a left *and* a right neighbour.

The automaton was implemented using periodic boundary conditions. This is a convenient way to guarantee the "law" of sand-conservation, which is enforced in our experiment by using vertical barriers.

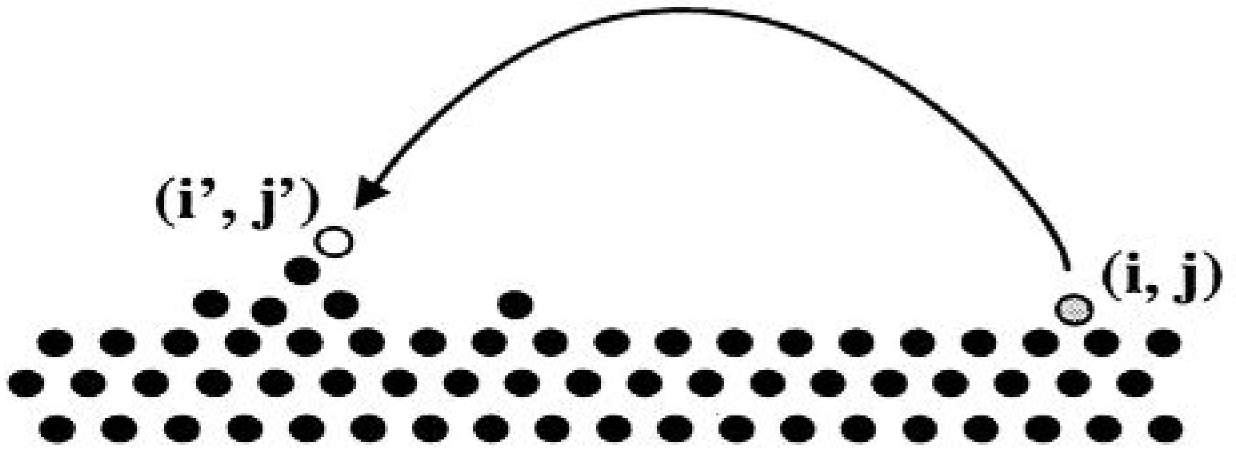

Figure 3: Illustration of the ideas of the cellular automaton

Special efforts had to be made to allow the quasi-simultaneity of the flying phase described at 1. At the beginning of iteration N two identical matrices $\tilde{L}$ and $\tilde{M}$ hold the configuration of the grains of sand (see figure 3). The matrix $\tilde{L}$ stays unaltered and is only used to check if any grain bumps against another on his flying phase (see 1.). The element $\tilde{M}_{ij}$ which corresponds to the starting position (i, j) of the grain currently watched is decremented and the element $\tilde{M}_{i'j'}$ of the final position $(i'j')$ of the grain is incremented. Applying the second rule to $\tilde{M}$ the new configuration $(N+1)$ is calculated and $\tilde{L}$ is updated: $\tilde{L} = \tilde{M}$.

Both matrices consist of $640 \cdot 120$ elements which has been proved to be a bearable compromise between the calculation time and the fineness of resolving patterns quantitatively. The code is compiled for the use on an *Intel 568* computer as a 32bit DOS target mainly to avoid problems concerning the management of the two

640 · 120 float arrays. The program is optimized for time. On our 90MHz *Pentium* computer it typically takes 0.3 – 0.7 seconds between two iterations including the display in a 256 color SVGA mode.
The parabolic trajectories $y = x \cdot \tan(\alpha) - \frac{g}{2v_0^2 \cos^2(\alpha)} \cdot x^2$ are parametrized by the angle α and the absolute value of the velocity $v_0 = |\vec{v}|$.

3.2 Results

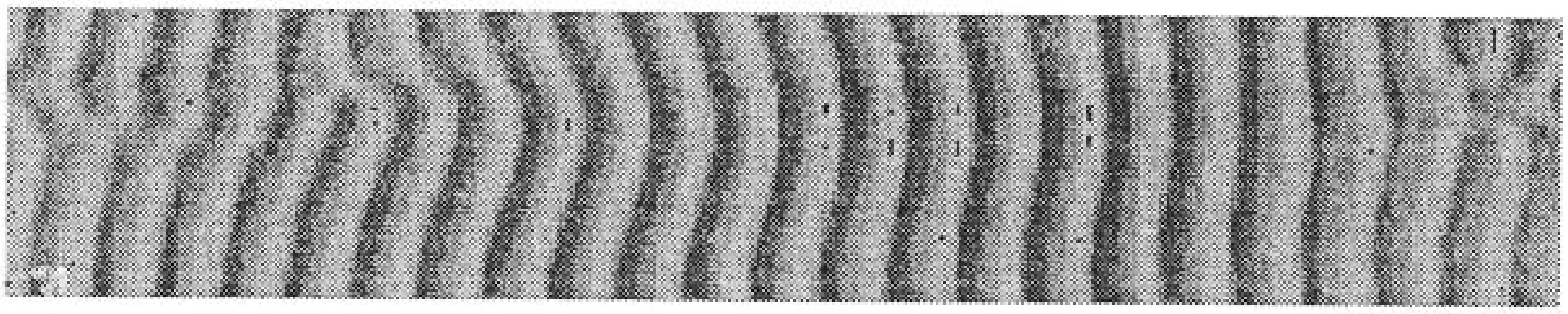

Figure 4: Patterns created by the Cellular Automaton ($\alpha = \pi/3, v_0 = 12.0$)

To visualize the configuration at any moment the height of each grain is depicted into a discrete set and the resulting value is color coded and displayed. A typical picture showing ripple-like patterns is given in figure 4.

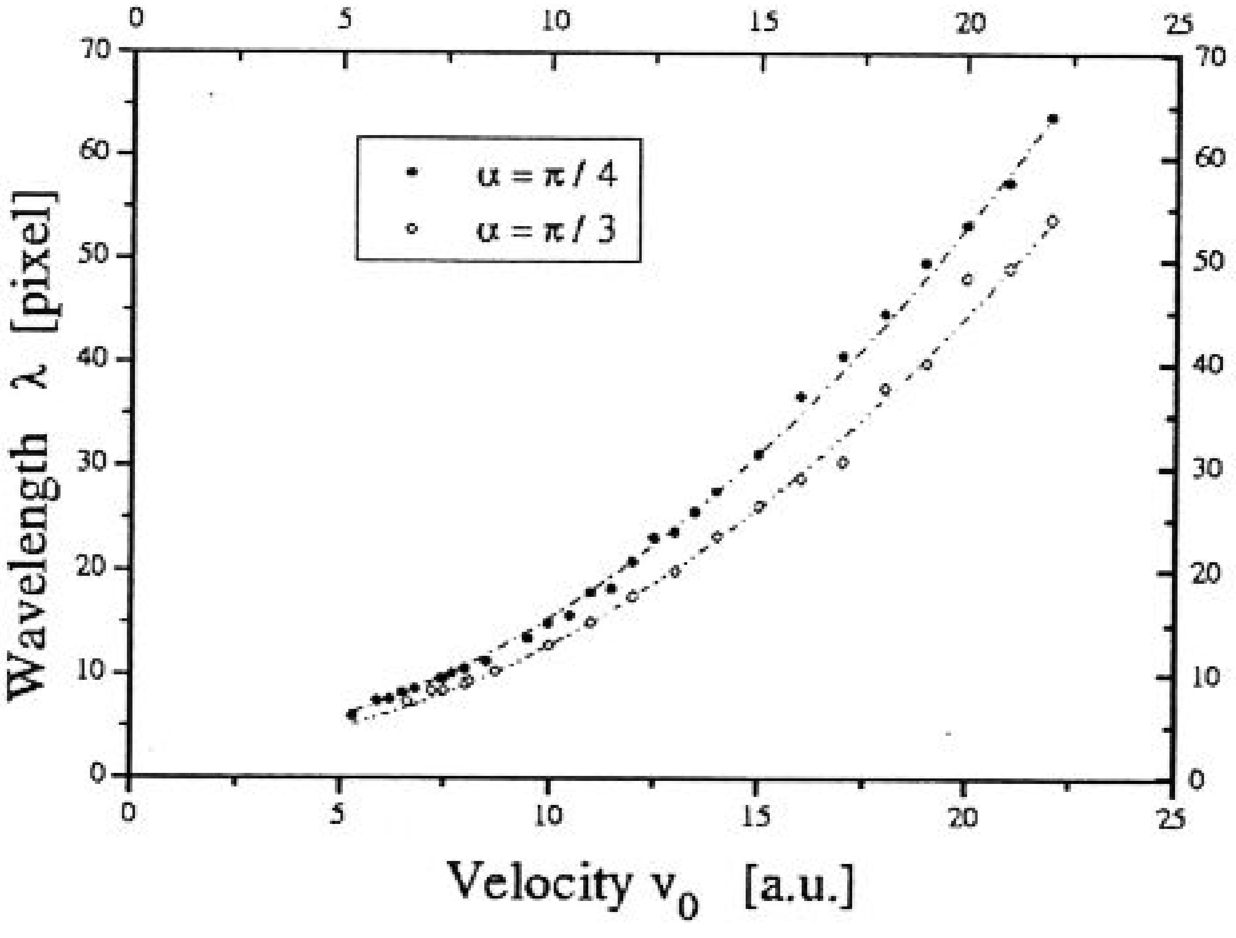

Figure 5: Wavelength of patterns versus the control parameter v_0

One interesting question both experimentally and principally concerning nonlinear dynamics is the dependence of a measured quantity from a control parameter. A generic measure to characterize pattern formation in granular media like ripples or dunes is the wavelength, i. e. the typical length between two crests. In our cellular automaton there are two control parameters introduced above as the angle α and the absolute value of the velocity v_0. First examinations relating to this aspect have been made and the results are shown in figure 5.

The qualitative statement "the higher the velocity v_0 the higher the wavelength of the patterns" of figure 3 is directly plausible, because –as a well known mechanical rule– the maximal displacement x_{max} of a body thrown at a fixed angle is proportional to the squared absolute value of the velocity $\vec{v}_0$, i. e. $x_{max} \propto |\vec{v}_0|^2$. Parabolas shown as dash-dotted lines are fitted to both curves belonging to two different angles α.

4 Discussion

We have presented the first experimental studies on pattern formation in horizontally vibrated dry granular media. These experimental results show qualitative analogies to a cellular automaton introduced above. A more quantitative examination is currently in progress.
Finally we want to emphasize that the experiments can be done in a narrow channel instead of a plate or a channel. This would limit the system to two dimensions and decrease the number of particles. Both features are very attractive for a quantitative comparison with molecular dynamic simulations.

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