Divergence of Coherence Length and Excitation of Resonance in Taylor Vortex Flow

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1. Introduction

Within the field of synergetics, considerable insight has been gained in hydrodynamic instabilities. Experimental work on the Rayleigh-Bénard instability is described by BERGÉ and LIBCHABER in chapters of this book. The present understanding of the Taylor instability is reviewed in a previous volume of this series [1, 2] and a topical book [3]. This communication deals with two aspects of the physics of the Taylor instability.

2. Divergence of Coherence Length

Theoretical treatments often deal with Taylor vortex flow between infinitely long cylinders. Experiments necessarily involve cylinders of finite length. Suppose the outer cylinder and both end plates are stationary while the inner cylinder rotates. Then, in the Couette flow regime, azimuthal velocity is reduced near the end plates. This implies a deficit of centrifugal pressure near both ends and results in inward radial secondary flow there. Fig. 1 [5] shows plots of radial velocity versus the axial coordinate as measured by laser Doppler equipment [4]. The lower two tracings refer to subcritical rotation rates, i. e. to conditions where no vortices should appear in infinitely long cylinders. Regions of inward flow, i. e. of negative radial velocity occur near both ends. Alternating velocity components adjacent to these indicate a system of vortices which decays towards the interior. The decay constant is called coherence length. The Ginsburg-Landau theory predicts divergence of the coherence length with critical exponent - 1/2. Our results agree with this prediction to high accuracy. To obtain satisfactory agreement, however, a simple exponential fit for obtaining the coherence length is not sufficient.

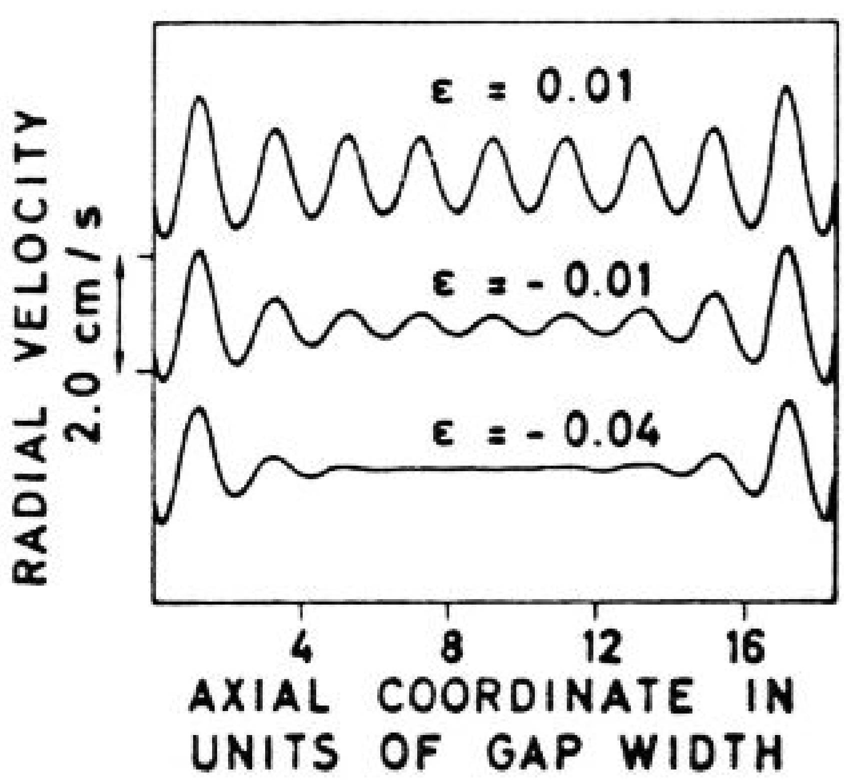

Fig. 1 Profiles of radial velocity versus axial coordinate as measured by laser Doppler equipment. The ε-values indicated were obtained by fitting procedure described in the text

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Our analysis [5] proceeds from the Ginsburg-Landau equation

$$\xi_o^2\, d^2V/dx^2 + \varepsilon V - V^3/V_o^2 = 0 \,, \tag{1}$$

where we identify the order parameter V with maximal radial velocity within a vortex. Thus $V(x)$ is defined at discrete points only. x is axial coordinate, ξ_o and V_o are scale factors for coherence length and order parameter, respectively. $\varepsilon = (R-R_c)/R_c$, R and R_c are actual and critical Reynolds numbers, the latter referring to an infinitely long cylinder. Appropriate solutions of (1) are [6]

$$V(x) = V_m \,\mathrm{nc}\left(\frac{x}{\xi_o} \sqrt{\frac{V_m^2}{V_o^2} - \varepsilon} \;\middle|\; \frac{V_m^2 - 2\varepsilon V_o^2}{2V_m^2 - 2\varepsilon V_o^2} \right) \tag{2}$$

if $\qquad \varepsilon V_o^2 < V_m^2/2$ and (2a)

$$V(x) = V_m \,\mathrm{dc}\left(\frac{x}{\xi_o} \frac{V_m}{V_o\sqrt{2}} \;\middle|\; \frac{2\varepsilon V_o^2 - V_m^2}{V_m^2} \right) \tag{3}$$

if $\qquad V_m^2/2 < \varepsilon V_o^2 < V_m^2 \,.$ (3a)

Here nc and dc are Jacobian elliptic functions whose second argument is named parameter. The origin of the x-coordinate is midway between both ends, where $V(x)$ attains its minimum V_m. Fig. 2 shows two examples of least squares fits of measured velocity data points in terms of functions (2) and (3). This fit yields the parameter which in turn yields εV_o^2. Fig. 3 shows the values εV_o^2 so determined versus rotation rate of inner cylinder. They fall on a straight line. Its intercept with the abscissa gives the critical rotation rate for the onset of stationary Taylor vortices in the infinite geometry. The numerical value $f_c = 4.233\ (\pm\ 0.001)$ Hz is compatible with the computed $R_c = 68.0$ for our wide gap geometry (with radii $R_2 = 2R_1$) to within the accuracy of our viscosity measurement.

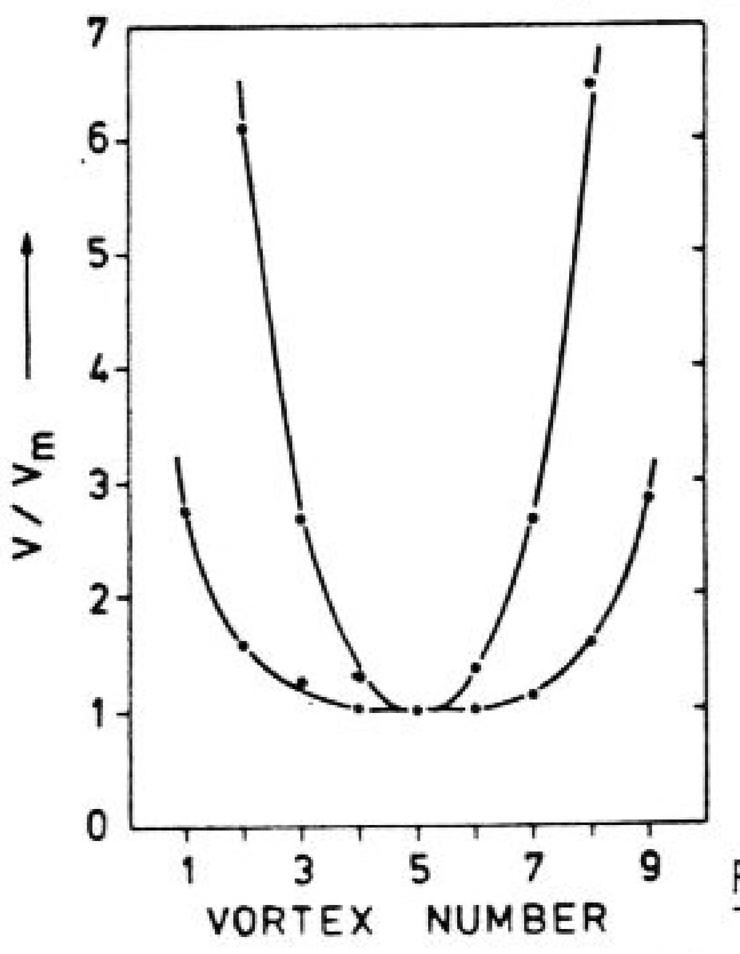

Fig.2

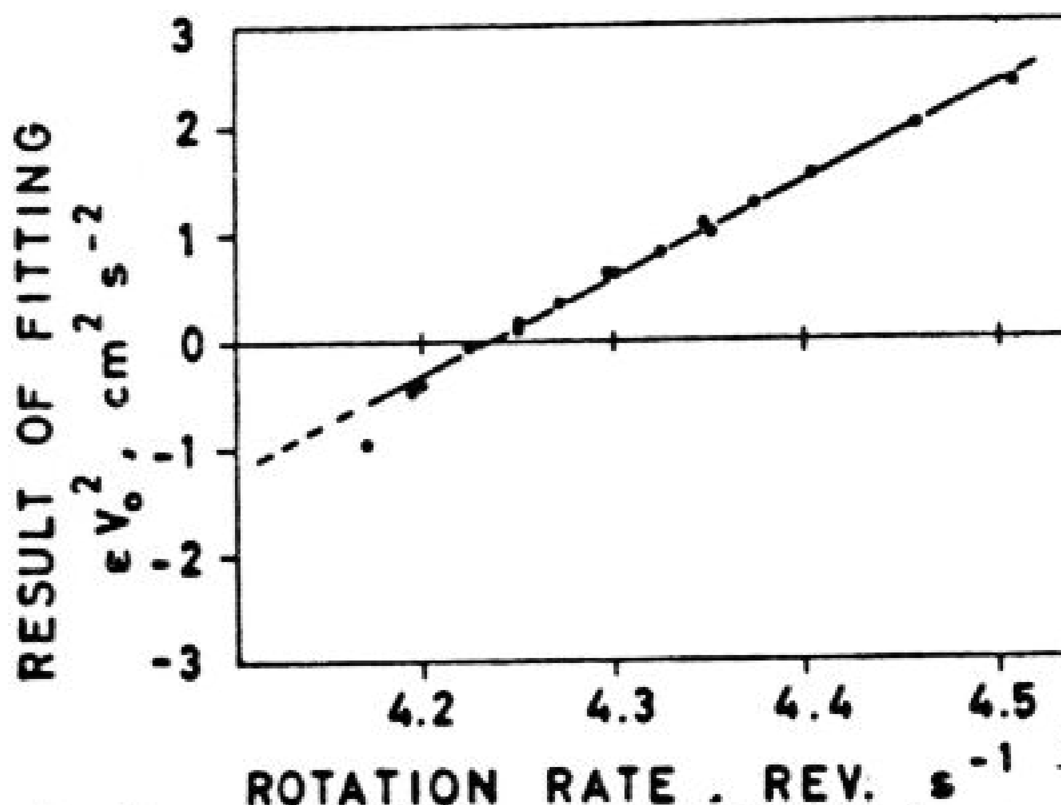

Fig.3

Fig.2 Examples of fitting of measured velocity maxima in terms of elliptic functions. Top curve: nc-function, parameter 1.00. $V_m = 0.07$ cm/s, rotation rate 4.174 Hz. Bottom curve: dc-function, parameter 0.22, $V_m = 0.48$ cm/s, rotation rate 4.249 Hz

Fig.3 Values of εV_0 obtained from curve fitting versus rotation rate. Intercept of straight line with abscissa yields critical rotation rate for infinite cylinder

Note that fig. 3 includes data points at negative ε. These refer to vortices generated by centrifugal deficit and inward flow at either end. Such data would not occur in infinitely long cylinders. Our procedure allows the evaluation of critical phenomena in infinitely long cylinders by equipment of finite length.

3. Excitation of Resonance

Here we report preliminary results of a forced oscillation experiment in which resonant response of the Taylor vortex system is observed. In our wide gap geometry, the first instability at $R_{c1} = 68$ involves the transition from Couette flow to stationary Taylor vortices. The second instability near $R_{c2} \approx 560$ leads to time dependent wavies with $m = 1$. In this flow pattern the vortices are tilted with respect to the plane normal to the cylinder axis, and precess about it with rotation rate $\omega_o \approx 0.08\ \Omega$ where Ω is the rotation rate of the inner cylinder.

Starting point for the experiment is the assumption that a tilted and rotating end plate should be able to induce tilted and precessing wavies. The experimental setup is sketched in fig. 4. For $R > R_{c2}$ available evidence indicates the simultaneous presence of natural and exciting frequencies, ω_o and ω, respectively. At $R < R_{c2}$ the oscillatory vertical velocity component measured by laser Doppler anemometry shows only the exciting frequency ω. However, this component becomes larger in a resonant fashion as ω approaches ω_o. The resonance is sharper and amplitude larger as R approaches R_{c2} from below. This is seen in the experimental curves of fig. 5. Note that similar behavior is also found in electronic oscillators. With subcritical positive feedback they are just resonant narrow-band amplifiers.

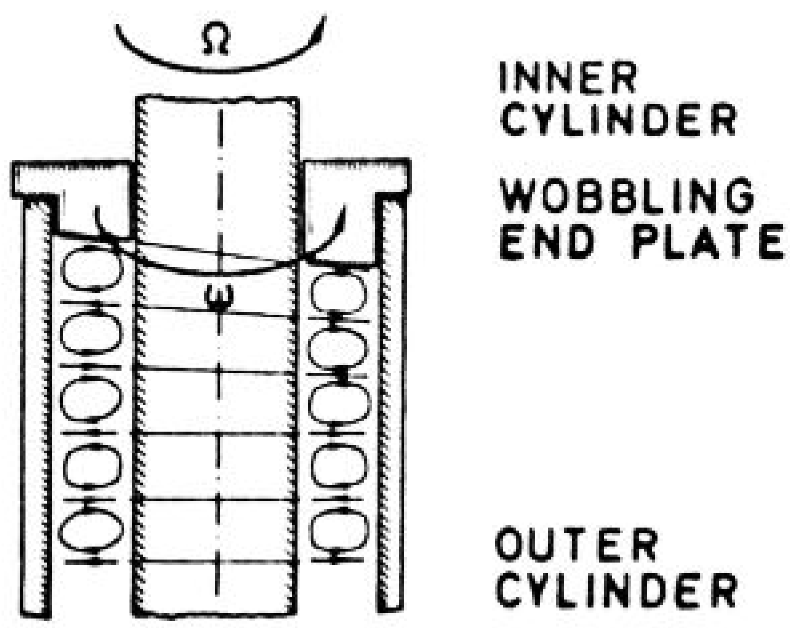

Fig. 4 Apparatus for inducing $m = 1$ wavies of externally controled frequency ω

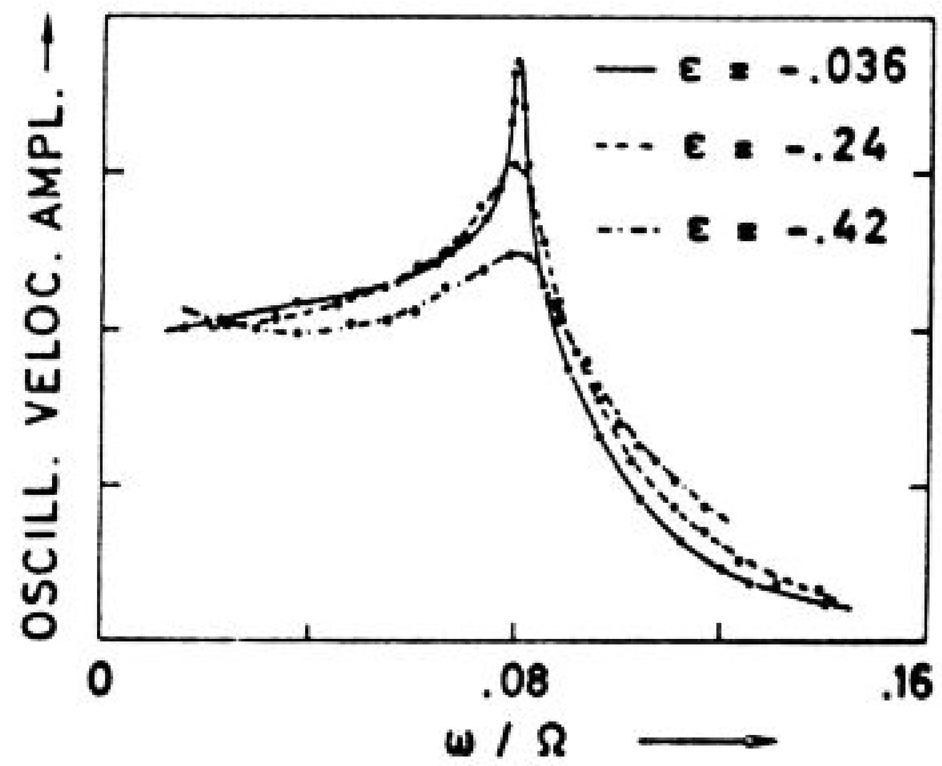

Fig. 5 Oscillatory vertical velocity amplitude measured at third vortex below top

For theoretical description we assume a complex Ginsburg-Landau equation

$$\tau_o \partial V/\partial t - i\omega_o \tau_o V = \varepsilon V - V^3/V_o^2 + \xi_o^2\ \partial_o^2 V/\partial x^2 . \qquad (4)$$

It may be obtained by transforming from the corotating to rest frame. Here $\varepsilon = (R - R_{c2})/R_{c2}$. For $\varepsilon < 0$, small order parameter and prescribed boundary value at $x = 0$, $V = V_1 \exp(i\omega t)$ the solution is

$$V = V_1 \exp\left[i\omega t - (x/\xi_o) \left\{ |\varepsilon| - i(\omega - \omega_o)\ \tau_o \right\}^{1/2} \right] \qquad (5)$$

This functional dependence accounts for the observed sharpness of resonance. The exponential x-dependence is not in disagreement with preliminary data. The phase shift versus x as implied in (5) has not yet been measured. The asymmetry of observed resonance curves fig. 5 is not in line with (5) and needs further explanation. We feel, however, that this kind of experiment will help to clarify mechanisms of hydrodynamic instability.

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