Phase Diffusion in a Ramped Convection Channel

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Summary

The drift of transverse convection rolls in a channel heated from below has been studied experimentally. Owing to the gradual decrease of the height at the ends of the channel, the Rayleigh number exhibits a ramplike decay toward subcritical values. Such a configuration selects a small wavenumber band. But the wavenumber bands selected by the two ramps may cease to overlap at supercritical Rayleigh numbers. The resulting wavenumber gradient causes phase diffusion, i.e. the convection pattern drifts from the high wavenumber end to the low wavenumber end. Various observational data on this phenomenon are presented.

Introduction

The question of wavelength selection is common to many nonequilibrium pattern forming systems. At the threshold of the control parameter the wavenumber is well defined, but above the critical point a band of stable wavenumbers opens up. Experiments with the usual sidewall boundaries do not indicate a preferred wavelength. All wavenumbers within the Eckhaus band are stable /1,2/, and the wavelength is selected by the fact that an integer number of rolls must fit into the box. This restriction can be overcome by a ramp: The local value of the control parameter is gradually changed from supercritical to subcritical values. Such a ramp does indeed select a single, but not necessarily unique wavelength /3,4/. In the case of Rayleigh-Bénard convection, the selected wavelength has been calculated as a function of the geometrical and thermal properties of the ramp /5/. It has also been shown by a general argument that any wavelength within the Eckhaus band can be reached by means of a suitable ramp /5/. This nonuniqueness leads to an interesting prediction: If the two ramps in a convection channel disagree on the wavelength to be selected, a wavelength gradient is established in the homogeneous (nonramped) part of the system. A steady state is not possible in the presence of a wavelength gradient, and phase diffusion ensues/6/. In the present paper observations are reported of phase diffusion in the form of a drifting convection pattern in a ramped convection channel.

Experimental Setup and Procedure

The convection channel is cut out of a copper plate (Fig. 1). The bulk part of the channel is 3mm high, 1.5mm thick and 18mm long. The ramps adjacent to this central part decrease the height of the channel from 3mm to 1mm over a length of 26mm by means of a parabolic curvature of the top and bottom. For most of our measurements one of these ramps has been replaced by a ramp with a pinning center /2,7,8/ as indicated by the dashed line in Fig. 1. The experiment takes place in an aluminum box regulated to 20±0.02°C forming a thermal and electrical shield. The top of the convection channel is attached to this isothermal box. The bottom is heated with an electric heater with a voltage stability of better than 0.01% and is thermally isolated by a

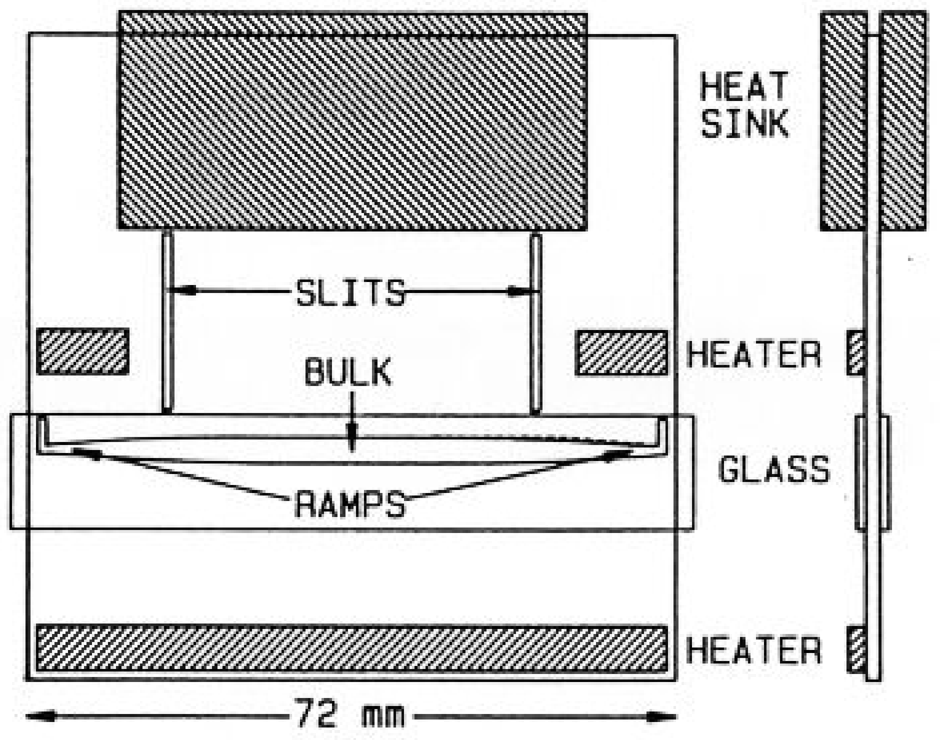

Fig. 1
The convection channel
(to scale) is cut out of a
72x72x1.5mm copper plate

styrofoam mantle. The temperature difference between top and bottom is measured with a pair of thermocouples and has a stability of better than 0.3%. Additional side heaters are placed above the ramps in order to influence their thermal properties. In order to minimize the horizontal component of the temperature gradient across the bulk part of the channel, slits of 1mm thickness are cut into the copper plate. Glass plates of 1mm thickness are attached to the sides of the copper plate in order to allow flow visualisation from the side. Water or purified cyclohexan are used as working fluids.

We use the shadowgraph technique to visualize the transverse convection rolls. A horizontal beam of parallel light crosses the channel and is deflected locally by the density gradients of the fluid. For most of the measurements presented here we used a photodiode, movable parallel to the

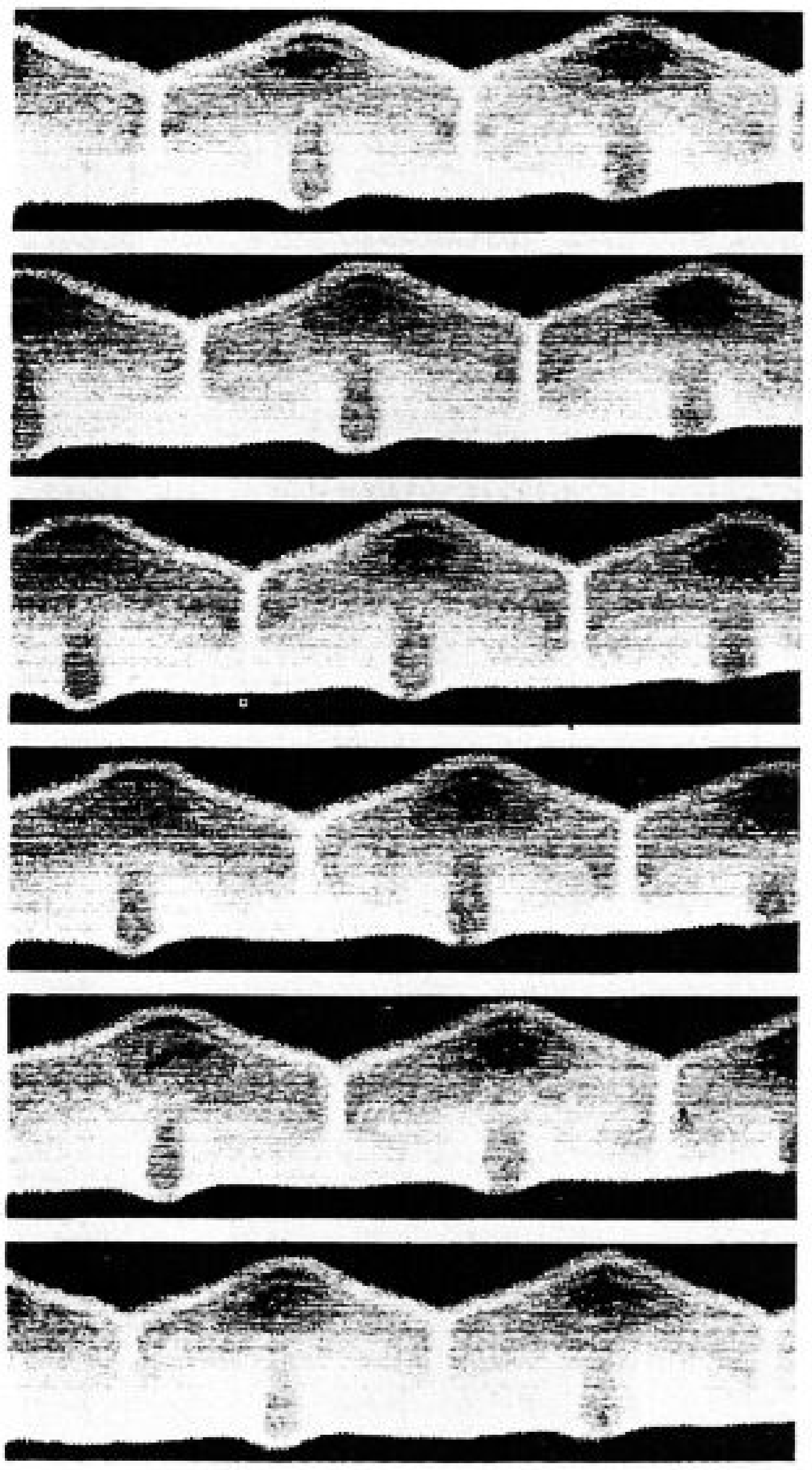

Fig. 2
Images of the drifting
convection pattern obtained
with a video camera.
The images are digitized
with a spatial resolution of
256x256 pixels of 16 grey
levels and displayed with
8 grey levels.
The time interval between
2 pictures is 40 minutes

long dimension of the channel by means of a stepping motor. The light intensity is scanned at a distance of typically 50 cm behind the channel. In order to eliminate effects due to the inhomogeneities in the light beam and in the glass boundaries, a horizontal line scan is taken in the absence of convection. All subsequent scans are then divided by this zero image with the result that the remaining spatial variations of the measured light intensity represent the effect of convection. This method is convenient for measuring the critical point for the onset of convection /9/. The critical temperature difference of 2.63K measured for cyclohexane in our channel is well within the range given by the theoretical predictions for perfectly insulating and perfectly conducting sidewalls /10/.

Figure 2 demonstrates the drifting convection pattern in the water-filled channel. The image of the temperature distribution has been recorded with a video camera. Sharp lines of high intensity (caustics) correspond to cold, sinking fluid, the darker zones indicate rising fluid. The convection pattern drifts from the left to the right. The time period for one wavelength to pass a fixed point is approximately 2 hours and 20 minutes.

Results

For measuring details of the convection structure we used a photodiode instead of the digitized camera images. Figure 3 demonstrates the existence of a drifting convection pattern in the channel without the pinning center. Line scans of the light intensity along the channel are shown. A sequence of 61 scans, taken 20 minutes apart, has been plotted in this figure thus providing a record of the drifting convection pattern over a period of 20 hours. Convection rolls are generated in the left ramp, drift throughout the channel (position 0 indicates the middle of the channel) and decay on the right hand side. Note that the rolls become visible at a position close to -30mm, but decay already at a position of +20mm. This difference in the position of the critical points is caused by the extra heater which was placed on top of the right hand ramp in order to change the property of this ramp. Without this additional heater the ramps are nearly symmetric and no drifting pattern can be observed.

By repeating the experiment shown in Fig. 3 at different heating rates the drift rate of the pattern can be measured as a function of the reduced Rayleigh number $\epsilon=(R-R_c)/R_c$. The frequency was actually measured by a photodiode located at a fixed position. The periodic time signal generated by

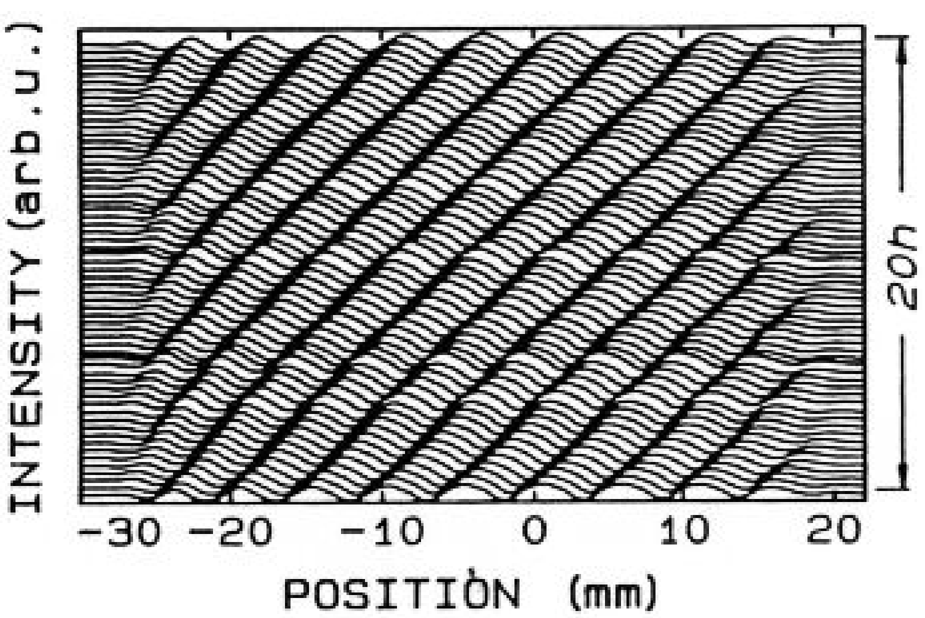

Fig. 3
Images of the drifting convection pattern obtained with a photodiode at the reduced Rayleigh number $\epsilon=(R-R_c)/R_c=0.33$

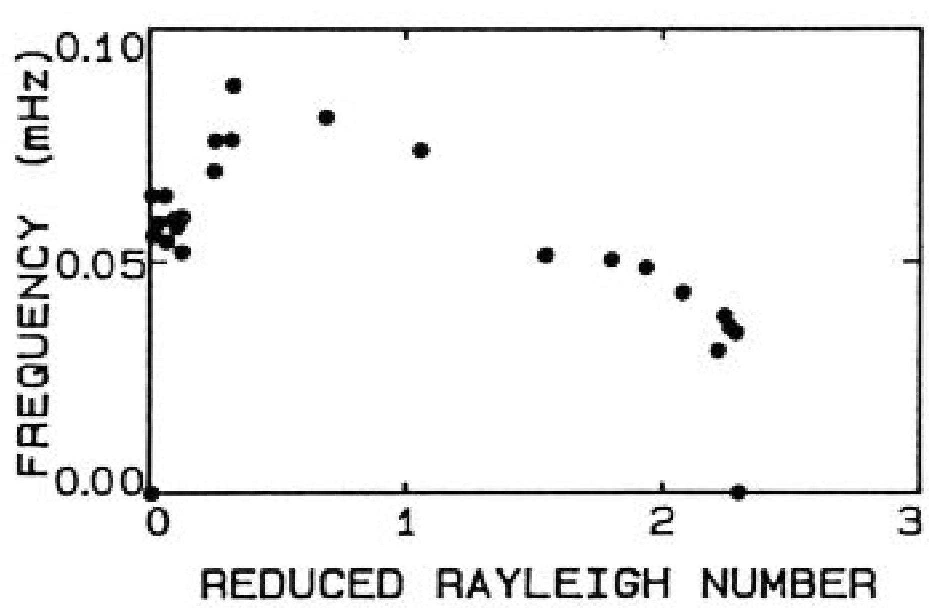

Fig. 4
Frequency of the light intensity modulations as a function of $\epsilon=(R-R_c)/R_c$ for the cell without the pinning center

the drifting convection pattern was analyzed by a fitting procedure in order to determine the frequency more accurately than is possible from the scanning plots. The results are shown in Fig. 4 for the case without pinning center. Because the small but finite wavenumber bands selected by the two ramps tend to overlap near the threshold R_c for the onset of convection/5/, the onset of a pattern drift is expectd to occur at a value R_d of the Rayleigh number above R_c. A steady pattern was indeed observed very close to R_c as shown in Fig. 5. But it was not possible to determine R_d accurately. The reduced difference $(R_d-R_c)/R_c$ was of the same order as the experimental resolution of about 0.3%. This fact is demonstrated by the drifting pattern shown in Fig. 6 which was observed at a Rayleigh number which exceeds the value of Fig. 5 by less than 0.3%. Clearly, the convective motion extends over a larger region and the amplitude also has increased, although the different units that have been used in Figs. 5 and 6 tend to exaggerate this effect. Fig. 7 shows the drifting convection pattern at a slightly higher Rayleigh number when the drift velocity exhibits less variation. As the ramps become supercritical their effect begins to resemble the effect of the usual sidewalls and the drift ceases quite suddenly as shown in Fig. 4.

In order to increase the difference between the critical Rayleigh numbers R_c and R_d for the onset of convection and the onset of the drift, respectively, a pinning center /7,8/ (see Fig.1) similar to the one used in Taylor-Couette flow experiments /2/ was added to the channel. Fig.8 shows the movement of the pattern obtained in this case: It is less uniform as compared to Figs.3 and 7. Note that the heater at the left hand side is used here.

Figure 9 gives information about the drift velocity as a function of the reduced Rayleigh number $\epsilon=(R-R_c)/R_c$ for the channel with the pinning center. These data were obtained in the same way as the data of Fig.4.

As indicated in Fig. 8, the velocity within one period of the movement is not constant. In order to measure this velocity we take the intensity data out of the range from -9mm to +9mm (the bulk part of the channel). The distance travelled within 20 minutes is obtained by means of the spatial cross-correlation function between two consecutive lines. In addition the wavelength gradient of the convection rolls can be extracted from the line scans. For this purpose we measure the distance between two intensity maxima, which gives the wavelength. Within the range of ±9mm at least 3 maxima can be

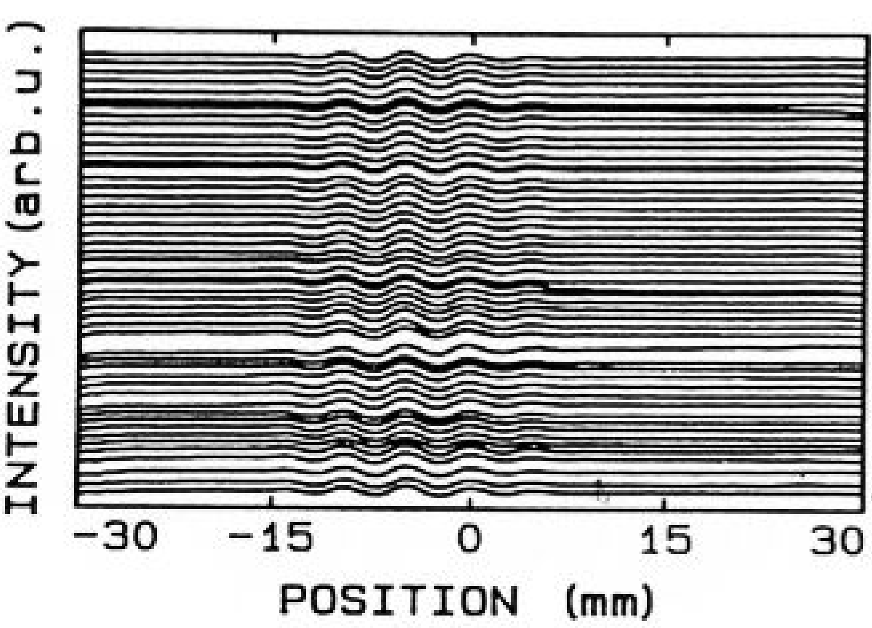

Fig. 5
Steady pattern observed at $\epsilon=0.01$

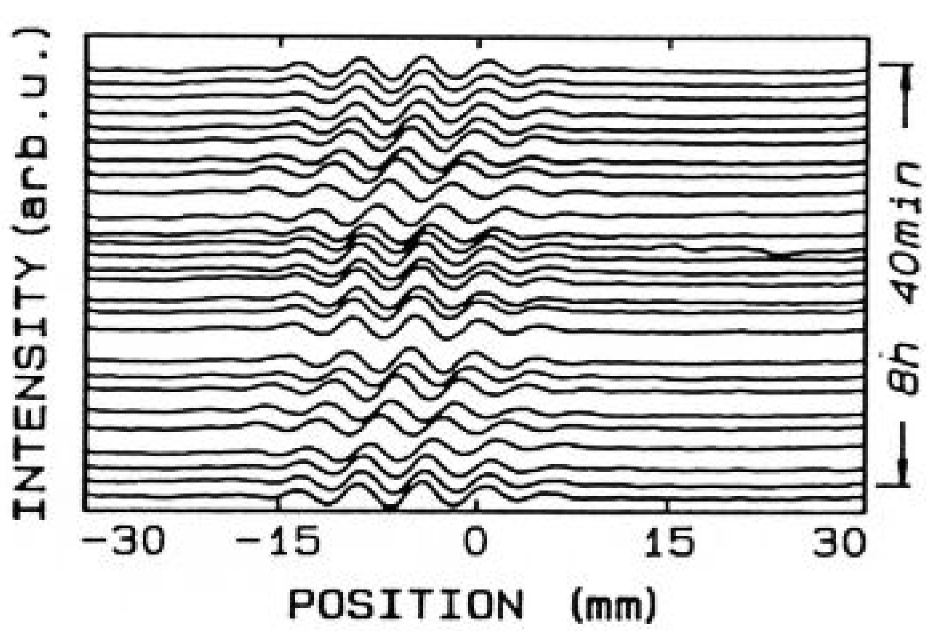

Fig. 6
Drifting pattern observed in the cell without pinning center at $\epsilon=0.01$

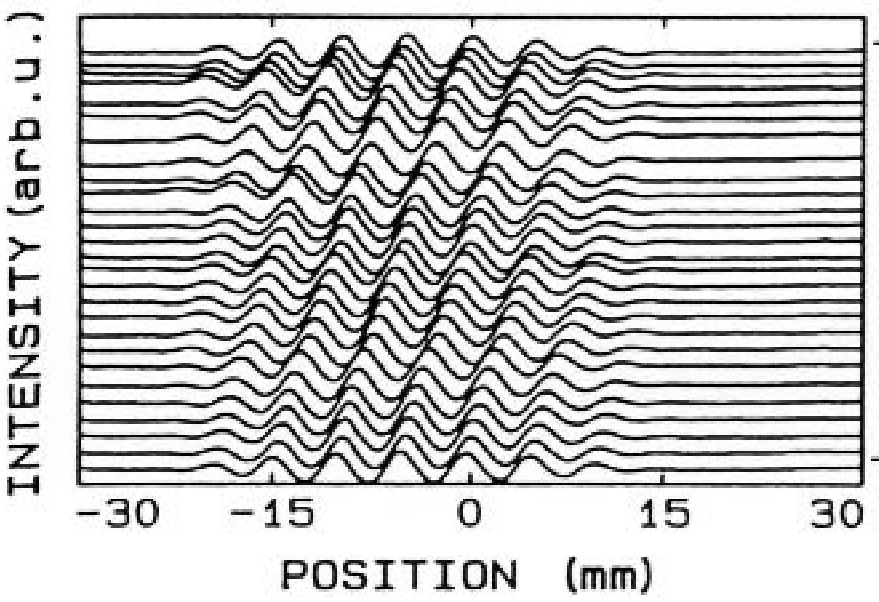

Fig. 7
Drifting pattern in the cell without pinning center at $\epsilon=0.06$

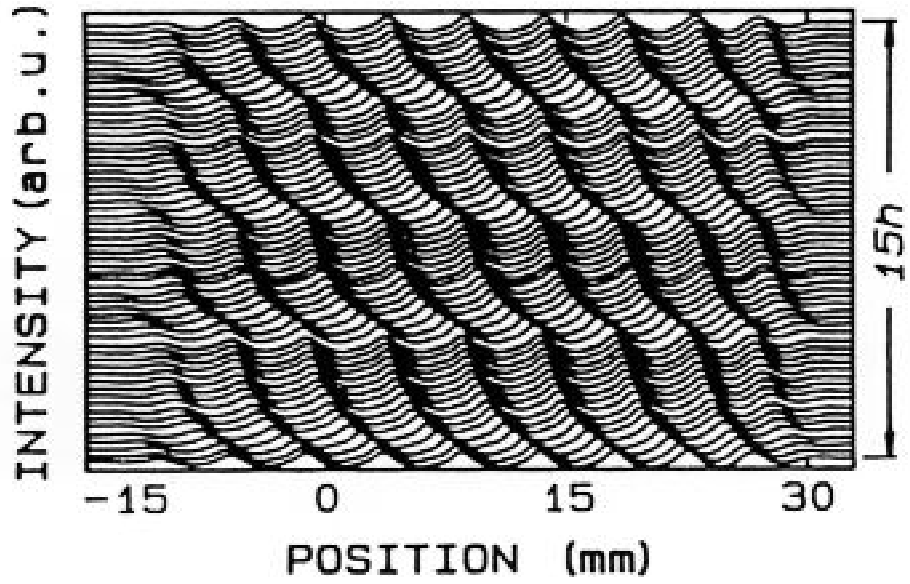

Fig. 8
Drifting convection pattern influenced by the pinning center. $\epsilon=0.05$

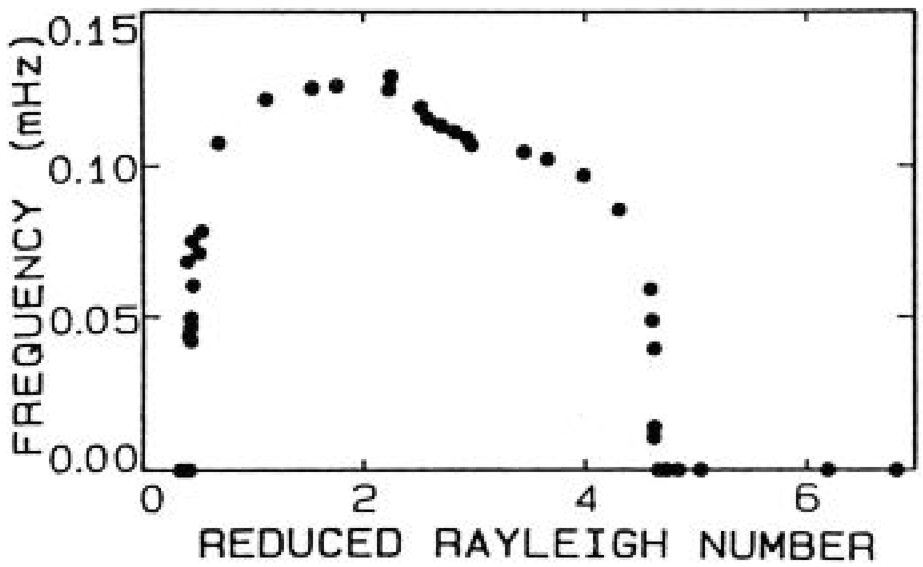

Fig. 9
Frequency of the light intensity modulations as a function of $\epsilon=(R-R_c)/R_c$

measured, and a wavelength gradient can thus be determined. Fig. 10 shows these two measurements for a point above R_d($\epsilon=2.52$, f=0.12mHz) in Fig. 8. The positive sign of the gradient indicates that the wavelength is shorter at the right hand side, i.e. the pattern drifts from the short wavelength end to the ramp selecting the longer wavelength.

Figures 11 and 12 give similar information as Fig. 10, but for a point close to the onset of the drift ($\epsilon=0.42$, f=0.05mHz) and close to the point

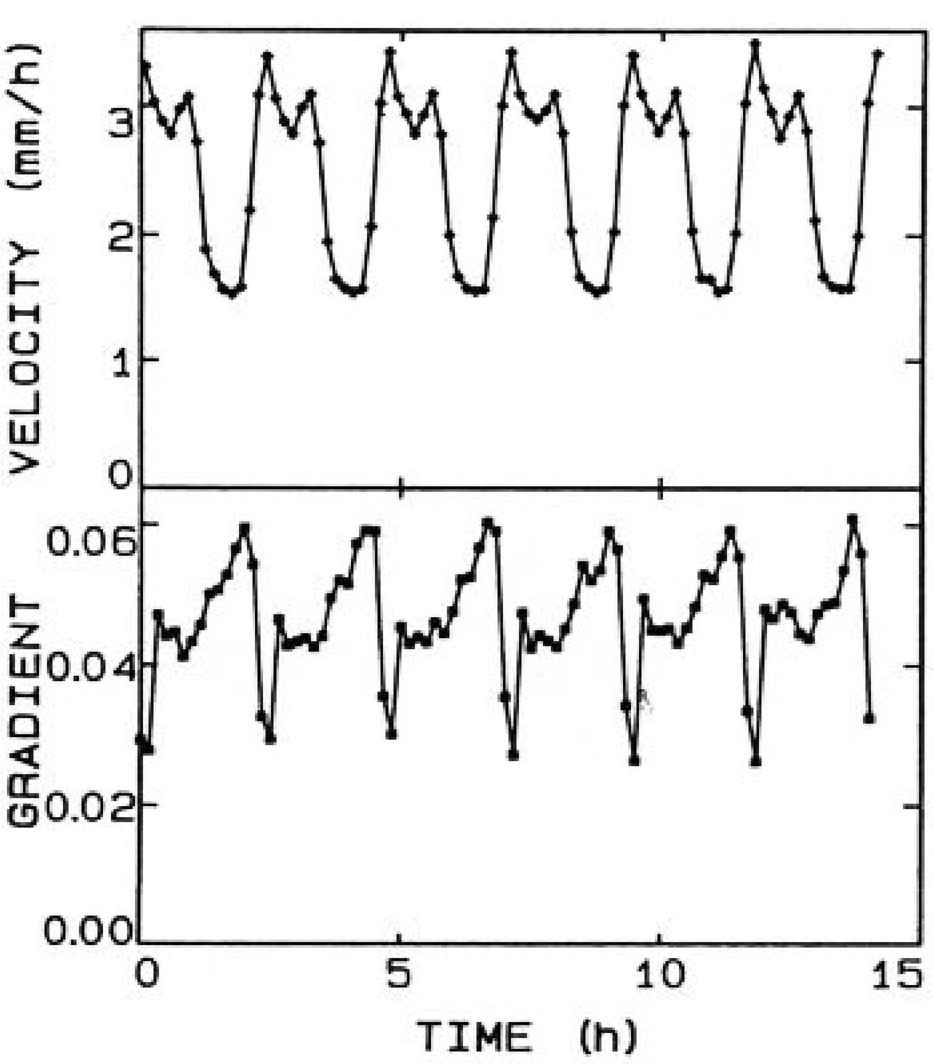

Fig. 10
The drift velocity and the wavelength grad nt as a function of time for $\epsilon=2.52$

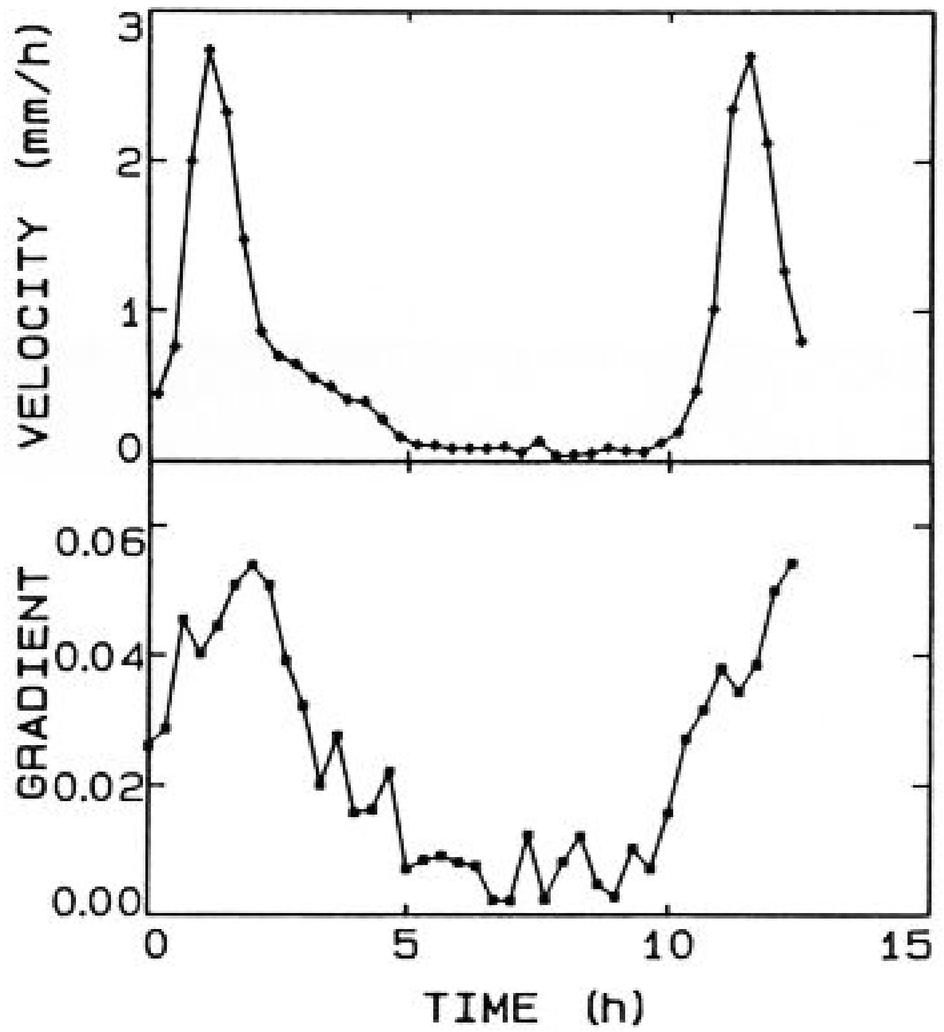

Fig. 11
The drift velocity and the wavelength gradient as a function of time for $\epsilon=0.42$

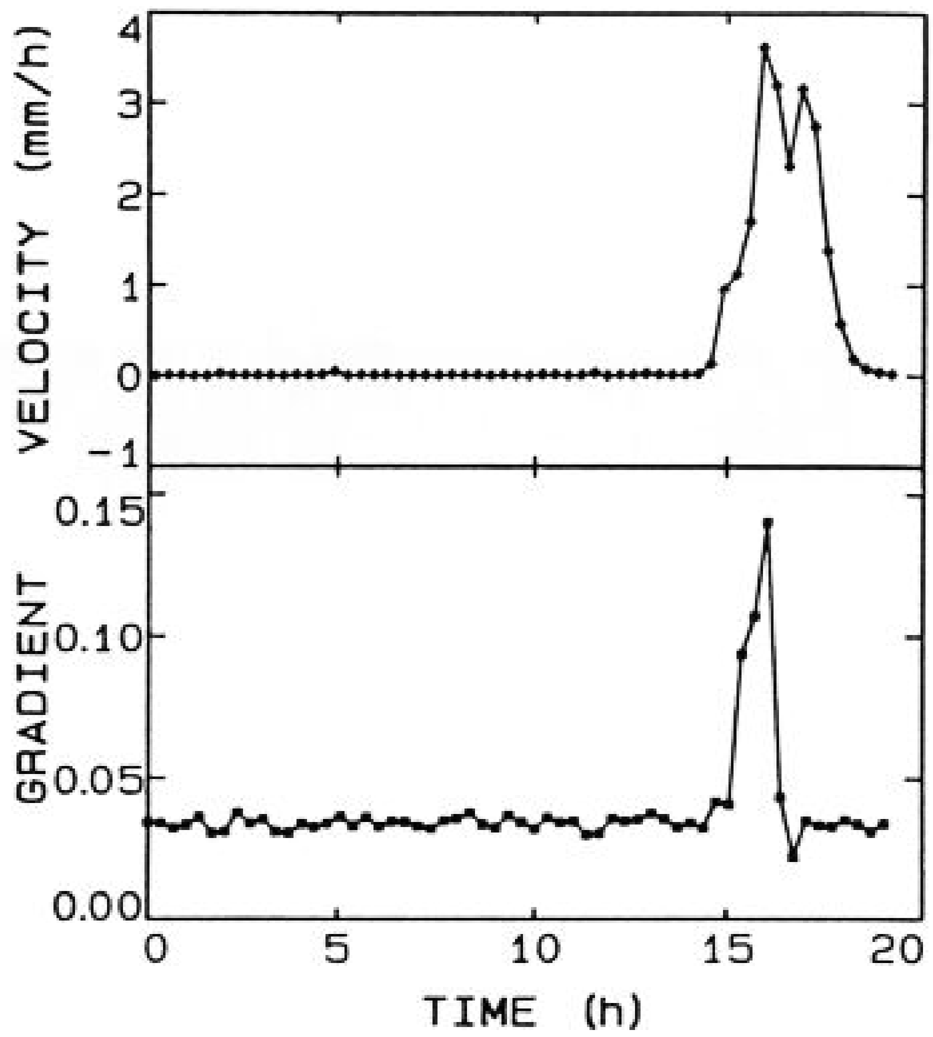

Fig. 12
Drift velocity and wavelength gradient for $\epsilon=4.64$.

where the drift stops ($\epsilon=4.64$, f=0.01mHz). The striking feature in these two cases is the strong nonuniformity of the movement.

Discussion

The theoretical interpretation of the observed phenomena cannot go beyond qualitative aspects. The theory of phase diffusion in Rayleigh-Bénard convection /5,7/ suggest the existence of two different selection bands as indicated by the two shaded regions in Fig. 13. In an experimental situation with ramps of a finite length, these bands are not infinitely small as in the case of perfect selection. Their finite width opens up in the neighborhood of the critical point (anomalous bands) /7,8,9,11/ and leads to the overlap of the bands within the range $R_c<R<R_d$. Because both bands can accommodate the same wavenumber no pattern drift occurs. Beyond the critical point R_d the overlap ceases and the different wavelengths selected by the two ramps give rise to a wavelength gradient and thus to drifting patterns. Pinning effects, however, may have an overriding influence in typical experimental situations. A similar picture can be drawn for the upper range of the drift in the neighborhood of $\epsilon=5$. Here the subcritical part of the ramp decreases in size with increasing ϵ and finally vanishes with the result that the width of the anomalous bands increases strongly/5/. As the two bands overlap again, the drift stops. Because of the relatively short size of the ramps it could also be said that the convective system becomes less flexible in accommodating wavelength changes in the case of supercritical ramps and thus begins to resemble a convection box with vertical side walls. The nonuniformity of the movement as indicated in Figs. 10-12 cannot be explained by Fig.13. It is believed to be an effect of the finite size of the ramp. In the case of a long ramp the phase diffusion would smooth both the velocity and gradient variations. Thus a longer channel will be preferable for a more quantitative comparison with the theoretical predictions. In that case, however, the time scales for the drift increase because the drift velocity is roughly proportional to the wavelength gradient.

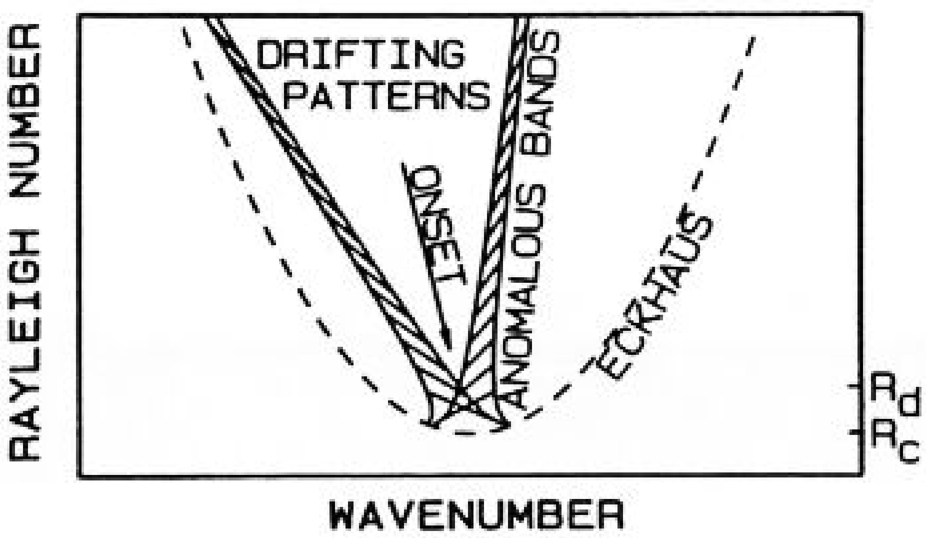

Fig. 13
Qualitative explanation of the observed drift. The two shaded regions indicate the wavenumber bands selected by the two ramps

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Note added in proof:
A brief account of some results described in this paper has been published in the meantime /12/.

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