Using Thermal Noise to Measure the Correlation Lengths of Electroconvection

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Abstract

The correlation lengths of electroconvection in a nematic liquid crystal both parallel and perpendicular to the director are measured by analyzing the correlation functions of thermally induced director fluctuations in the subcritical regime.

1 Introduction

Pattern forming instabilities in anisotropic systems can be described by an amplitude equation in the neighborhood of their critical point [1]:

$$\tau_0 \partial_t A = \epsilon A + \xi_{\|}^2 \partial_{xx}^2 A + \xi_{\perp}^2 \partial_{yy}^2 A$$

which contains two length scales $\xi_{\|}$ and $\xi_{\perp}$. Within this linear approximation, the correlation lengths parallel and perpendicular to the convection rolls diverge near the transition ($\epsilon = 0$) according to $\xi_P^2 = \xi_{\|}^2/(-\epsilon)$ and $\xi_N^2 = \xi_{\perp}^2/(-\epsilon)$, respectively. An example for an anisotropic system is electroconvection in a thin layer of nematic liquid crystal. It offers the possibility to study the influence of thermal noise on a nonequilibrium phase transition directly [2, 3]. This fact should make it possible to address a number of fundamental questions experimentally [2], but in addition it also offers a convenient tool to study the linear response of the hydrodynamic system to small perturbations. The applicability of this noise method to measure the decay rate τ_0 has already been demonstrated in Ref. [4]. In the present paper the thermal noise is used to measure the correlation lengths $\xi_{\|}$ and $\xi_{\perp}$.

2 Experimental Setup and Procedure

The experimental setup is similar to the one described in Ref. [2]. We used a thin layer of the nematic liquid crystal N-(p-methoxybenzylidene)-p-butylaniline (MBBA). A lower bound for the thickness is given by the use of $d = 8\,\mu$m mylar spacers between the transparent electrodes. The actual thickness was measured by optical inspection with a microscope, a procedure which is believed to be reliable within an error of $\pm 2\,\mu$m at best, which gave a cell thickness of $d = (14 \pm 2)\,\mu$m. The director alignment is enforced by rubbing of the polymer-coated glass slides which confine the sample. The temperature is controlled to (21 ± 0.01) °C. The mean-square value of the AC-voltage applied to the transparent electrodes serves to define the control parameter $\epsilon = V^2/V_c^2 - 1$, where V_c is the frequency dependent critical voltage for the onset of convection.

A light bulb illuminates the sample through a polarizer. The microscope objective forms an image of the layer located 0.5d below the upper boundary of the liquid crystal, where the linear approximation of the optical response is expected to have the largest range of applicability [5]. This image is detected by a charge-coupled device array (CCD)-Camera of 768×512 quadratic pixels thermostatted at (5 ± 0.05) °C, with a spacing of 0.011 mm. The exposure time for the diodes is 0.04 seconds. The light intensity is digitized with 8 bits, which corresponds to 256 gray levels.

The camera is aligned such that the camera lines are located parallel to the director of the nematic liquid crystal, i.e. perpendicular to the convection rolls, and the camera columns parallel to the rolls. The camera works in an interlaced modus, thus only the even lines are used to ensure simultaneous illumination. When measuring the correlation length parallel to the director, the lines over one entire image are added in order to restrict the measurement to long wavelength modes perpendicular to the director, as explained in Ref. [3]. When measuring the correlation length perpendicular to the director, one central column of the camera is used.

We determine the structure function of 2048 lines for measuring $\xi_{\|}$, and 2048 columns for measuring $\xi_{\perp}$, as explained in detail in Ref. [5]. The correlations lengths ξ_P and ξ_N are obtained by fitting exponential decays to the structure functions.

3 Experimental Results

In order to characterize the liquid crystal sample used, Fig. 1 presents the phase diagram of our convection cell. The cutoff-frequency can be seen to be at about 200 Hz. Above this frequency the motionless ground state goes unstable with respect to the dielectric rolls.

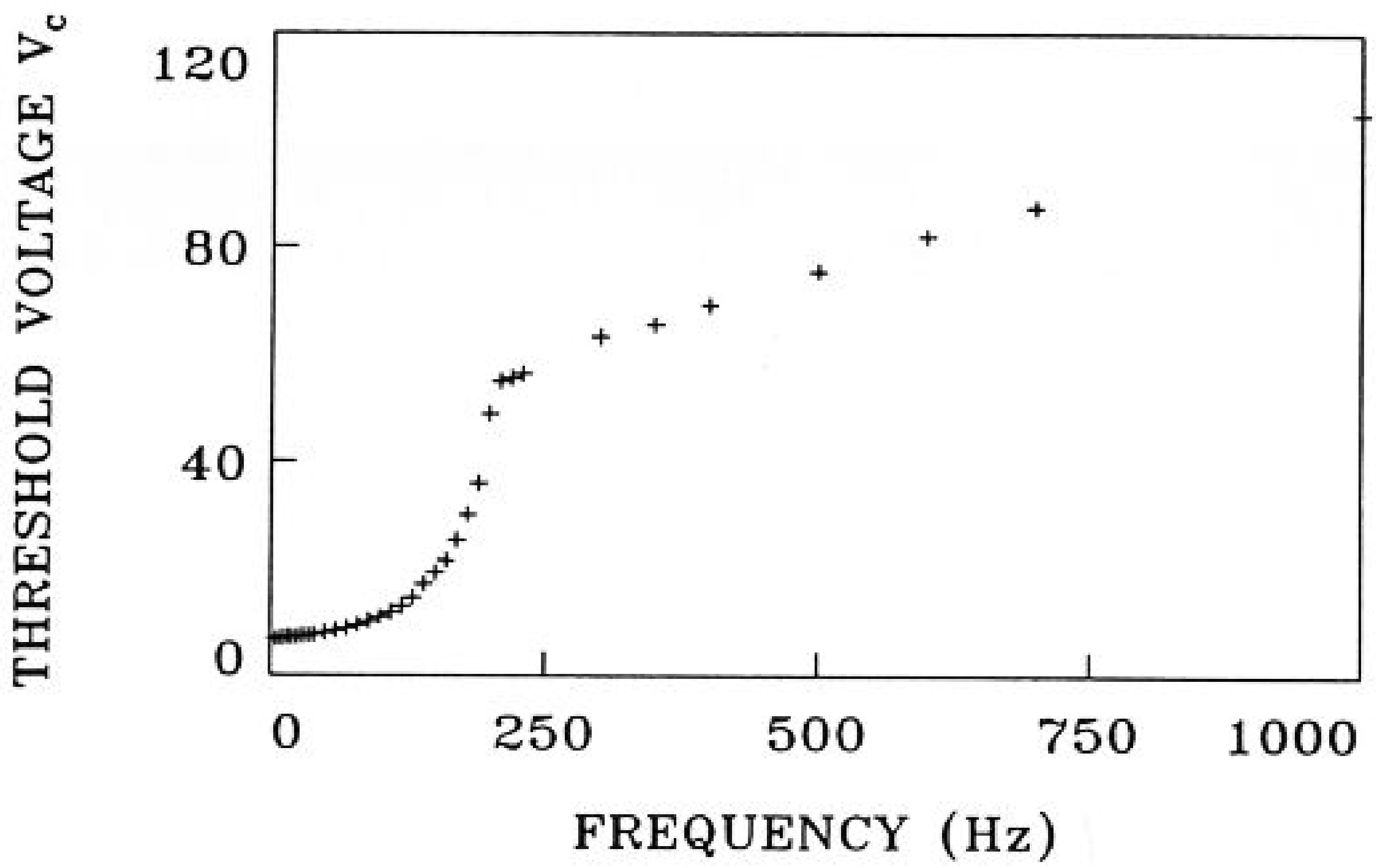

Figure 1: *The phase diagram. The crosses indicate the critical voltage for the onset of convection.*

Further characterization of the sample is provided by measuring the wave number of the rolls in the conduction regime. This has been done by measuring fully developed convection rolls slightly above threshold. The results are presented in Fig. 2. The data are larger than the numbers presented in Ref. [1], at least when comparing with the limit of thick cells or high electrical conductivity. The fact that the numbers do not match might be attributed to the fact that our sample is not within this limit. It is not known whether this assumption can also explain the qualitative disagreement observed in the curvature of the line, when comparing our Fig. 2 to Fig. 3 of Ref. [1]. In order to make better contact with the linear stability analysis it might be advantageous to measure the critical wave number by analyzing subcritical fluctuations. That problem is currently under investigation.

For determining the correlation lengths, we measure spatial structure functions at different subcritical values of ϵ and at a frequency of the driving voltage of 25 Hz. For determining ξ_P we fit the structure function obtained along the director orientation to $S(\Delta x) \sim \exp(-\Delta x/\xi_P)\cos(k\Delta x)$ similar to Ref. [2], and the results are shown in Fig. 3. The structure function obtained along the roll orientation is fitted by $S(\Delta y) \sim \exp(-\Delta y/\xi_N)$, and the resulting values for ξ_N are shown in Fig. 4. From the slope of the fitted straight lines in Figs. 3, 4 we get $\xi_{||} = 0.26$d, and $\xi_{\perp} = 0.09$d. Both numbers are smaller than the ones which can be read of Fig. 15 of Ref. [1] to be

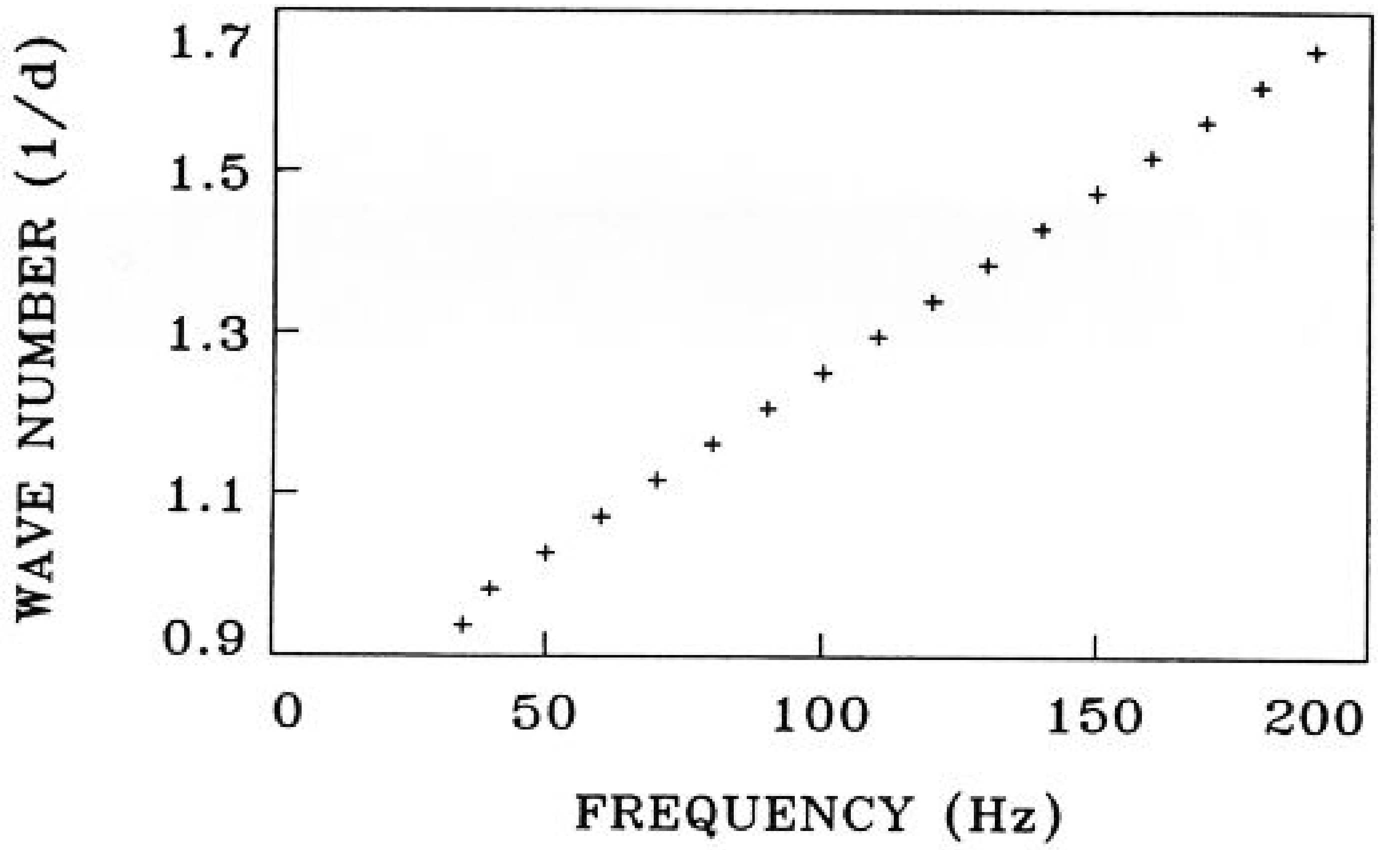

Figure 2: *The crosses indicate the wave number as measured slightly above the onset of convection.*

$\xi_{\|} = 0.28$d and $\xi_{\perp} = 0.11$d. The ratio $\xi_{\|}/\xi_{\perp}$ of the measured correlation lengths is larger by about 10% than the theoretically expected ratio. A possible explanation for these discrepancies might be the fact that we are dealing with a relatively pure and thin sample, which tends to change the theoretically expected numbers as indicated by Fig. 12 of Ref. [1]. However, a detailed calculation of the correlations lengths as a function of the cell thickness does not seem to be available.

4 Summary and Discussion

In summary, we have shown that correlation lengths can be measured by means of analyzing subcritical hydrodynamic fluctuations caused by intrinsic thermal noise. An elegant deterministic alternative for measuring these numbers is to analyze the neighborhood of a defect [6]. One might argue, however, that this procedure is not a direct method, but it rather relies on the applicability of the amplitude equation in the neighborhood of the defect, and on a linear response of the optical signal, which can be guaranteed only in a very small $\epsilon-$ interval [5]. The noise method offers a fairly large working interval free of these restrictions, because the fluctuations are so small that the optical response to director fluctuations remains linear, and also the applicability of a linear stability analysis cannot be questioned.

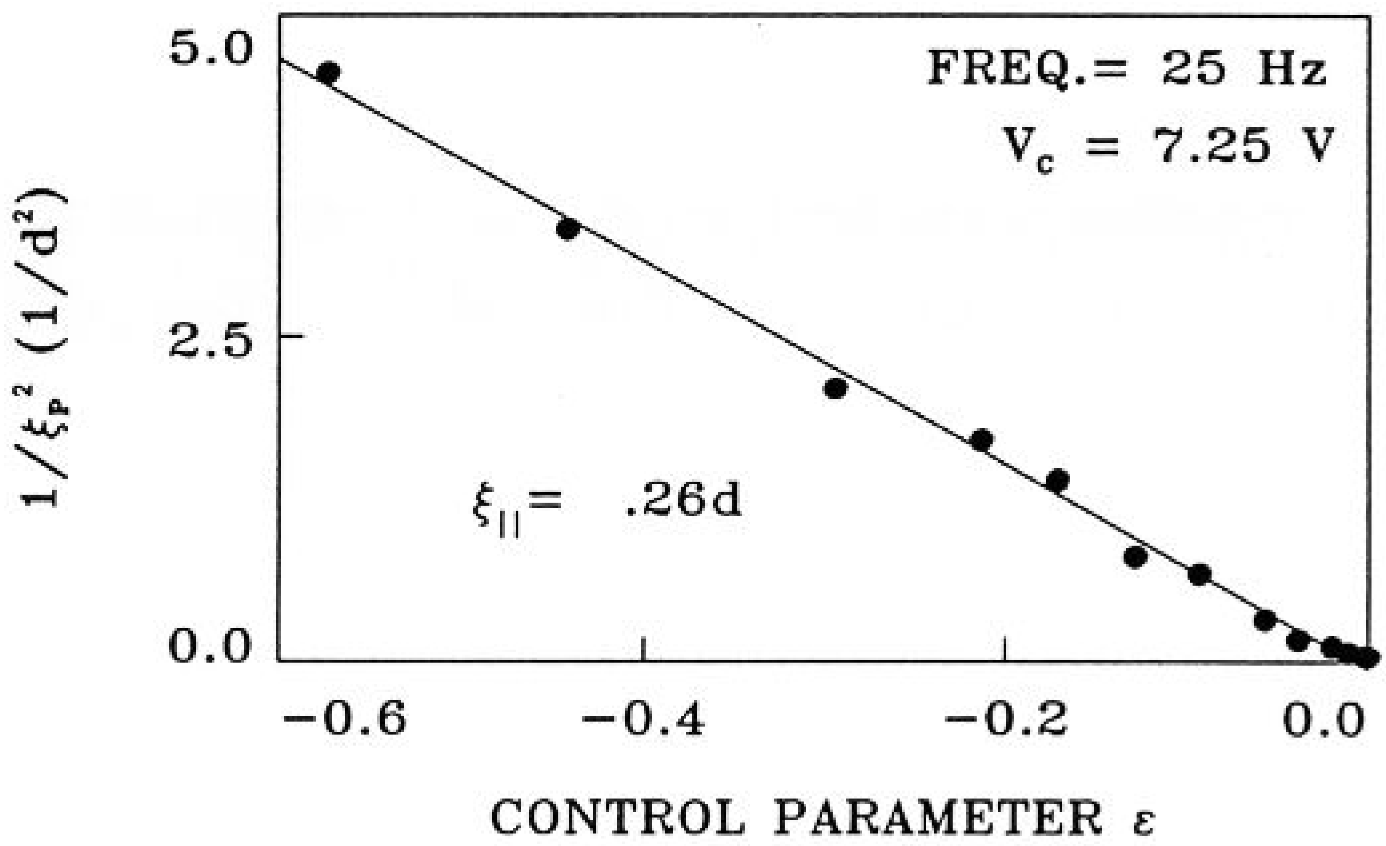

Figure 3: *The correlation length as measured parallel to the director orientation at a driving frequency of 25 Hz. The slope of the fitted straight line yields the value of* $\xi_{\|} = 0.26d$.

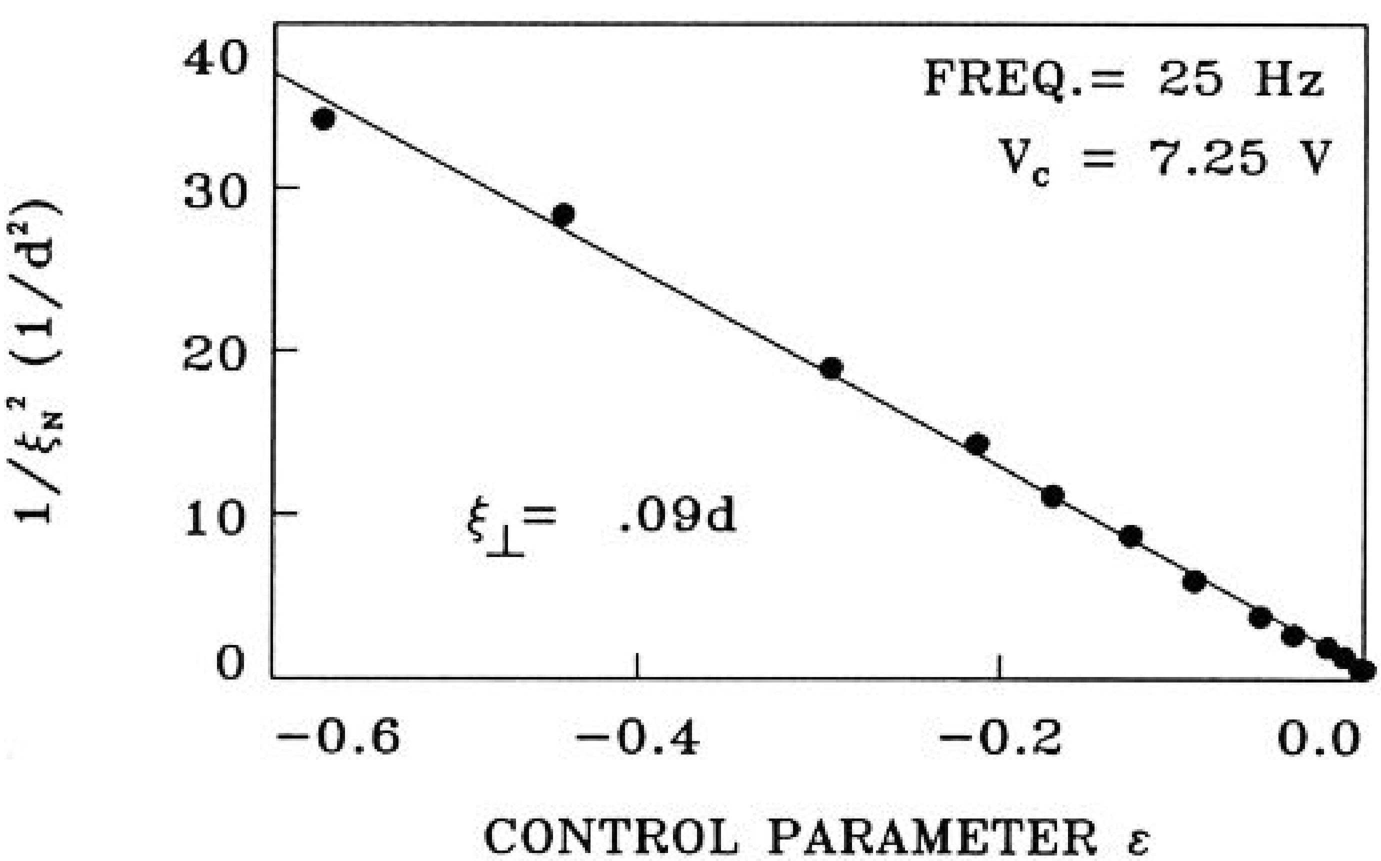

Figure 4: *The correlation length as measured perpendicular to the director orientation at a driving frequency of 25 Hz. The slope of the fitted straight line yields the value of* $\xi_{\perp} = 0.09d$.

5 Acknowledgements

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References

[1] E. Bodenschatz, W. Zimmermann, and L. Kramer, J. Phys. France **49** (1988) 1875.

[2] I. Rehberg, S. Rasenat, M. de la Torre Juárez, W. Schöpf, F. Hörner, G. Ahlers, and H.R. Brand, Phys. Rev. Lett. **67** (1991) 596.

[3] I. Rehberg, F. Hörner, L. Chiran, H. Richter, B. L. Winkler, Phys. Rev. **A44** (1991).

[4] B.L. Winkler, W. Decker, H.Richter, and I. Rehberg, submitted to Physica D.

[5] I. Rehberg, F. Hörner, G. Hartung, J. Stat. Phys. **64** (1991) 1017.

[6] S. Rasenat, V. Steinberg, and I. Rehberg, Phys. Rev. **A42** (1990) 5998.