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Magnetic Faraday instability

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Abstract. – In a magnetic fluid, parametrically driven surface waves can be excited by an external oscillating magnetic field. A static magnetic field changes the restoring forces and damping coefficients of the various surface waves. This property enables the excitation of both subharmonic and harmonic responses of the standing waves.

Introduction. – "Most fluids, if not all, may be used to produce these crispations, but some with particular advantages; alcohol, oil of turpentine, white of egg, ink and milk produce them". What Michael Faraday describes here is the parametric excitation of surface waves, nowadays known as the Faraday instability [1]. Today it is a popular experimental system for the study of parametrically excited instabilities, pattern formation and spatio-temporal chaos [2]. Following Faraday's suggestion we use a fluid which was not available to him: a superparamagnetic fluid, best popularly known as ferrofluid or magnetic fluid. Its particular advantage is that the properties of the fluid can be tuned in a wide range by an external magnetic field, and that Faraday's parametric excitation can be achieved by temporal modulation of that magnetic field, *i.e.* without a mechanical driving. The mechanism of the parametric driving is however very similar to the mechanic case: the spontaneously created surface deformation of a standing wave is amplified by the concentration of the magnetic field in the neighborhood of a wave crest (demagnetization). This amplification takes place twice during the oscillation cycle of the standing wave: peaks which are half a wavelength apart are amplified at time differences of half a driving period T_D.

The idea to excite surface waves magnetically in a ferrofluid is fairly similar to the electric excitation of dielectric fluids [3] and thus not new [4, 5]. In those experiments a magnetic field perpendicular to the surface of the fluid was used. Related to this idea is the excitation of surface waves with a horizontal magnetic field, which however leads to oscillations not described by the Mathieu equation [6]. In the experiment presented in this paper, we use

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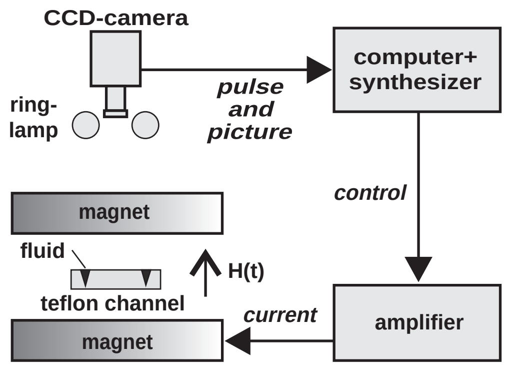

Fig. 1. – Experimental setup.

a steady magnetic field to tune the fluid parameters, and an oscillating field perpendicular to the surface to produce a parametric excitation of standing surface waves. With this magnetic excitation we are able to obtain the classical subharmonic response, and to measure its threshold curve. More importantly, we succeed in measuring for the first time threshold values for higher resonance tongues, which are not easily accessible for ordinary liquids [7]. This is possible because magnetic fluids have a particular advantage: They allow for a tuning of the restoring force of the surface waves by means of a steady external magnetic field. In particular, surface waves with the critical wave number of the Rosensweig instability [8] have zero frequency and are thus undamped at the critcal field H_c for the onset of the instability.

Experimental setup. – The experimental setup is shown in fig. 1. The magnetic fluids are poured into a V-shaped circular Teflon channel of 60 mm diameter [9]. The cylindrical geometry prevents disturbances by meniscus-induced surface waves, and allows an evaluation of the wave number by means of a Fast Fourier Transformation, FFT [10]. The channel has a depth of 5 mm and upper width of 4 mm which is smaller than the typical wavelengths in the experiment. The upper part of the channel has a slope of 15° in order to damp out surface waves in radial direction.

We use the commercially available magnetic fluids EMG 705 and EMG 909 (Ferrofluidics). The properties of the water-based fluid EMG 705 are: density $\rho = 1190\ \mathrm{kg\,m^{-3}}$, surface tension $\sigma = 4.75 \cdot 10^{-2}\ \mathrm{kg\,s^{-2}}$, initial magnetic permeability $\mu = 1.56$, magnetic saturation $M_\mathrm{S} = 1.6 \cdot 10^4\ \mathrm{A\,m^{-1}}$, dynamic viscosity $\eta = 5 \cdot 10^{-3}\ \mathrm{N\,s\,m^{-2}}$, yielding a critical field for the onset of the Rosensweig instability $H_\mathrm{C} = 9.9 \cdot 10^3\ \mathrm{A\,m^{-1}}$. The properties of the oil-based fluid EMG 909 are: density $\rho = 1020\ \mathrm{kg\,m^{-3}}$, surface tension $\sigma = 2.65 \cdot 10^{-2}\ \mathrm{kg\,s^{-2}}$, initial magnetic permeability $\mu = 1.8$, magnetic saturation $M_\mathrm{S} = 1.6 \cdot 10^4\ \mathrm{A\,m^{-1}}$, dynamic viscosity $\eta = 6 \cdot 10^{-3}\ \mathrm{N\,s\,m^{-2}}$, and the critical field for the onset of the Rosensweig instability is $H_\mathrm{C} = 7.9 \cdot 10^3\ \mathrm{A\,m^{-1}}$.

The channel is placed in the center of a pair of Helmholtz coils (Oswald-Magnetfeldtechnik), with an inner diameter of 40 cm. One coil consist of 474 windings of flat copper wire with a width of 4.5 mm and thickness 2.5 mm. A current of about 5 A is then sufficient to produce the magnetic field of about $8 \cdot 10^3\ \mathrm{Am^{-1}}$ used in this experiment. The static field is monitored by means of a hall probe (Group 3 DTM-141 Digital Teslameter) located near the surface of the channel.

The light from a tungsten bulb, placed in the center of the top of the channel, is reflected at the surface of the fluid and directed towards the CCD-camera placed 80 cm above the center

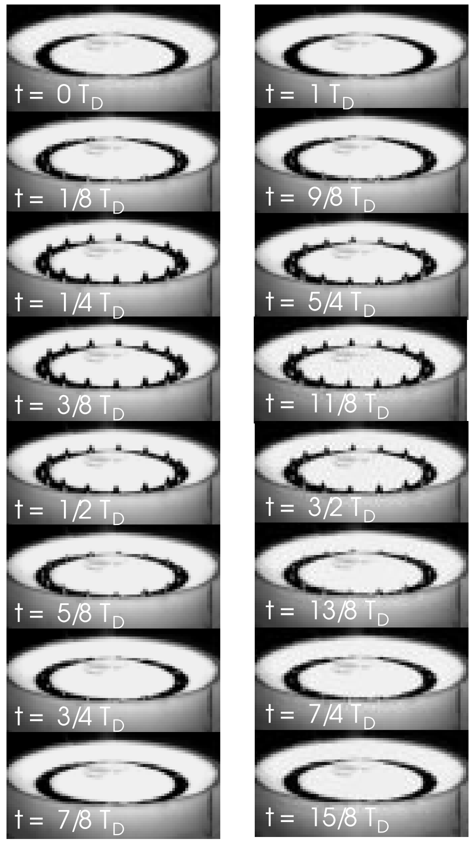

Fig. 2. – 16 snapshots of a parametrically excited standing wave at $T_{\mathrm{D}} = 0.1$ s, $H_0 = 0.95 H_{\mathrm{C}}$, $\Delta H = 0.21 H_{\mathrm{C}}$, $H_{\mathrm{C}} = 6.8 \cdot 10^3$ A m^{-1} during a time of $2T_{\mathrm{D}}$. The phases are indicated in the left-bottom corner of each picture. The fluid is EMG 909.

of the channel. The camera (Philips LDH 0600/00) works in the interlaced mode at 50 Hz using an exposure time of 40 ms.

The analysis of the images and the control of the experiment is done with a 90 MHz Pentium-PC, equipped with a 512×512 8-bit frame grabber (Data Translation DT2853), a programmable counter (8253) located on a multifunction I/O-board (Meilhaus ME-30), and a synthesizer-board (WSB-10). The counters are used to keep track of the pacing frequency of the synthesizer-board. Their output is used to trigger the camera in any desired phase of the driving oscillation of the magnetic field. This phase-locked technique between the driving and the sampling ensures a jitter-free measurement of the amplitude. By keeping track of the synthesizer pace, the computer moreover manages the writing of the data into the synthesizer memory at times when no conflict with the DA-converter arises. This allows for smooth switching of the amplitude of the AC-component of the magnetic field [11].

The wave signal is amplified by a linear amplifier (fug NLN 5200 M-260). The resulting driving magnetic field is $H(t) = H_0 + \Delta H \sin 2\pi t f_{\mathrm{D}}$, with H_0 as the static and ΔH as the oscillating part of the magnetic field; f_{D} is the driving frequency and $H_0 > \Delta H$.

Experimental results. – Using only a static magnetic field, H_0, leads to the Rosensweig instability at the threshold H_{C}. In fig. 2 we demonstrate the existence of parametrically excited

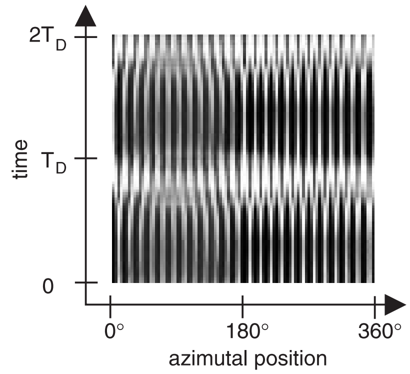

Fig. 3. – Space-time plot for $f_\mathrm{D} = 15$ Hz, $H_0 = 0.98H_\mathrm{C}$, $\Delta H = 0.24H_\mathrm{C}$, $H_\mathrm{C} = 8.6 \cdot 10^3$ A m^{-1}. The fluid is EMG 909.

surface waves for $H_0 < H_\mathrm{C}$: when increasing the oscillating part, ΔH, of the driving above a certain threshold the flat surface of the magnetic fluid becomes unstable and a standing wave ensues, as illustrated by a series of snapshots of the behavior of the surface of the fluid during the time interval $t = 2T_\mathrm{D}$. Here the oil-based fluid EMG 909 is excited at $f_\mathrm{D} = 10$ Hz, $H_0 = 0.95H_\mathrm{C}$ and $\Delta H = 0.21H_\mathrm{C}$. Each of the 16 pictures in fig. 2 corresponds to a certain phase which is labeled in the left-bottom corner of each picture. The first driving period, T_D, is seen in the left column, the following second period in the right. Starting at a nearly flat surface at $t = 0$, wave crests with wavelength λ grow to a maximum amplitude at $t = 3/8T_\mathrm{D}$ and decrease to a flat surface at $t = 7/8T_\mathrm{D}$. At the next driving period the wave crests grow to a maximum amplitude at $t = 11/8T_\mathrm{D}$ with a $\lambda/2$ shift of the position of the peaks with respect to the preceding period. This clearly indicates that we observe a standing wave with the response period $T_\mathrm{resp} = 2T_\mathrm{D}$.

For a more detailed characterization of the spatio-temporal behavior, we measure the light intensity reflected from the surface along the annulus as described in detail in [9]. The resulting spatio-temporal representation is shown in fig. 3. The dark zones correspond to wave troughs and a high light intensity represents crests of the standing waves. A time interval of two driving periods is shown and the standing-wave character with a period of $2T_\mathrm{D}$ is clearly visible.

In the following, we examine the stability of the flat surface in dependence on the driving frequencies, f_D, and we present measurements of the threshold for the onset of the Faraday instability. To do so, we fix the static part of the magnetic field and the driving frequency. The oscillation amplitude, ΔH, is increased until the flat surface becomes unstable. The criterion for instability is obtained from the FFT analysis of the light intensity measured along the channel: after switching to a new amplitude we wait 15 s, and if then the intensity of the peak in the power spectrum is larger than its mean value by more than a factor of five, we conclude that the instability threshold has been crossed. We pinpoint the threshold by means of binary interval search algorithm with an accuracy corresponding to one bit of the 12-bit synthesizer card, which corresponds to an accuracy better than one percent. The advantage of this procedure is that it works both for sub- and supercritical bifurcations of the surface instability. These threshold values are plotted for the oil-based fluid EMG 909 at $H_0 = 0.97H_\mathrm{C}$ as a function of the driving frequency in the lower part of fig. 4.

This clearly shows that the response with $2T_\mathrm{D}$ is dominant, a fact which can be explained on the basis of the theory presented in fig. 4 of ref. [5]. According to that theory, the half-frequency regime is indeed the dominant one, a behavior which is typical for the parametric driving of surface waves.

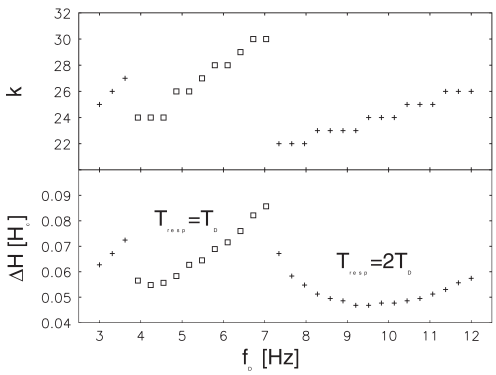

Fig. 4

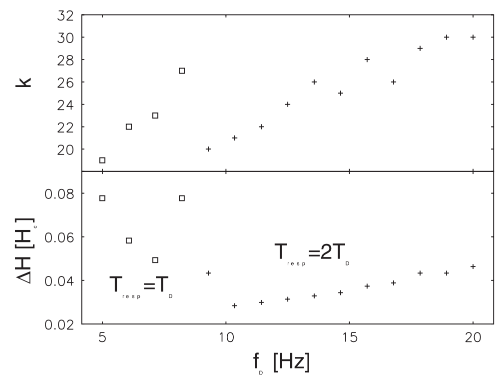

Fig. 5

Fig. 4. – At a fixed value of $H_0 = 0.97H_\mathrm{C}$ for each value of the driving frequency f_D the oscillating part is increased until the flat surface becomes unstable. While in the lower diagram these thresholds are indicated, in the upper diagram the wave numbers of the resulting standing wave are shown. The step $\Delta k = \mp 1$ corresponds to the destruction or creation of one wavelength. Squares mark the $1T_\mathrm{D}$-states and crosses the $2T_\mathrm{D}$-states. The fluid is EMG 909. $H_\mathrm{C} = 7.9 \cdot 10^3$ A m^{-1}, $k_\mathrm{C} = 25$.

Fig. 5. – At a fixed value of $H_0 = 0.93H_\mathrm{C}$ for each value of the driving frequency f_D the oscillating part is increased until the flat surface becomes unstable. While in the lower diagram these thresholds are indicated, in the upper diagram the wave numbers of the resulting standing wave are shown. Squares mark the $1T_\mathrm{D}$-states and crosses the $2T_\mathrm{D}$-states. The fluid is EMG 705. $H_\mathrm{C} = 9.9 \cdot 10^3$ A m^{-1}, $k_\mathrm{C} = 20$.

In addition, the FFT yields the wave number k of the surface wave which is presented in the upper part of fig. 4. This plot clearly indicates wave number jumps along the subharmonic and harmonic branches. The step $\Delta k = \mp 1$ corresponds to the destruction or creation of one wavelength. In principle one would expect to observe minima of the critical ΔH for a fixed value at k, as shown in fig. 20 of ref. [12]. The resolution in ΔH of our apparatus is not sufficiently fine for this purpose. Moreover a spatio-temporal spectral analysis allows for discrimination between harmonic and subharmonic responses: for frequencies $f_\mathrm{D} > 7$ Hz the surface responds with $T_\mathrm{resp} = 2T_\mathrm{D}$.

We have performed similar measurements with a less viscous fluid, namely the water-based EMG 705. The results are presented in fig. 5. We find the two modes $T_\mathrm{resp} = T_\mathrm{D}$ and $T_\mathrm{resp} = 2T_\mathrm{D}$. Note that around the minimum at $f_\mathrm{D} = 10.4$ Hz of the subharmonic tongue, *i.e.* the line in the f_D-ΔH plane where the instability sets in with a frequency of $2T_\mathrm{D}$, the total magnetic field $|H_0| + |\Delta H|$ is well below the critical value H_C of the Rosensweig instability. This means that the parametric driving is more efficient in this fluid, which is easily explained by its smaller viscosity.

The threshold of the amplitude ΔH in fig. 4 shows a minimum at about $f_\mathrm{D} = 9.3$ Hz. For even lower frequencies the harmonic response has a lower stability threshold than the subharmonic response. To some extend, this might be due to the fact that for increasing wavelengths the dissipation in the bottom layer is enhanced [7]. More importantly, this might be a manifestation of the advent of the Rosensweig instability which occurs at about $k_\mathrm{C} = 25$. This damping favors the higher resonance tongue with $T_\mathrm{resp} = T_\mathrm{D}$ which has a larger wave number k. The harmonic resonance tongue is observed in the range between 3.6 Hz $< f_\mathrm{D} <$ 7.0 Hz. For driving frequencies lower than 3.6 Hz the first instability is again the subharmonic response with $T_\mathrm{resp} = 2T_\mathrm{D}$. The qualitative behavior of the threshold graphs $\Delta H(f_\mathrm{D})$ and $k(f_\mathrm{D})$ are similar to the numerically obtained linear threshold amplitude and critical wave number shown in [13]. Nevertheless, there is a striking difference when comparing our data in fig. 4 with fig. 3 of ref. [13]: the linear theory for mechanically excited parametric resonance predicts $T_\mathrm{resp} = 2/3T_\mathrm{D}$, while we measure a response period $T_\mathrm{resp} = 2T_\mathrm{D}$. We believe that this is due to a nonlinear effect, the $2/3T_\mathrm{D}$-mode is presumably not stable. Our measurement procedure only allows for a determination of the linear threshold, while the nonlinear aspects of magnetically driven oscillations are many and various and are presented elsewhere [11].

Summary and conclusion. – In summary, we have obtained parametrically driven higher-resonance tongues by means of tuning the viscous damping of a magnetic fluid with an external magnetic field. This is made possible by a specific peculiarity of magnetic fluids, namely a very weak damping of surface waves when the wave number is close to the critical wave number of the Rosensweig instability. One might expect that the interaction between the $2T$ with the $1T$ resonance leads to complicated spatial patterns, a supposition which is currently under investigation.

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