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# On the consistency of the standard model for magnetoviscosity in an alternating magnetic field

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**Abstract**

Recently, a model for the negative viscosity effect was proposed in form of a set of ordinary differential equations (ODEs). We transform this set of ODEs in order to simplify its numerical integration. We investigate the limits of these equations for the case of small frequencies and small amplitudes of the exciting magnetic field and compare the numeric results with the analytical formulas. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The increase of the viscosity of a magnetic fluid under the influence of a magnetic field has been known since the measurements of McTague in 1969 [1]. A theoretical treatment for dilute dispersions and accounting for the Brownian rotational motion with their relaxation time $\tau_B$ was first presented by Shliomis in 1972 and later refined in a review article in 1988 [2]. For the reduced viscosity $\eta_r$ he obtained

$$\eta_r = \frac{\Delta\eta}{\eta} = \frac{3}{2}\,\phi_h \frac{\xi L^2}{\xi - L}\sin^2\beta. \qquad (1)$$

Here $\phi_h$ denotes the volume fraction of the hydrodynamic effective volume of the particles. The Langevin parameter $\xi = mH/kT$ characterizes the ratio between the energy of the magnetic moment $m$ of the particle in the magnetic field $H$ and its thermal energy $kT$. The Langevin function $L$ is given by $L(\xi) = \coth(\xi) - 1/\xi$. The angle between the direction of the field $H$ and the local angular velocity of the particle is measured by $\beta$.

Later Shliomis and Morozov [3] investigated the additional viscosity generated in a flow with vorticity due to an alternating magnetic field. They postulated a negative viscosity contribution ($\Delta\eta < 0$) for a certain range of the frequency and field strength of the applied field. The negative viscosity effect has recently been detected by Bacri et al. [4]. We present an investigation of the model proposed

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in Ref. [4] in order to simplify the quantitative comparison between theory and experiment.

## 2. Model equations and analytical results

The uncoupled system of ordinary differential equations (ODEs) suggested in Ref. [4] is

$$\tau_B \frac{d\xi_e}{dt} = -\left(\frac{d\ln L_e}{d\xi_e}\right)^{-1}\left(1 - \frac{\xi_0}{\xi_e}\cos(\omega t)\right), \tag{2}$$

$$\tau_B \frac{dF}{dt} = 1 - \frac{1}{2}\left(\frac{1}{L_e} - \frac{1}{\xi_e}\right)\xi_0 F \cos(\omega t). \tag{3}$$

Here $\xi_e = mH/kT$ denotes the effective field, $L_e = L(\xi_e)$, and $\xi_0$ is defined over the amplitude of the external magnetic field. The product $F\tau_B \boldsymbol{\Omega} \times \boldsymbol{H}/H$ determines the ratio between the magnetization parallel and perpendicular to the applied field, where $\boldsymbol{\Omega}$ is the averaged angular velocity of the liquid. Solving this equations with the initial conditions $\xi_e(0) = \xi_0$, $F(0) \approx 0$ up to relaxation and averaging the relaxed solution over one period of the external magnetic field $\xi = \xi_0 \cos(\omega t)$ the function

$$g(\omega\tau_B, \xi_0) = \tfrac{1}{2}\,\xi_0\,\overline{\cos(\omega t) L(\xi_e) F(\xi_e)}, \tag{4}$$

was obtained. It determines the reduced viscosity $\eta_r$ by means of the equation

$$\eta_r = \eta_r(\omega\tau_B, \xi_0) = \tfrac{3}{2}\,\phi_h g(\omega\tau_B, \xi_0). \tag{5}$$

(i) *Transformation:* Solving the system (2,3) becomes complicated, because the poles of $F$ at $\xi_e = 0$ have to be taken into account. We prefer to solve a mathematical equivalent system (2,6). Introducing $A = LF$ yields

$$\tau_B \frac{dA}{dt} = L_e + A\left(\xi_0 \frac{3L_e - \xi_e}{2\xi_e L_e}\cos(\omega t) - 1\right). \tag{6}$$

In contrast to Eq. (3), Eq. (6) has no poles. The function $g$ is now be determined by

$$g(\omega\tau_B, \xi_0) = \frac{1}{2}\frac{\xi_0\omega}{2\pi}\lim_{t\to\infty}\int_{t=x}^{t=x+2\pi/\omega} \cos(\omega t) A(t)\,dt. \tag{7}$$

The transformed initial values are $\xi_e(0) = \xi_0$ and $A(0) = 0$.

(ii) *Limit for zero frequency:* For the limit of very small frequencies of the alternating magnetic field the system of ODEs (2,6) degenerates to an algebraic system. After the transformation $t' = \omega t$ and inserting $\omega = 0$ in the system we obtain

$$0 = -\left(\frac{d\ln L_e}{d\xi_e}\right)^{-1}\left(1 - \frac{\xi_0}{\xi_e}\cos(t')\right), \tag{8}$$

$$0 = L_e + A\left(\xi_0 \frac{3L_e - \xi_e}{2\xi_e L_e}\cos(t') - 1\right). \tag{9}$$

The solution of this uncoupled system yields $\xi_e = \xi_0 \cos(t')$ for the first equation. With this solution one obtains $A(t') = 2L_e^2/(\xi_e - L_e)$. After inserting $A(t')$ in Eq. (7) we get

$$g(0, \xi_0) = \frac{1}{T}\int_0^T \frac{\xi_e(t') L^2(\xi_e(t'))}{\xi_e(t') - L(\xi_e(t'))}\,dt', \tag{10}$$

which is the limiting solution of the system (2,6,7). It is obvious, that Eq. (10) is the mean over one period of the static formula (1) using $\xi = \xi_0 \cos(t')$.

(iii) *Limit of weak excitation:* The linearisation of the Eqs. (2) and (6) in the case $\xi_0 \ll 1$ yields the equations

$$\tau_B \frac{d\xi_e}{dt} = -\xi_e + \xi_0 \cos(\omega t) \tag{11}$$

$$\tau_B \frac{dA}{dt} = -A + \frac{1}{3}\xi_e(t). \tag{12}$$

These are two linear ODEs, which can be solved with standard mathematical methods. Inserting the solution for Eq. (11)

$$\xi_e(t) = \xi_0 \frac{e^{-t/\tau_B}\omega^2\tau_B^2 + \cos(\omega t) + \omega\tau_B \sin(\omega t)}{1 + \omega^2\tau_B^2},$$

in Eq. (12) yields the function

$$A(t) = A_0 e^{-t/\tau_B} + \frac{\xi_0}{3(1+\omega^2\tau_B^2)^2}$$
$$\times \{e^{-t/\tau_B}(\omega^4\tau_B^3 t + \omega^2\tau_B^2 + \omega^2\tau_B t - 1)$$
$$+ \omega\tau_B \sin(\omega\tau_B) - \omega^2\tau_B^2 \cos(\omega\tau_B)\}, \tag{13}$$

as solution $A(t)$ of the linearized Eq. (12). After neglecting all exponential decaying terms in Eq. (13), using the initial value $A(0) = 0$ and inserting in Eq. (7) one obtains

$$g(\omega\tau_B,\xi_0) = \frac{1}{12}\xi_0^2\frac{1-\omega^2\tau_B^2}{(1+\omega^2\tau_B^2)^2}\,. \tag{14}$$

This is in accordance with the formula presented by Shliomis et al. in Ref. [3].

## 3. Numerical Results

For the numerical solution of the system (2,6,7) we use a Runge–Kutta method of eighth order with a combined error predictor of fifth and third order as proposed by Hairer et al. in Ref. [5]. After solving the first two equations up to relaxation all three equations are solved over one period.

Because of the singular perturbation of the problem at $\omega\tau_B = 0$ the ODEs were only solved for values of $\omega\tau_B > 0.001$. For smaller values the approximation $g(\omega\tau_B, \xi) = (1 - 3\alpha\omega^2\tau_B^2)g(0, \xi_0)$ was used, which was obtained from a Taylor series expansion of the weak field limit Eq. (14) for small $\omega\tau_B$. The corrector value $\alpha$ is introduced to force a smooth solution between values $g_{\mathrm{ode}}$ calculated from the ODEs and values from the approximation above. Its value is determined by $\alpha = [1 - g_{\mathrm{ode}}(0.001,\xi)/g(0,\xi)]/(3\times 0.001^2)$.

In Fig. 1 the results of the integration of the ODEs for different values of $\omega\tau_B$ are plotted versus the amplitude of the external nondimensional field $\xi_0$. The values for $\omega\tau_B$ are 0.0, 0.001, 0.5, 1.0, 2.0 and 5.0 and marked with open squares, solid circles, open triangles, solid squares, solid triangles and open circles, respectively. The curve for $\omega\tau_B = 0$ and $\omega\tau_B = 0.001$ can not be distinguished by the naked eye. The curve for $\omega\tau_B = 5.0$ falls below $-1$. In this range also the total viscosity is negative. As a result a spontaneous flow instability is expected. Fig. 2 displays the isolines of the function $g$ in dependence of $\omega\tau_B$ (abscissa) and $\xi_0$ (ordinate) around $\omega\tau_B = 1$. Note, the changing slope of the isoline $g = 0$ in Fig. 2.

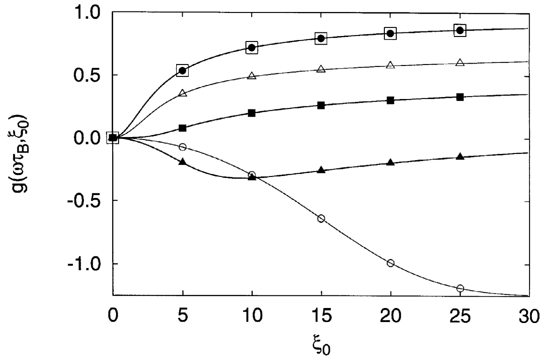


Fig. 1. $g(\omega\tau_B,\xi_0)$ for different values of $\omega\tau_B$: 0.0 (open squares), 0.001 (solid circles), 0.5 (open triangles), 1.0 (solid squares), 2.0 (solid triangles) and 5.0 (open circles) as function of the nondimensional field.

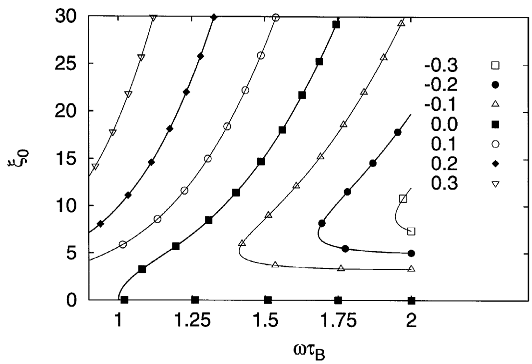


Fig. 2. Contour plot of $g(\omega\tau_B,\xi_0)$ near $\omega\tau_B = 1$.

We have presented a transformation which allows the exact solution of the standard model for magnetoviscosity in an alternating magnetic field. This is of crucial importance for fitting the model to experimental data [6].

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