Numerical investigation of the Melt Channel Instability

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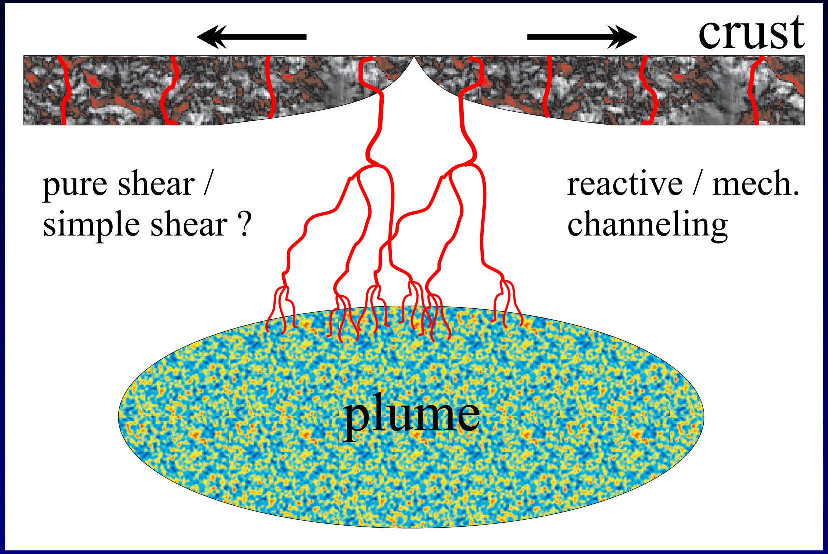
Topics

- Introduction
- Code, Boundary Conditions and Equations
- The Melt Channel Instability
- Results
- Conclusions

Open questions



- Could channeling occur in a matrix under a given stress field?
- Which orientation does it take?
- Is it possible to achieve a focussing of melt towards a Mid Oceanic Ridge (MOR)?
- Does applying different stress fields influence the formation/orientation of channels?

Code, boundary conditions and equations

- 2D-Finite-Difference-Code
- It solves the relevant fluid-dynamic equations (conservation of mass and momentum, according to McKenzie (84)) for melt and matrix respectively
- Solving: Stream function formulation and the Compaction Boussinesq Approximation for the momentum equation
- Simple Shear with no slip at all boundaries
- Pure Shear with free slip at all boundaries
- Simple and Pure Shear with no slip at all boundaries

Non-dimensionalization

Melt Rayleigh Number

$$Rm = \frac{\delta\rho\, g\, h^3}{\eta_0\, \kappa}$$

Melt "Retention" Number

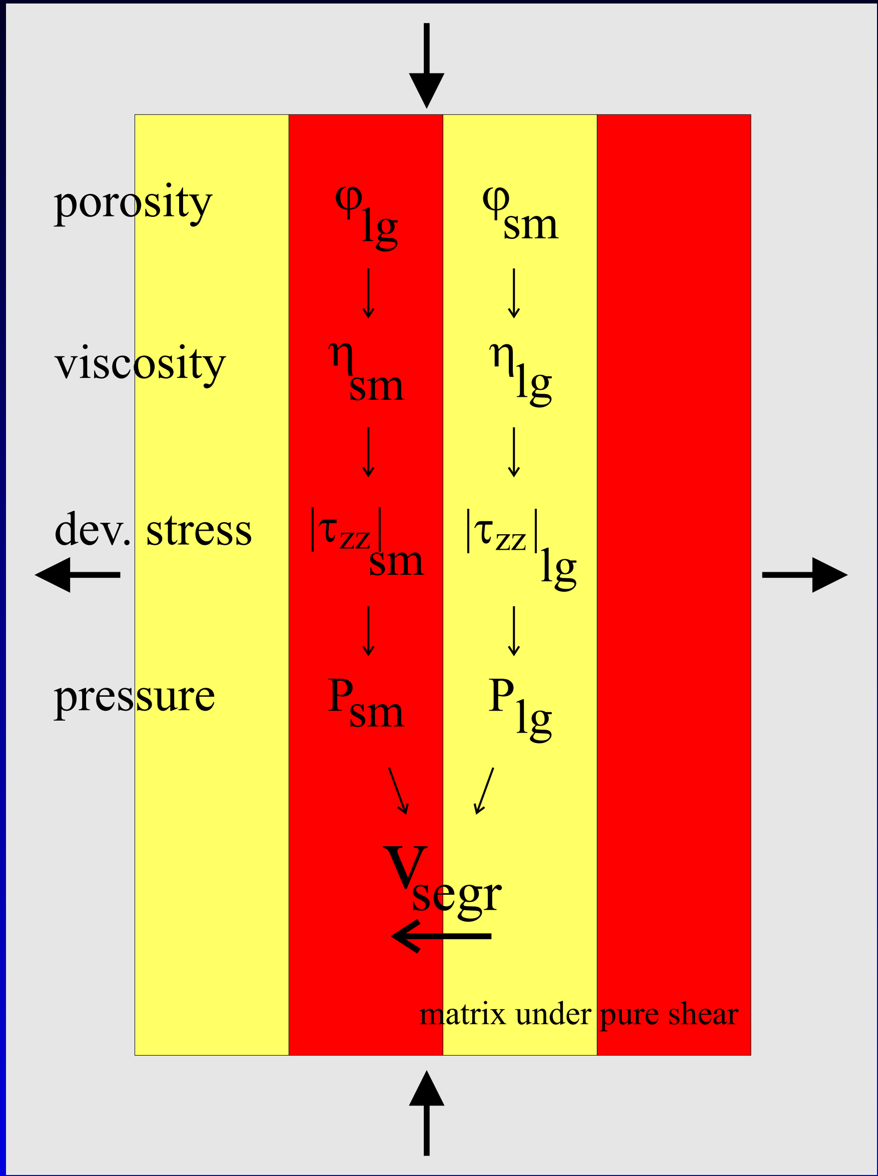
$$Rtn = \frac{\eta_f\, h^2}{\eta_0\, k_0} \qquad\qquad k_\varphi = \frac{a^2}{b}\, \varphi^n = k_0\, \varphi^n$$

Dimless. velocity

$$u' = u\, \frac{h}{\kappa} \overset{ps}{=} \dot{\varepsilon}\, \frac{h^2}{\kappa}$$

$\delta\rho$ density contrast, g gravity acceleration, h box height, κ diffusion constant, η_0 scaling viscosity, η_f fluid viscosity, a, b, n geom. factor, φ porosity, u velocity, $\dot{\varepsilon}$ strain rate

The Melt Channel Instability



The non. dim. growth rate α'

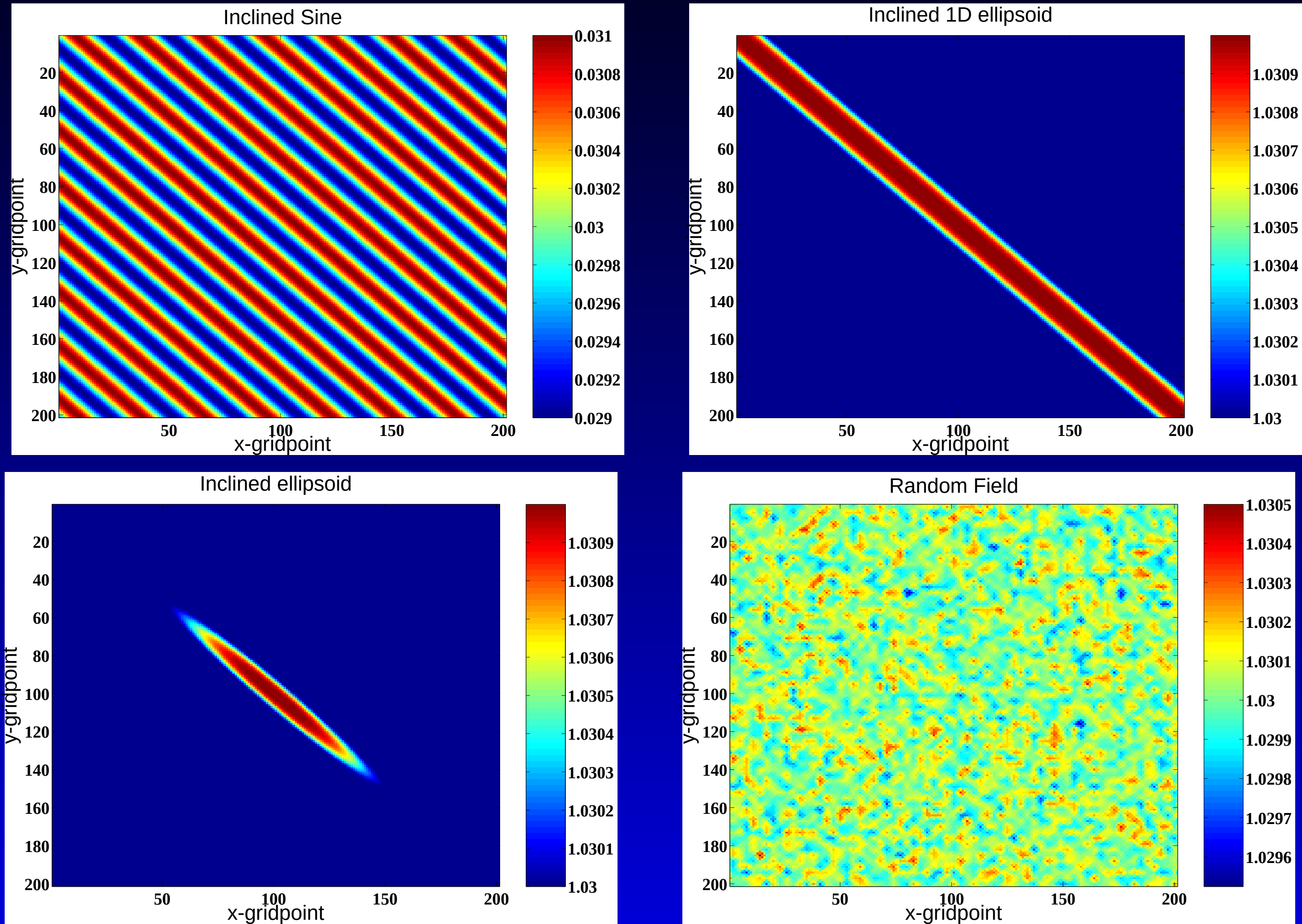
From a linear stability analysis of the governing equations, $\alpha'\,(non.dim.)$ comes out as

$$\alpha' = \frac{\alpha}{\dot{\varepsilon}_0} = \frac{2\,(1-\varphi_0)\,\frac{k_{\varphi_0}}{\eta_f}\,\eta_{s_0}\,a_1\,k^2}{1+\left(\eta_{b_0}+\frac{4}{3}\,\eta_{s_0}\right)\frac{k_{\varphi_0}}{\eta_f}\,k^2} \sim O\,(a_1)$$

$\eta_{\underline{s}hear} = \eta_0 e^{-a_1\varphi}$ after Kohlstedt 2000

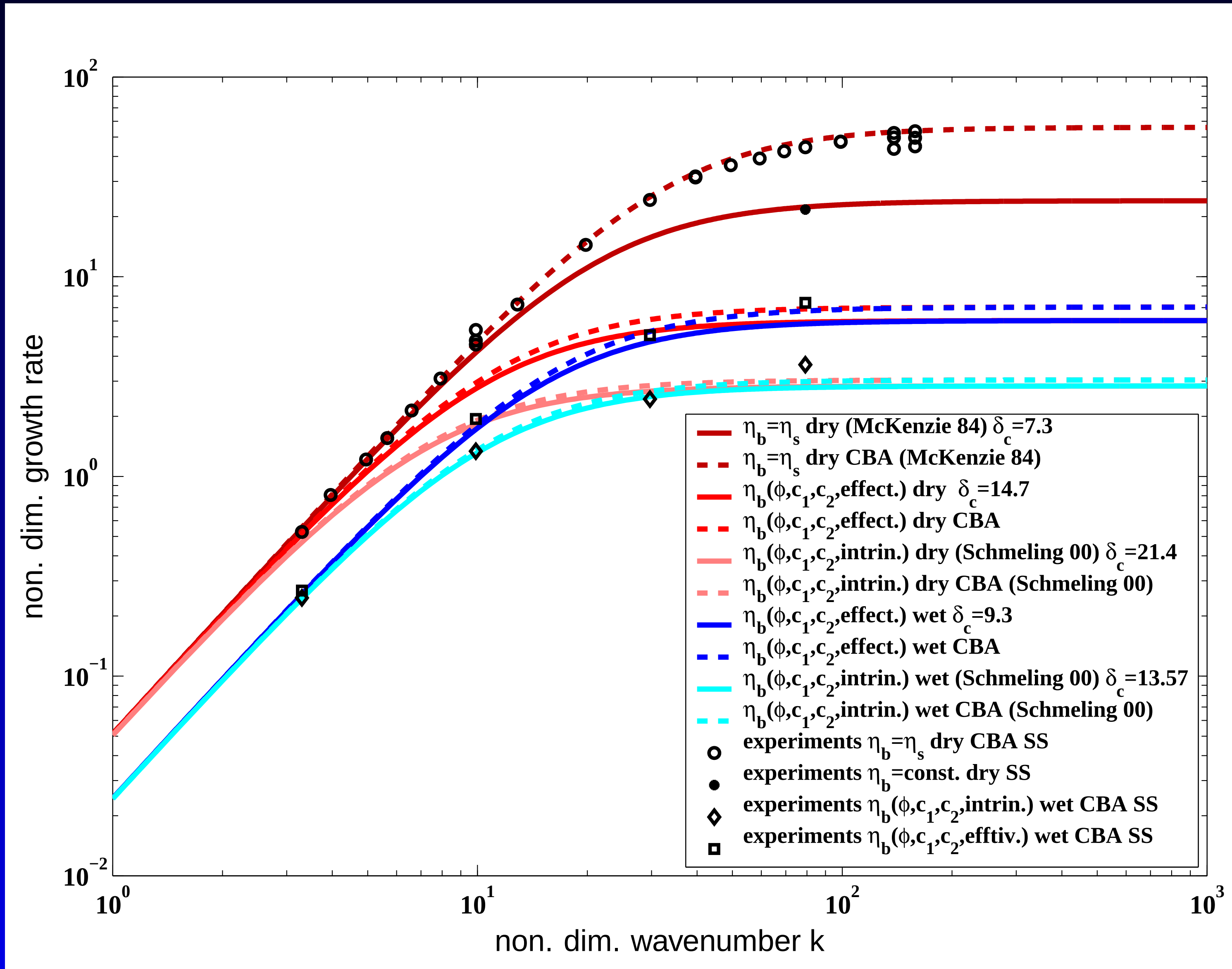
$\eta_{\underline{b}ulk} = \eta_0 c_1 \dfrac{c_2-\varphi}{\varphi}$ after Schmeling 2000

Initial field

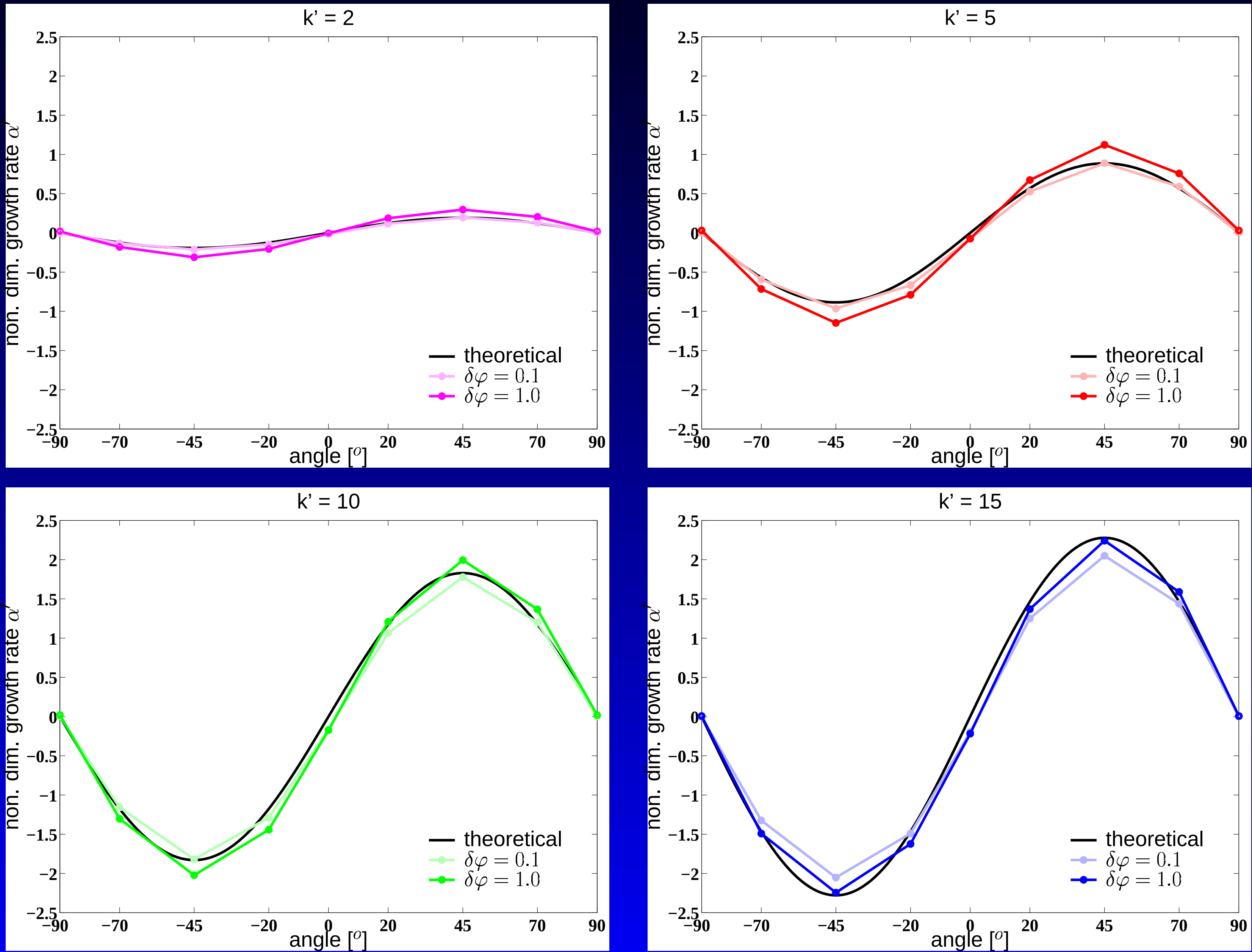


Coordinate system: 0^o denotes the vertical, CCW

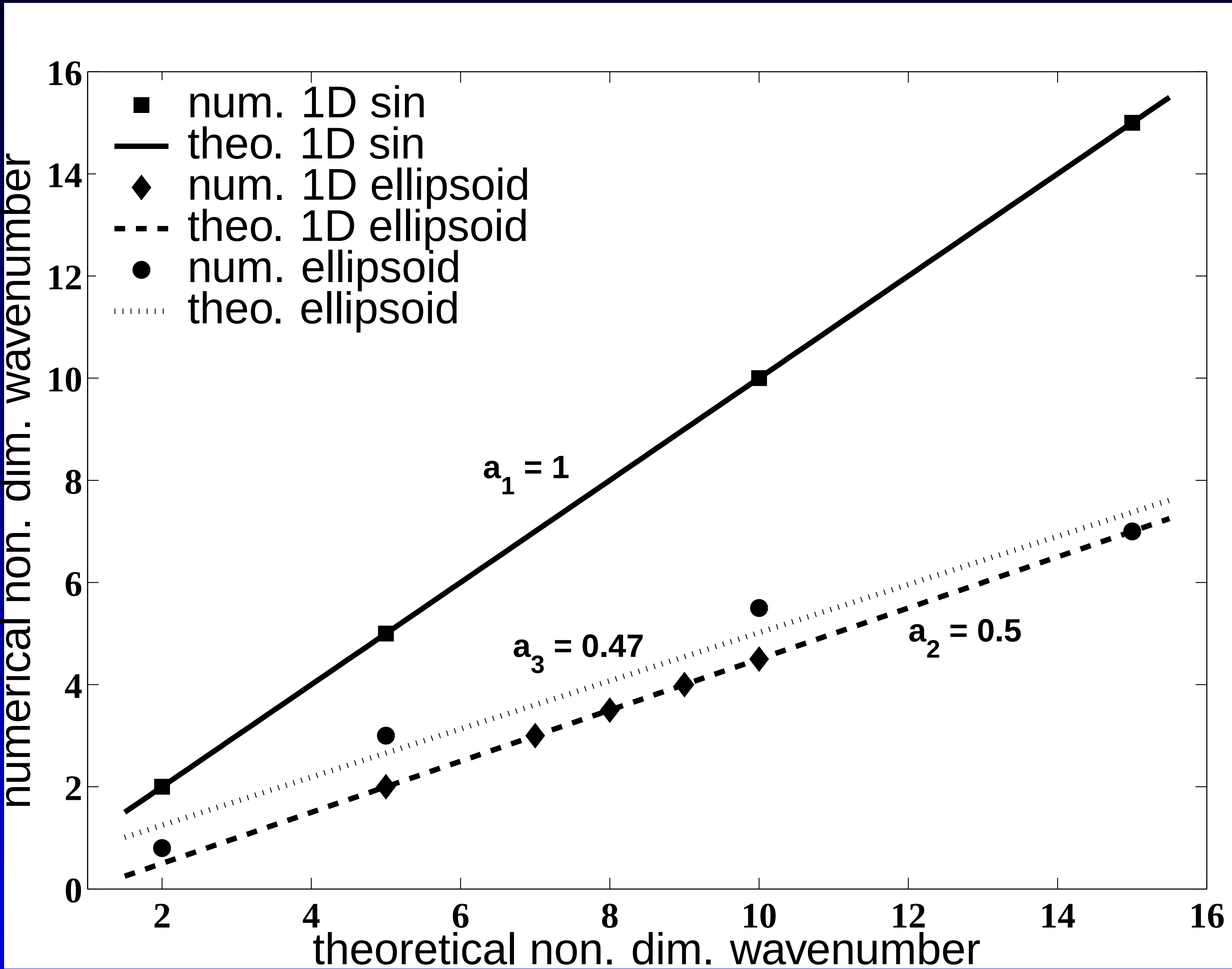
Does FDCON reproduce the theoretical α'



1D Sine $\Gamma' = 1 \; Rtn = 0.5 \; \dot{\varepsilon} = 1e-10$

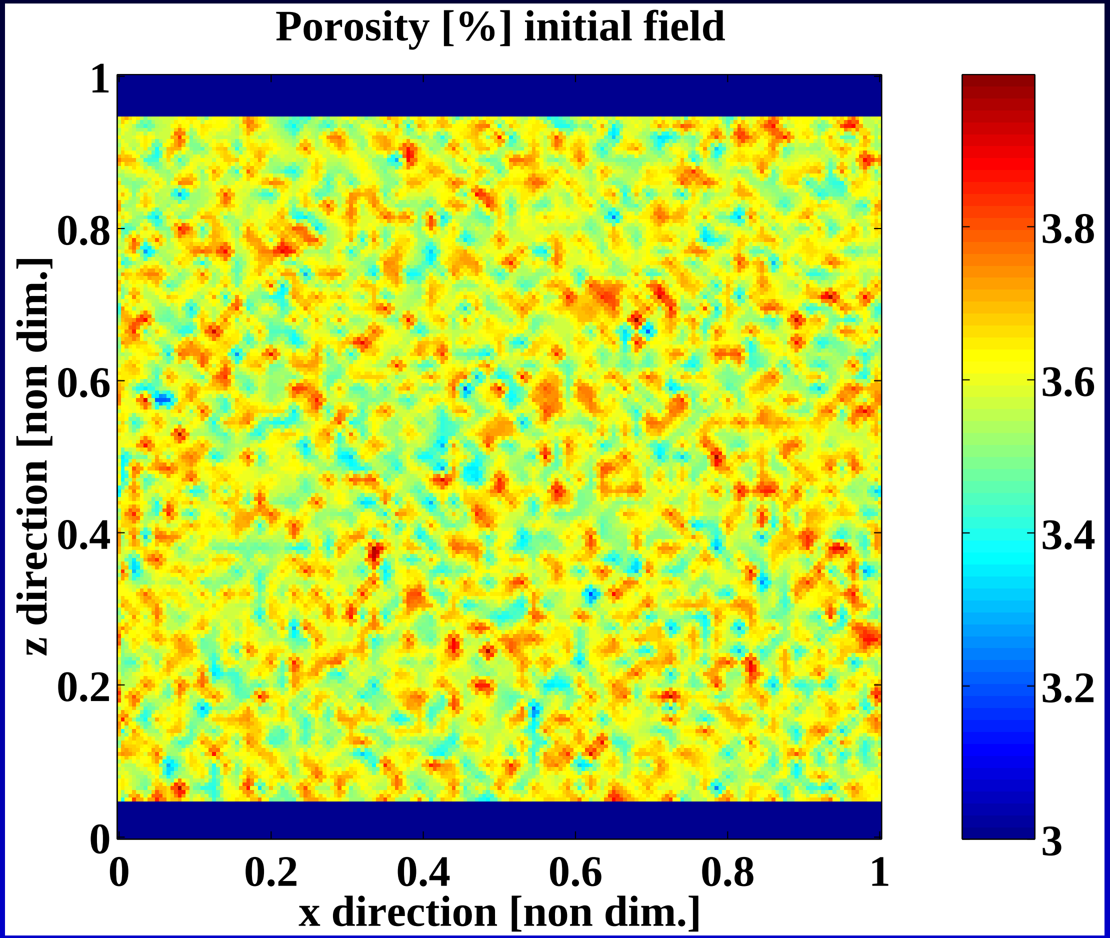


The effective wavenumber for simple shear



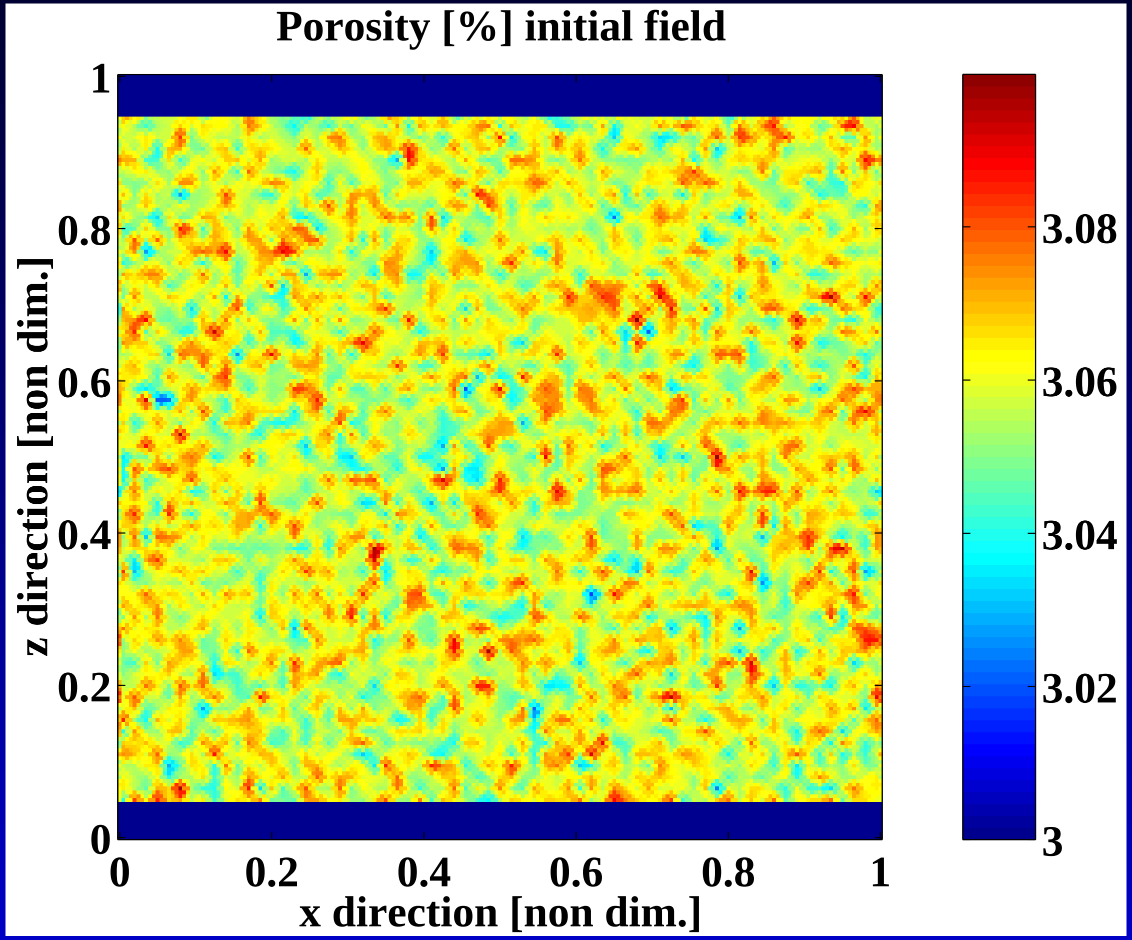
Superposition of pure and simple shear

$\frac{\dot{\varepsilon}_{ps}}{\dot{\varepsilon}_{ss}} = 2$ Rtn = 0.5 Rm = 0.0 $\varphi_{\sigma_{max}} = 13^{\circ}$ $\varepsilon \approx 1.5$



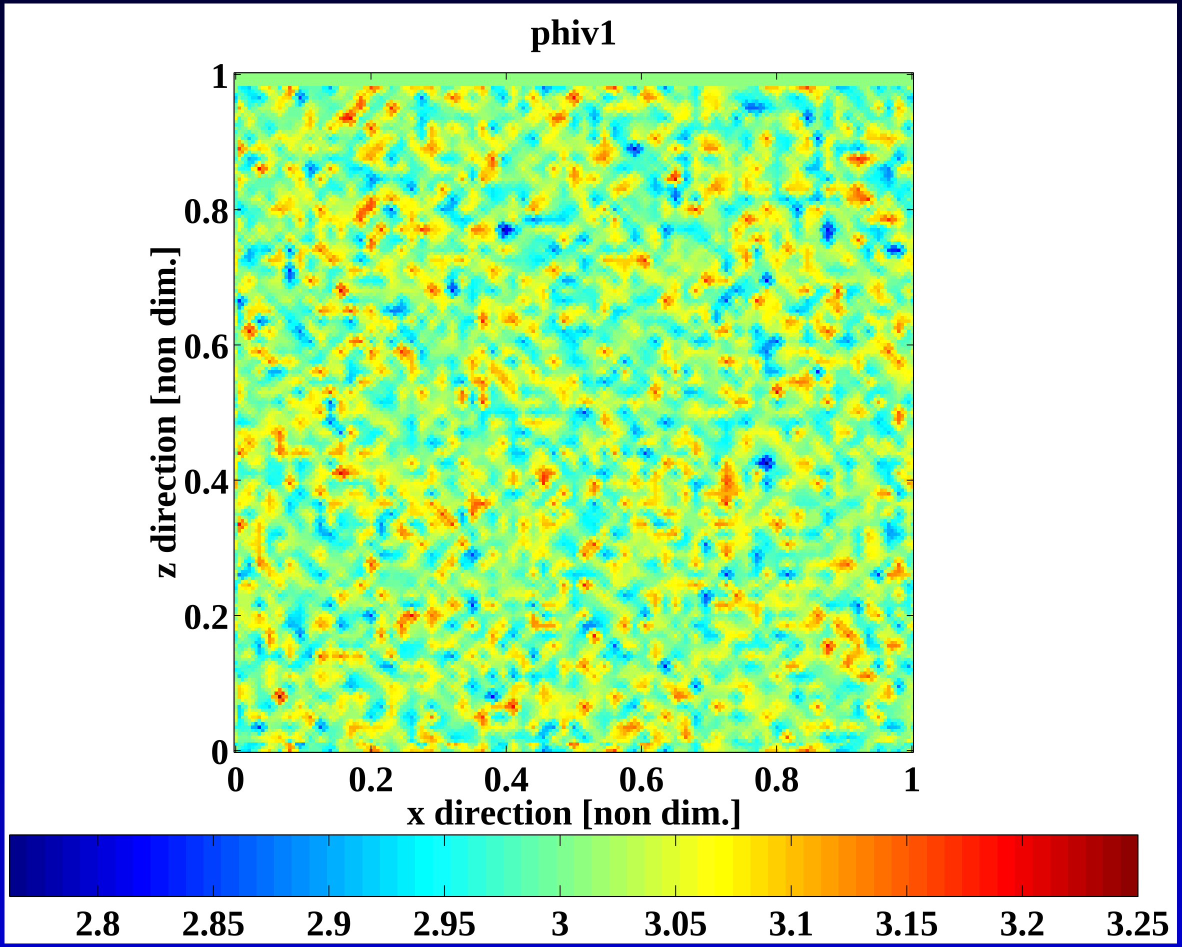
Simple shear

Rtn = 0.5   Rm = 0.0 $\varphi_{\sigma_{max}} = 45°$ $\varepsilon \approx 4$



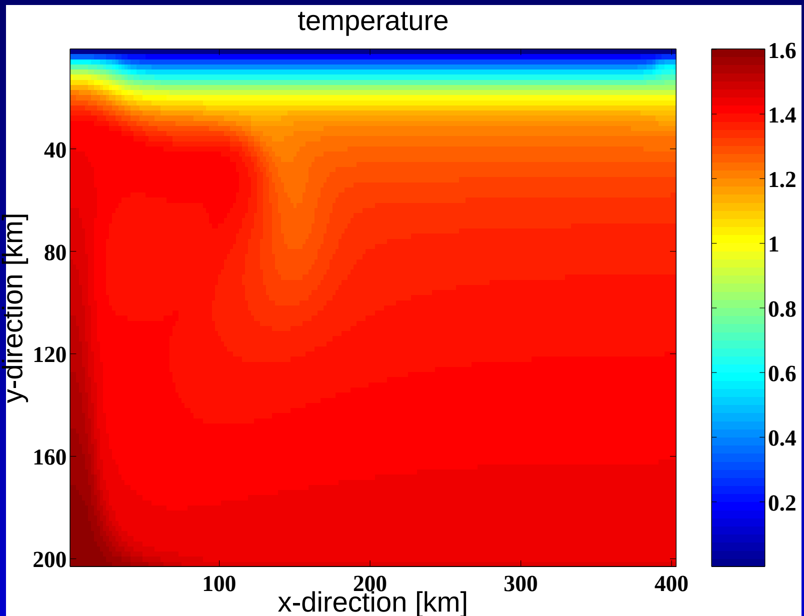
Simple shear with buoyancy

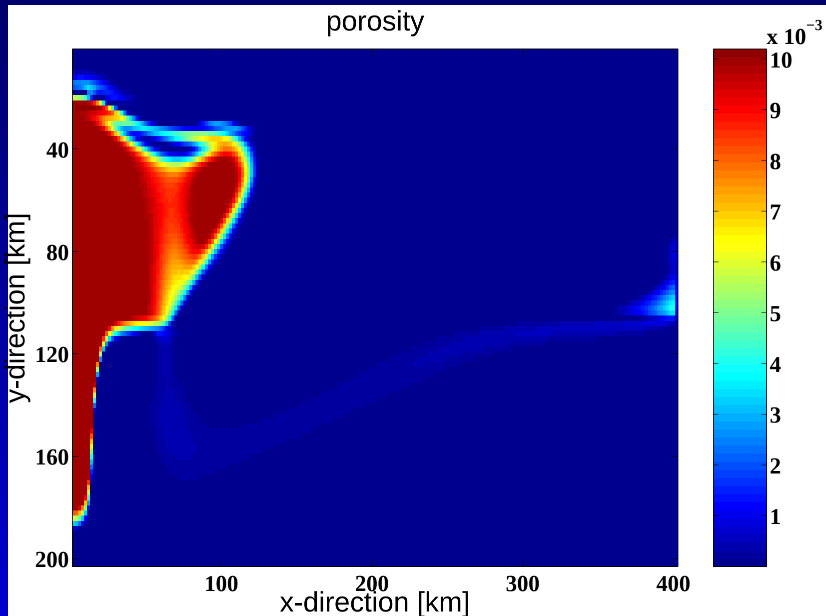
Rtn = 0.5 Rm = 2000.0 $\varphi_{\sigma_{max}} = 45°$ $\varepsilon \approx 4$



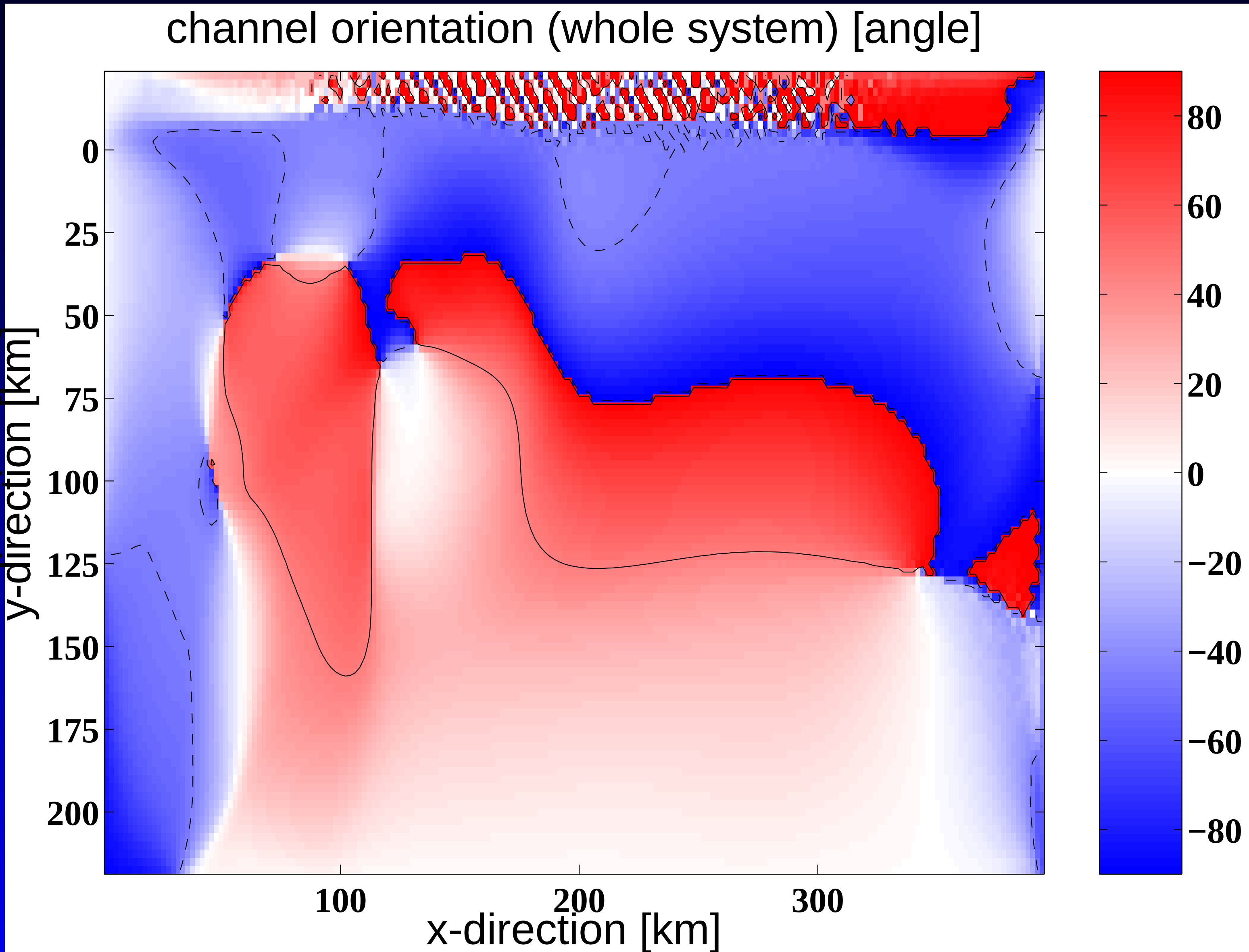
Application to the Earth

- Plume with power law after $1\ Ma$
- Sea floor spreading velocity $1\ cm/a$
- Finite strain after $1\ Ma = \varepsilon = 0.05$





Application to the Earth



Conclusion

- Mechanic channeling may occur.
- The porosity grows exponentially, with a growth rate which is proportional to $\dot{\varepsilon}$, $\frac{1}{Rtn}$.
- The analytical solution of the channeling problem as well as the simulations show that channeling occurs in an orientation parallel to the maximum compressive stress for all examined geometries.
- In a simple shear regime the analytical solution matches the 1D ellipsoid and 2D ellipsoid bodies, when the wavenumbers from the theory are divided by a factor off approx. two (rendering an effective wavenumber).
- In a pure shear regime an effective wavelength for a 1D ellipsoid could be specified, but only for elongation $\pm 45^o$ from the maximum compressive stress direction.
- We did not achieve a focussing towards the MOR.
- The CBA just influences small wavelengths, and may be considered when introducing an effective viscosity.

The End