A comparison of finite difference formulations for the Stokes equations in presence of strongly variable viscosity

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Numerical modeling of geodynamic problems typically requires the solution of the Stokes equations for creeping, highly viscous flows. Since material properties such as effective viscosity of rocks can vary many orders of magnitudes over small spatial scales, the Stokes solver needs to be robust even in the case of highly variable viscosity. Currently, a number of different techniques (e.g. finite element, finite difference and spectral methods) are in use by different authors. Benchmark studies indicate that the accuracy of the velocity solution is satisfying for most methods. The accuracy of deviatoric stresses and pressure, however, is typically less than that of velocity. In the case of highly variable viscosity, some methods even result in oscillating pressures. This problem becomes particularly important in coupled problems, where the pressure feeds back to the solution. An example of such coupled problems is melt migration through deforming viscous media.

The purpose of this study is therefore to evaluate the accuracy of the pressure solution for a number of numerical techniques. Thereby, we make use of a recently developed 2D analytical solution for the stress distribution inside and around a viscous inclusion in matrix of different viscosity subjected to pure-shear or simple-shear boundary conditions (Schmid and Podladchikov, 2003). Results will be presented for a staggered grid velocity-pressure finite difference method, a stream function finite difference approach and a rotated staggered grid velocity-pressure finite difference method.

The setup of the viscosity within the numerical grid is extensively discussed since it strongly affects the accuracy of the results. The problem is investigated by studying a simple physical 1-D model with a contact of two media representing the contact between an inclusion embedded in a matrix (2-D case). Mathematically and numerically, it is demonstrated that interpolation of the viscosity using a harmonic average leads to the best results.

The developed finite difference codes have furthermore been compared with analytical solutions for a constant viscosity mid-oceanic ridge and for density driven flow (the Rayleigh-Taylor instability).

Reference:

Schmid, D. W. and Podladchikov,Y. Y. (2003). Analytical solutions for deformable elliptical inclusions in general shear. *Geophysical Journal Internaltional*, Vol. 155:269-288.