Time-dependent solutions for a convection problem with variable viscosity in codim-2 zones

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The formation of the rocks indicates that viscosity in the interior of the Earth and planets strongly depends on temperature, and this influence is fundamental to understanding the mantle convection and subduction motions [1, 4, 7]. This is the goal for which a convection problem with temperature-dependent viscosity in the Navier-Stokes equations is studied. The dimensionless hydrodynamics equations considered are as follow:

$$\nabla \cdot \vec{v} = 0$$
$$\partial_t \theta + \vec{v} \cdot \nabla \theta = \Delta \theta$$
$$\partial_t \vec{v} + (\vec{v} \cdot \nabla)\, \vec{v} = P_r \left[(R_a \theta - b)\vec{e}_z - \nabla P + \nu_0^{-1} div \left(\nu(\theta \triangle T) \left\{ \nabla \vec{v} + (\nabla \vec{v})^t \right\} \right) \right],$$

where $\vec{v}$ is the velocity vector field, θ is the temperature field, P is the pressure, ν the variable viscosity, R_a and P_r the dimensionless Rayleigh and Prandtl numbers.

This work studies the time-dependent solutions for a convection problem with the viscosity as an exponential function of the temperature $\nu(\theta \triangle T) = \nu_0 \cdot exp(-\gamma \triangle T \theta)$ for different exponential rate γ.

The physical set-up considered is a two-dimensionless box [5, 2]. This box is subjected to an uniform heating from the bottom with a temperature higher than the upper layer. The boundary conditions for the velocity correspond to rigid and free-slip in the bottom and in the upper walls respectively.

The time-dependent solutions in codim-2 zones for different aspect-ratios, Rayleigh numbers and other parameters in which constant viscosity and strongly temperature-dependent viscosity appear are obtained.

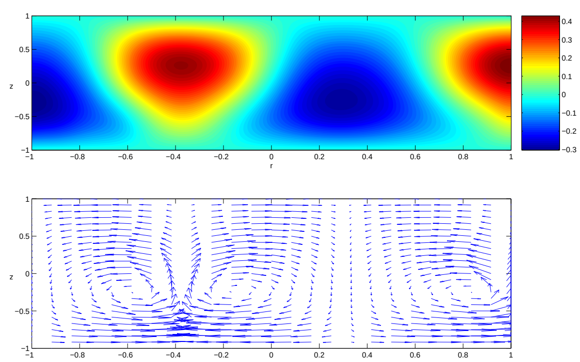

Figure 1. Time-dependent solution for $\gamma = 30000$, $R_a = 73.9497$ and $l = 0.1011$.

From the bifurcation study in a convection problem with temperature-dependent viscosity in a parallel infinity layer [6] and then with an appropriate change into the wave number of the stability curve the codim-2 zones are obtained.

A bifurcation study with the stability results for constant viscosity [3] and variable viscosity are also exposed [6].

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