All Bent Out of Shape: The Dynamics of Thin Viscous Sheets

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Although the earth's lithosphere is thermally, chemically and rheologically complicated, one can understand some important aspects of its dynamics by modeling it as a thin sheet of very viscous fluid. The key to the dynamics of viscous sheets is the competition between two distinct modes of deformation: stretching and bending. The partitioning of the deformation between these modes depends in a complicated way on the sheet's shape and on the distribution of the load applied to it. A simple way to quantify this is to calculate the deformation of a "shallowly" curved sheet with thickness h and principal curvatures κ_1 and κ_2 due to a load with wavelength λ applied normal to the sheet's midsurface. One thereby obtains the "rule of thumb" that stretching (bending) is dominant when $\lambda^2(\kappa_1^2+\kappa_2^2)^{1/2}/h \gg 1$ ($\ll 1$) (Ribe 2002.)

Because the resistances of a sheet to stretching and bending scale with the sheet thickness h in different ways ($\sim h$ and $\sim h^3$, respectively), loaded sheets exhibit a rich dynamics characterized by structures of boundary-layer type whose thickness is intermediate between h and the sheet's characteristic lateral dimension L. A remarkable example known to all amateur cooks is the periodic folding instability of a viscous sheet falling onto a surface (fig. 1.) As the height from which the fluid falls increases, the instability traverses four different dynamical regimes: a "viscous" regime in which folding is driven kinematically by the imposed fluid flux; a "gravitational" regime in which the viscous forces that resist folding are balanced by gravity; an "inertio-gravitational" regime in which the upper part of the sheet behaves as a swinging pendulum in resonance with the folding portion; and an "inertial" regime in which the viscous forces are balanced by inertia. The same sequence of regimes is also observed in the closely related phenomenon of "liquid rope coiling" that occurs when the fluid has the form of a thin filament (Ribe et al. 2006.) Because inertia is negligible in Earth's mantle, the viscous and gravitational regimes are the geodynamically relevant ones.

While folding instabilities are most easily observed at solid surfaces, they can also occur at fluid interfaces where the density and/or viscosity increases. It is therefore natural to ask whether such instabilities might occur in the mantle when subducted lithosphere encounters the transition zone or the CMB. Tomographic images under subduction zones such as Central America and Java show that the subducted slab appears to widen rapidly beneath the transition zone to form a broad (up to 700 km wide) wedge-shaped anomaly. To test the hypothesis that this apparent widening is the result of a folding instability, we used a numerically determined universal scaling law (Ribe 2003) to predict the folding amplitude δ for parameters (lithospheric thickness, subduction rate, etc.) appropriate to Central America and Java. The results ($\delta = 460$ and 400 km, respectively; Ribe et al. 2007) match well the widths of the velocity anomalies just beneath the transition zone, suggesting that the folding mechanism may be viable. As for the CMB, the possibility of slab folding there has recently been proposed based on seismic observations of a 100-km offset in the depth of a reflective layer (Hutko et al. 2006.) Because the scaling law of Ribe (2003) breaks down when the viscosity of the sheet is not much larger than that of the fluid surrounding it, we have performed laboratory experiments to determine how the viscous ambient mantle influences the folding dynamics of a slab encountering the CMB. When the sheet/ambient mantle viscosity contrast $\gamma > 500$, the sheet folds as if the ambient mantle were not there. For more moderate contrasts $\gamma \approx 10$, however, folding is suppressed, and the sheet impinges on the CMB as a stagnation-point flow with superimposed

Figure 1: Periodic folding of a sheet of corn syrup (fall height 7 cm.)

small-amplitude oscillations, creating a periodic undulating topography on the top of the fluid spreading laterally over the boundary.

In addition to the insight they provide into folding instabilities, thin-sheet models are useful tools for studying the dynamics of subduction more generally. One promising approach for modeling the coupled dynamics of a deforming sheet and the flow around it is the *boundary-integral method*, wherein a Stokes flow in a given region of space is represented in terms of the velocities and tractions at its boundaries. I will present results of numerical models of subduction obtained using this method, focusing on how concepts from thin-sheet theory can be used to derive scaling laws for important subduction parameters such as the surface plate speed, the rate of trench rollback, and the geometry of the subducted slab.

References

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