Dynamics & Consequences of Core Formation in Terrestrial Planets

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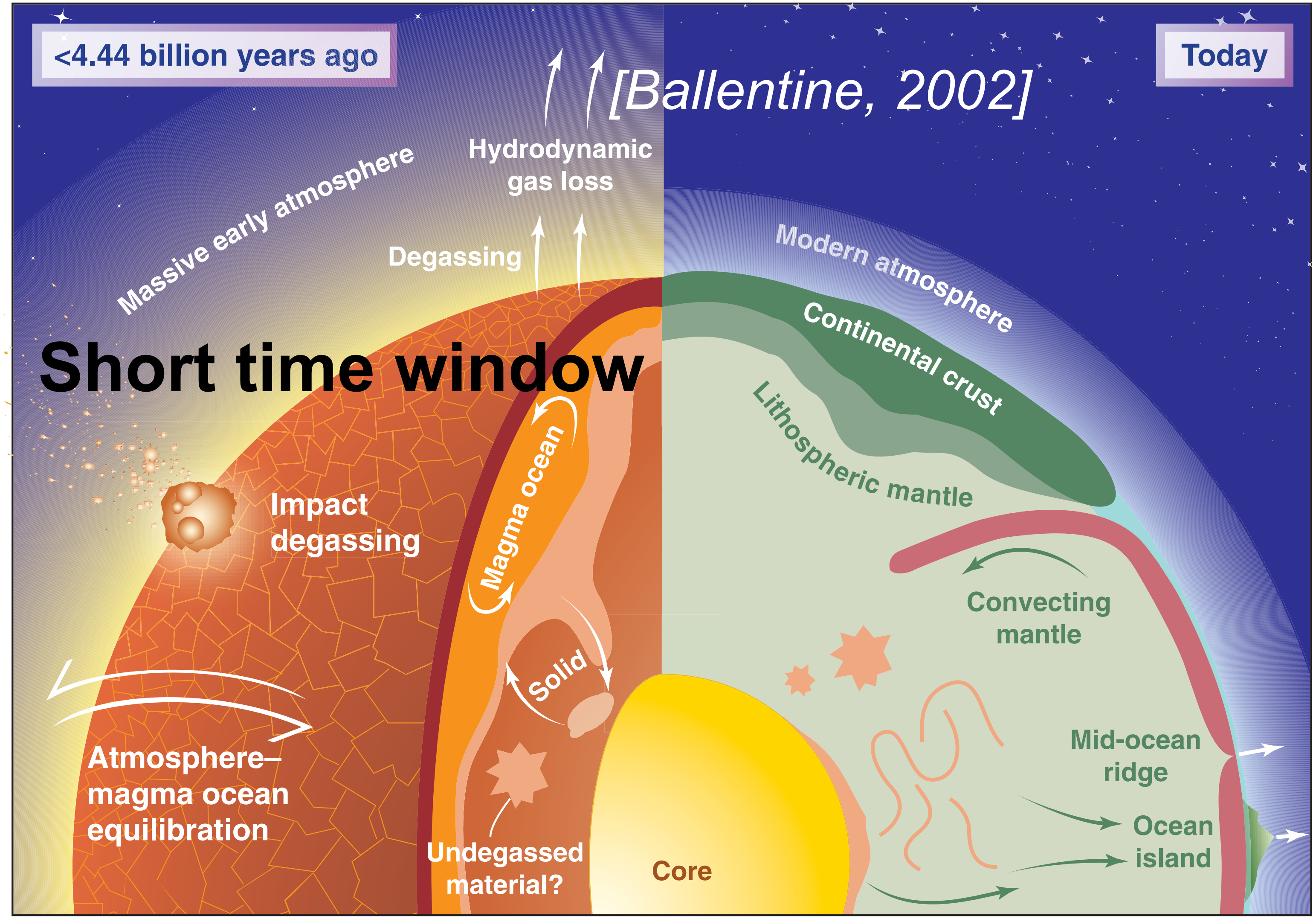
The early stages of planetary evolution

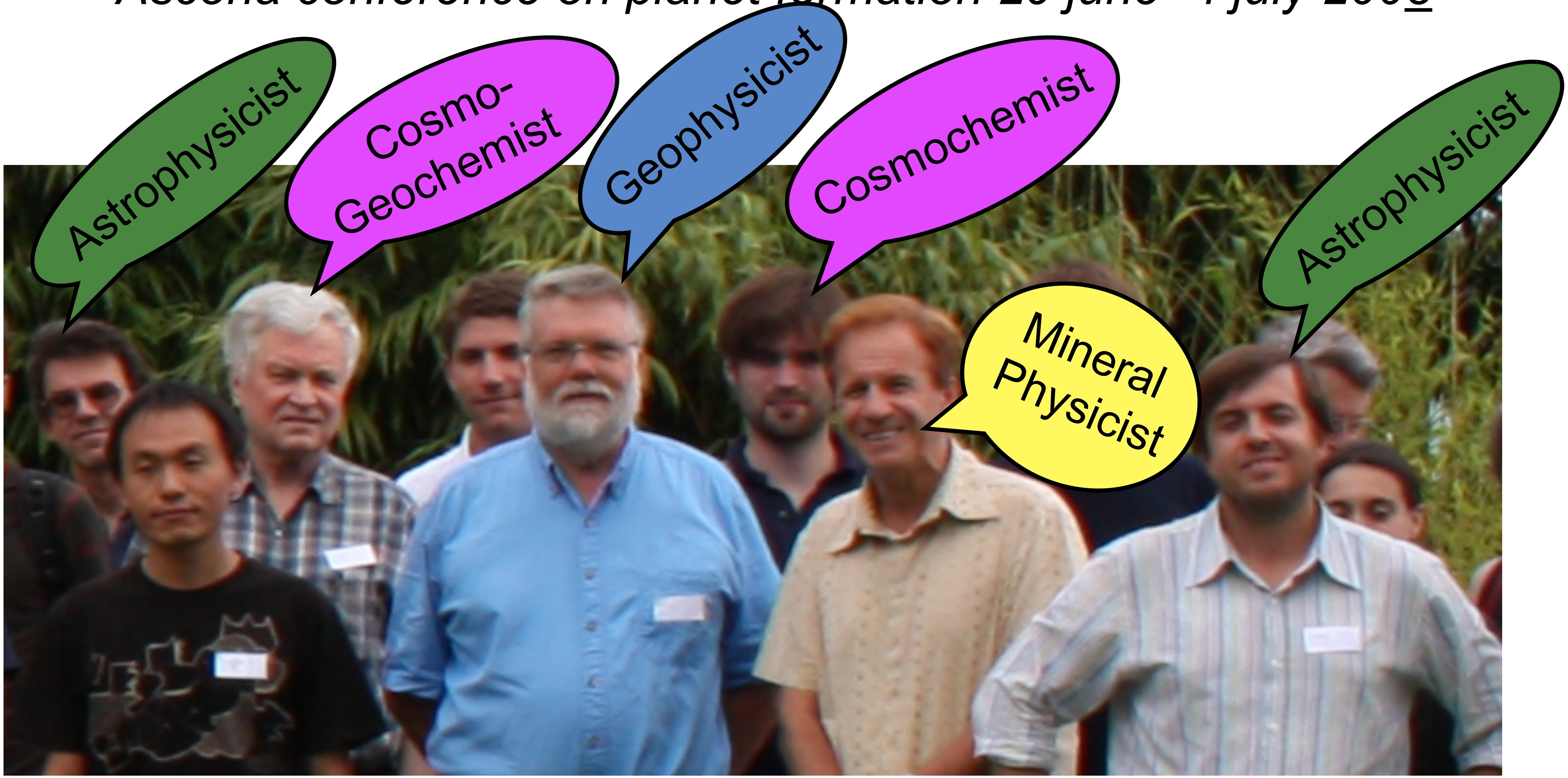


[Ballentine, 2002]

Early stages = Initial condition ➪ long term evolution

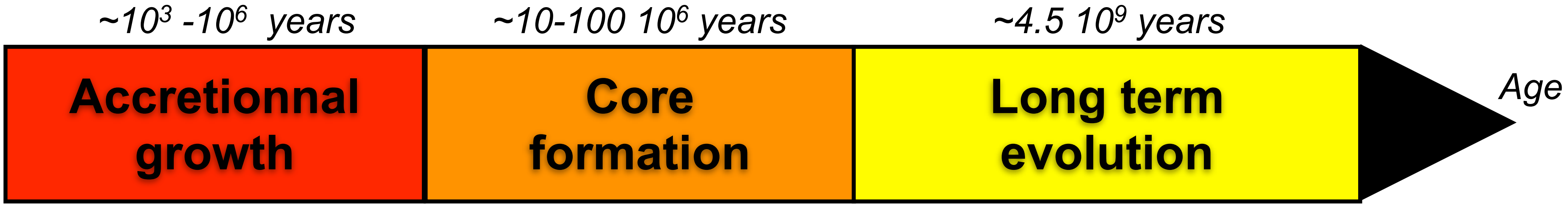
Who is concerned?

Ascona conference on planet formation 29 june- 4 july 2008



Early planetary evolution is everybody’s business

Early thermo-chemical state of terrestrial planets



$\sim 10^3$ -10^6 years

Accretionnal growth

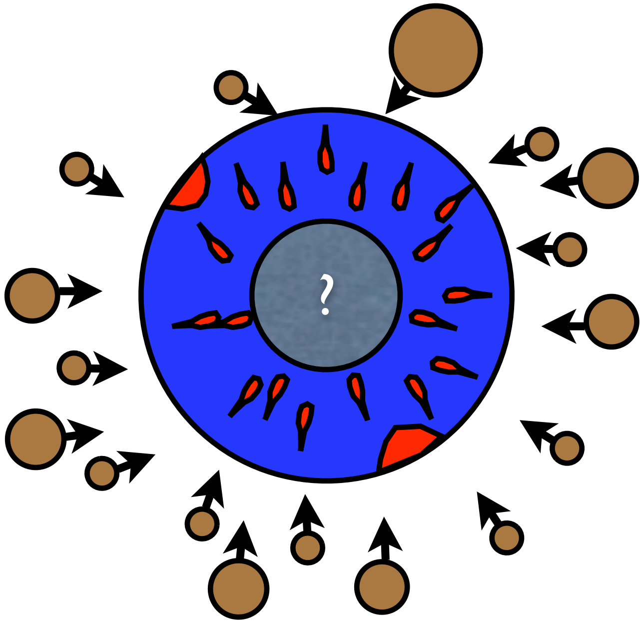
$\sim$10-100 10^6 years

Core formation

$\sim$4.5 10^9 years

Long term evolution

Age



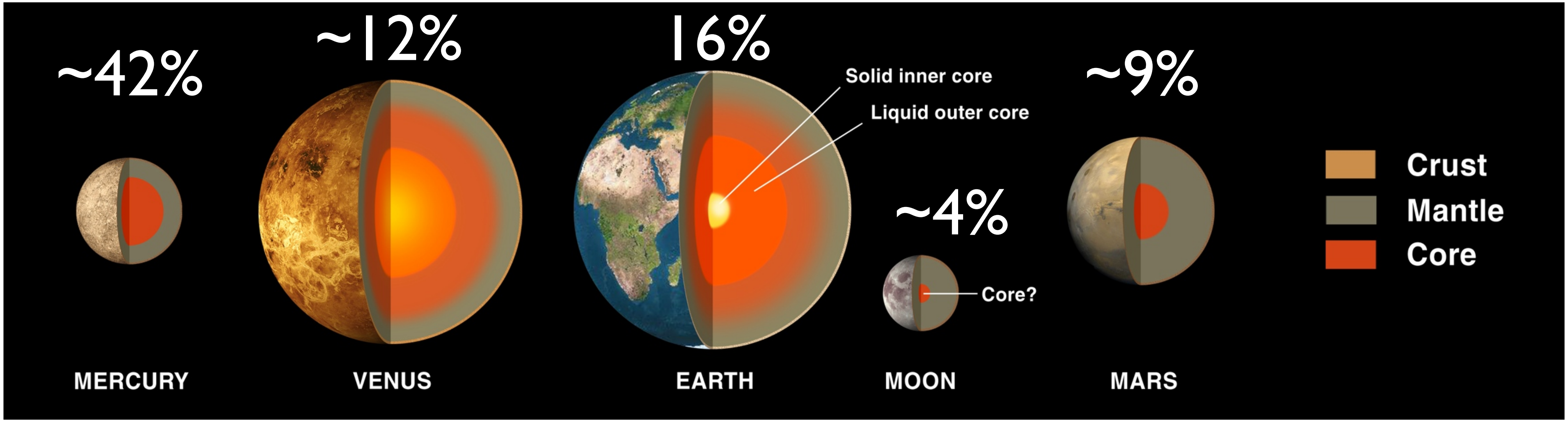
➪ Accretion
➪ Impact heating
➪ Isostatic readjustment
➪ Differentiation
➪ Convection
➪ Viscous heating
➪ Diffusion
➪ Radioactive heating ...

Major Difficulties

Many events & processes acting on various scales
➪ Difficult to model all at once
➪ Modeling using parameterized models (based on scaling laws)

Core Formation

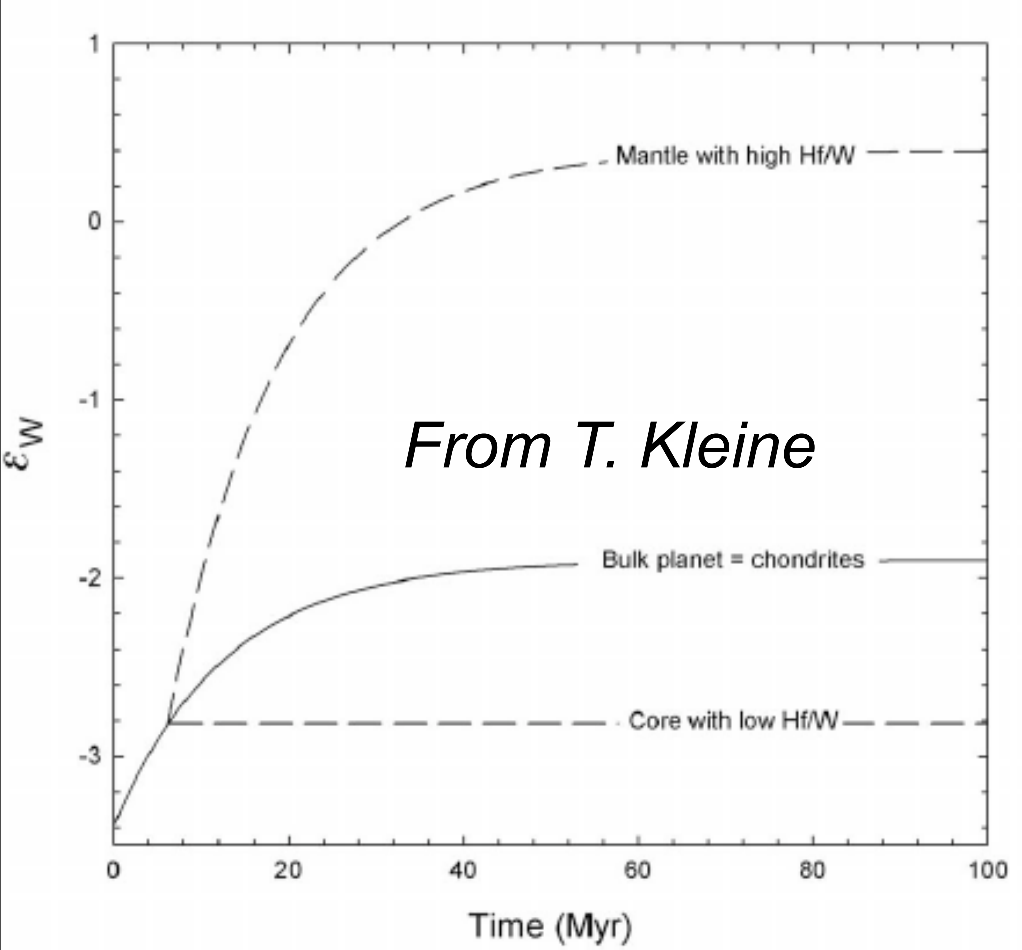
- ✓ Metallic core present on several terrestrial planets & satellites
- ✓ Core formation: First major differentiation event in terrestrial planets



NASA

What are the constraints?

Hf/W chronometry



From T. Kleine

^{182}Hf decays to ^{182}W

$$\varepsilon_W = \frac{(^{182}W/^{184}W)_{\text{sample}}}{(^{182}W/^{184}W)_{\text{standart}}} - 1]\times 1000$$

Earth’s mantle $\varepsilon_W = 0$

Hf :lithophile

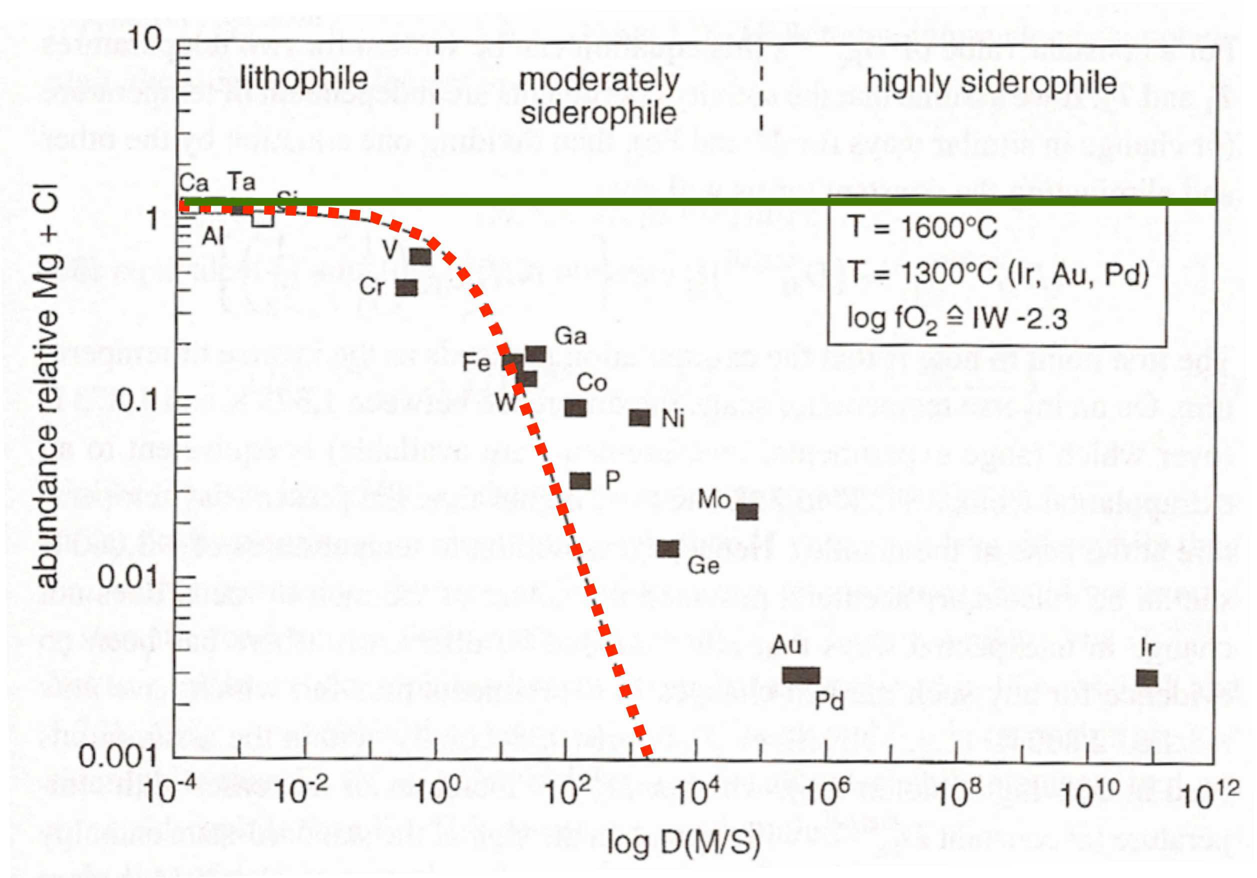
W: siderophile

➪ Hf-W fractionation during core formation

Core formation depleted Earth’s mantle in siderophile elements, resulting in elevated ratios of lithophile (e.g., Hf) to siderophile (e.g., W) elements in Earth’s mantle

➪ **Core formation is a Fast process: t <100 Myrs (e.g., [Kleine et al., 2004])**

Observations: Abundance of Siderophile Elements in Earth's Mantle



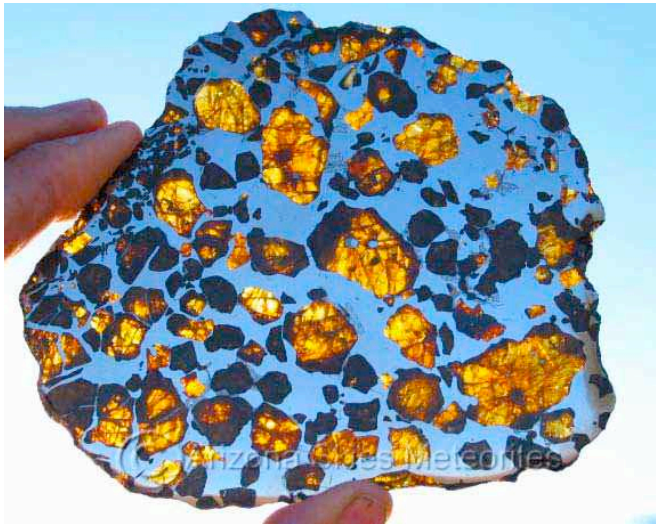
After [Oneill & Palme, 1998]

Core formation depletes Earth's mantle in siderophile elements (Fe, W, Ni, Co...)

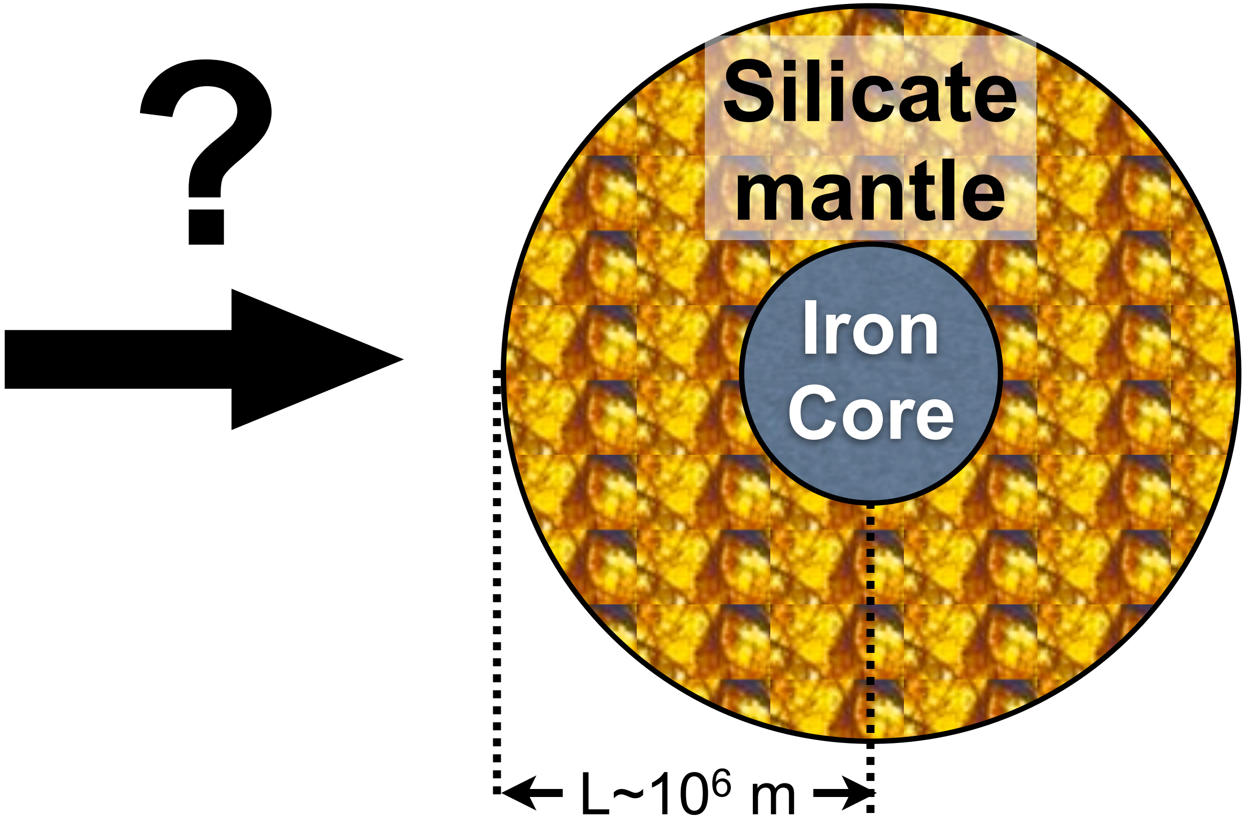
However, a few moderately & highly siderophile elements (Co, Ni, Ir...) remain overabundant in the Earth's mantle.

➪ **(high P & T) Metal-Silicate equilibration / chemical exchanges during core formation**

Metal-Silicates separation



$\rho_{Fe} - \rho_{Si} >> 1$



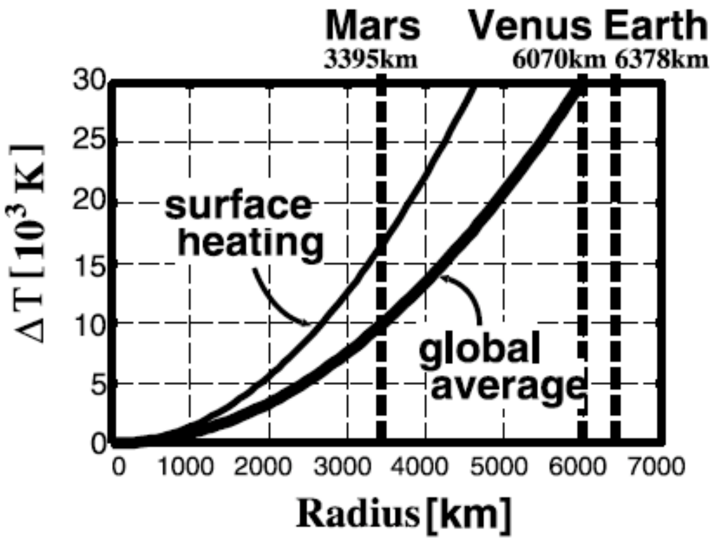
Gravitational segregation (of any kind)?

Yes, if melting is present (low viscosities)

⇨ High temperatures

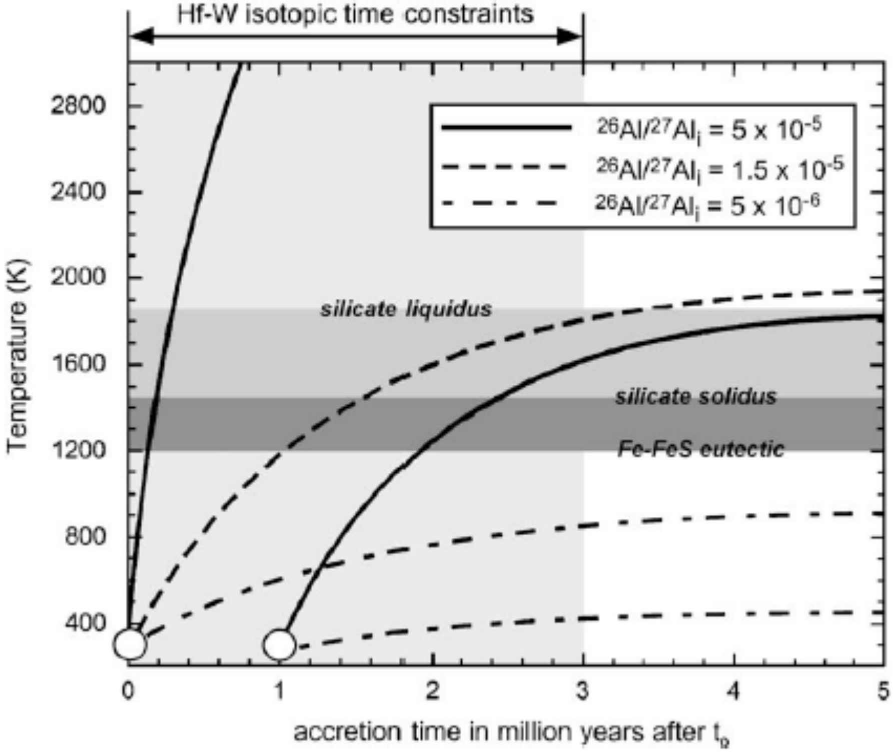
Heat sources during core formation

1. Impact: E_k ⇨ E_T



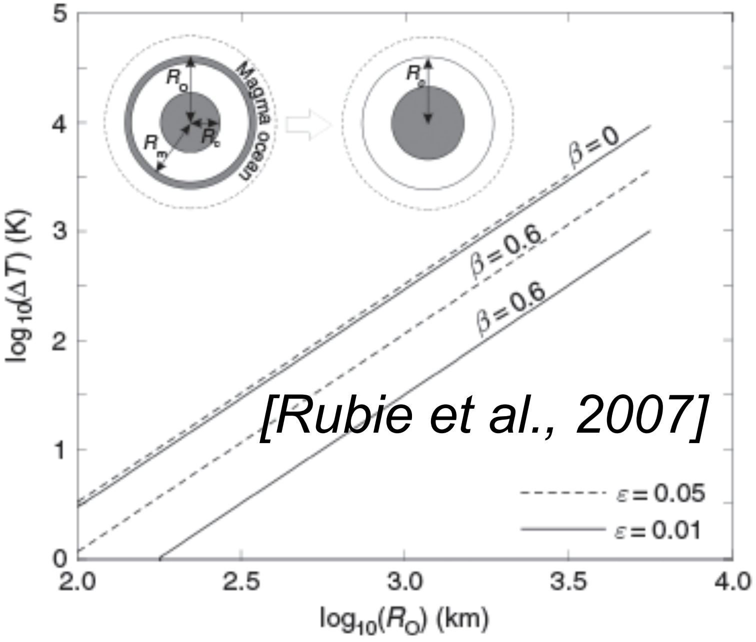
[Senshu et al. 2002]

2. Radioactive decay (^{26}Al, ^{60}Fe)



[Walter & Trønnes, 2004]

3. Viscous / gravitational: E_p ⇨ E_T



[Rubie et al., 2007]

Comparable amounts of heating

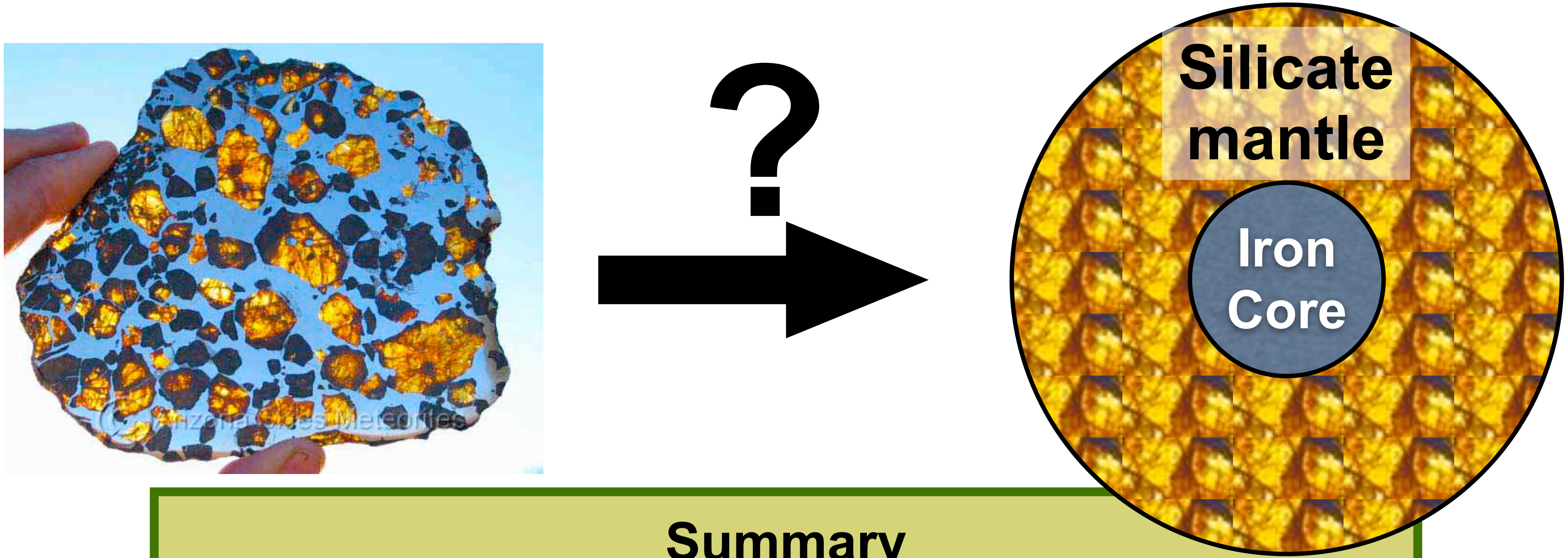
⇨ ΔT ~ 1000 K

⇨ Large scale melting likely

⇨ local & global magma oceans

⇨ Differentiation of planetesimals

Constraints on core formation



Summary

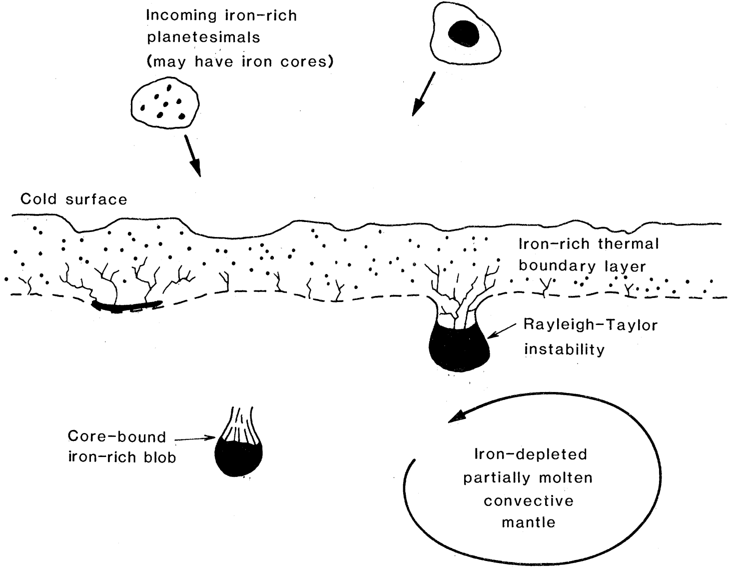
✓Hf/W chronometry
➪ Fast process: t <100 Myrs

✓ Overabundance of siderophile elements in mantle
➪ Requires Fe-Si equilibration

✓High T process (”Si-Fe” separation)
➪ Melting in magma ocean/ponds or elsewhere...

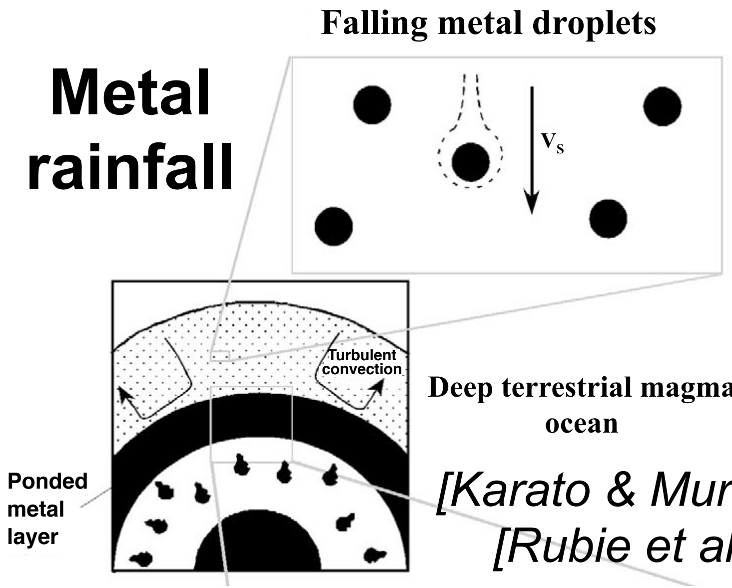
Several possible core formation scenarios

Sinking of iron-rich diapirs



[Stevenson,1981]

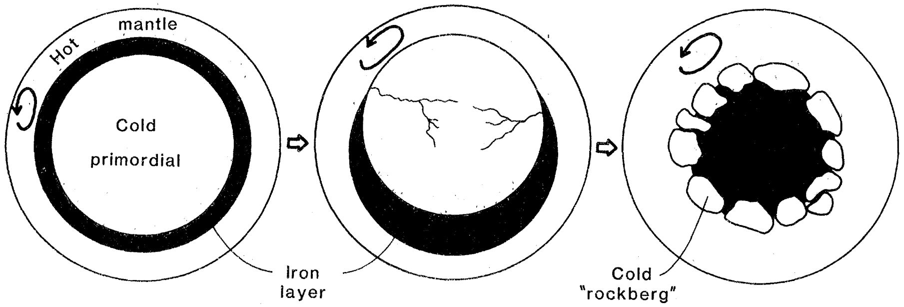
Metal rainfall



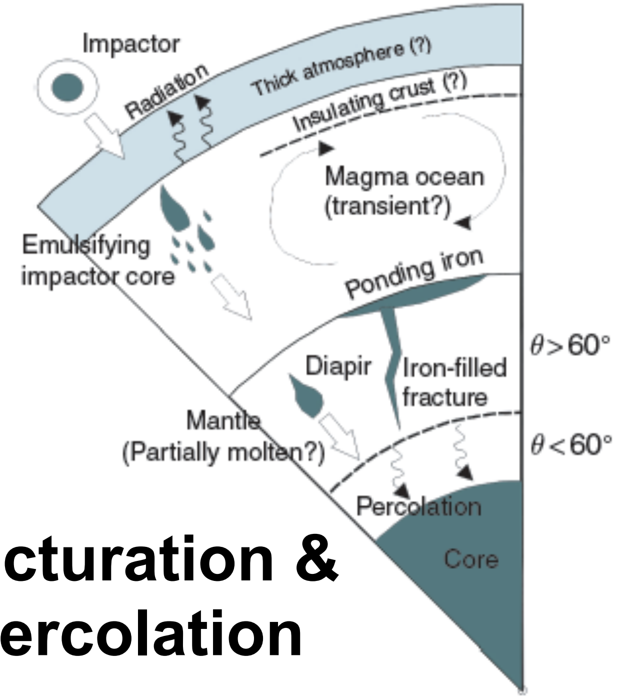
[Karato & Murthy, 1997]
[Rubie et al.,2003]

End-member cases!

Destabilization of a global iron layer



[Stevenson,1981]

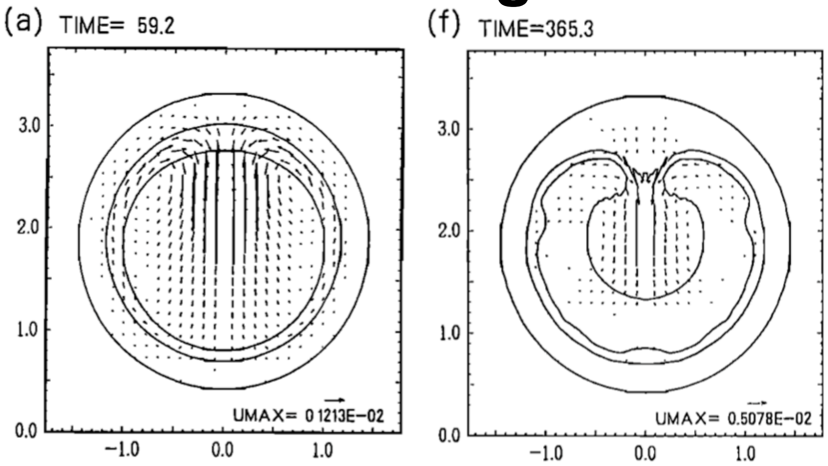


Fracturation & Percolation

[Rubie et al.,2007]

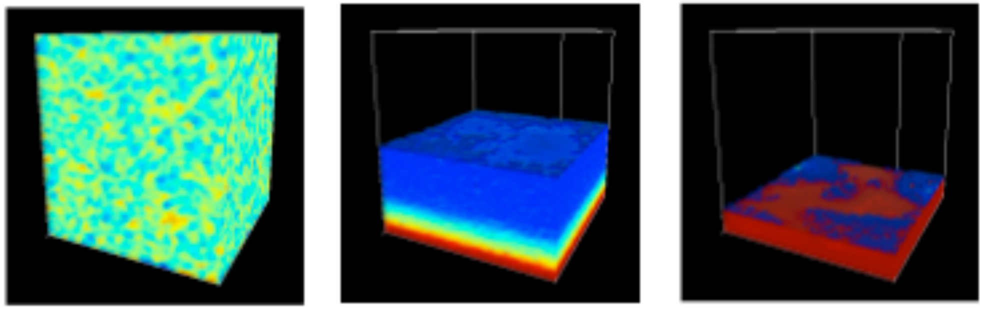
Core formation scenarios dynamically tested

Destabilization of a global iron layer



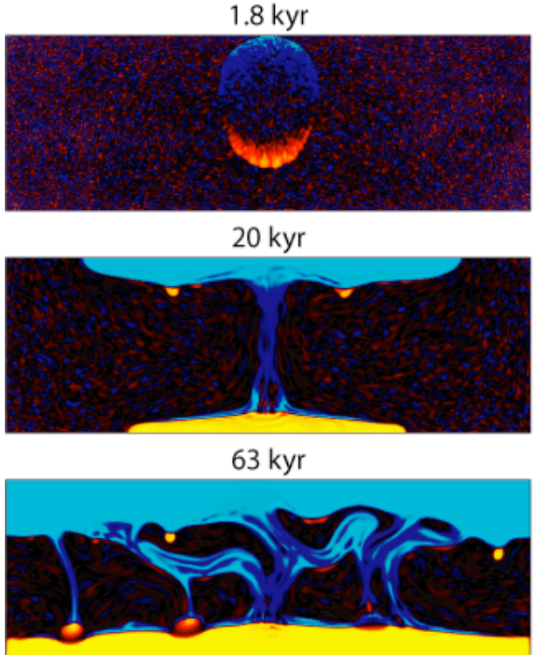
[Honda et al.,1993]

Metal rainfall

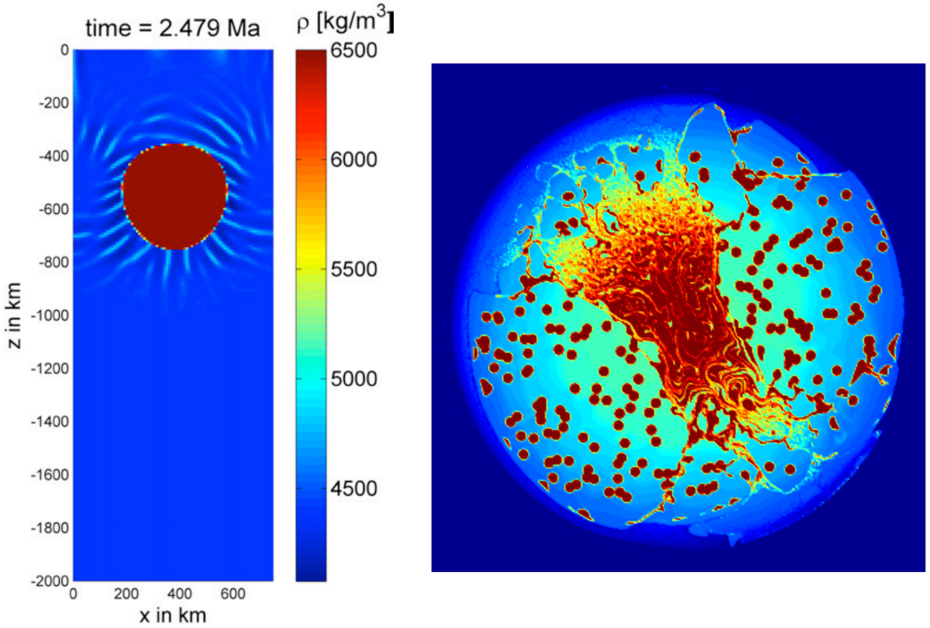


[Hoink et al., 2006]

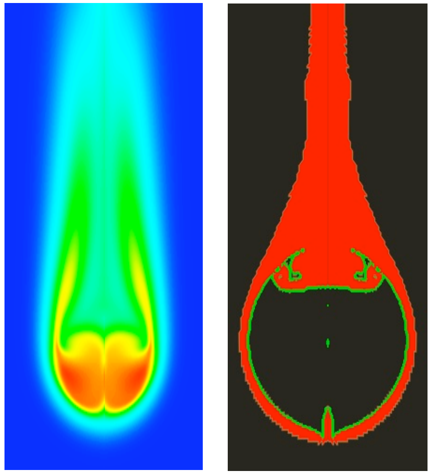
Negative diapirism (+ additional complexities)



[Ricard et al.,2009]



[Golabek et al., 2008, 2009]



...

[Samuel & Tackley, 2008]

What, Why, How?

What?

Focus on negative diapirism in:

✓ solid or partially molten proto-mantle

✓ magma ocean

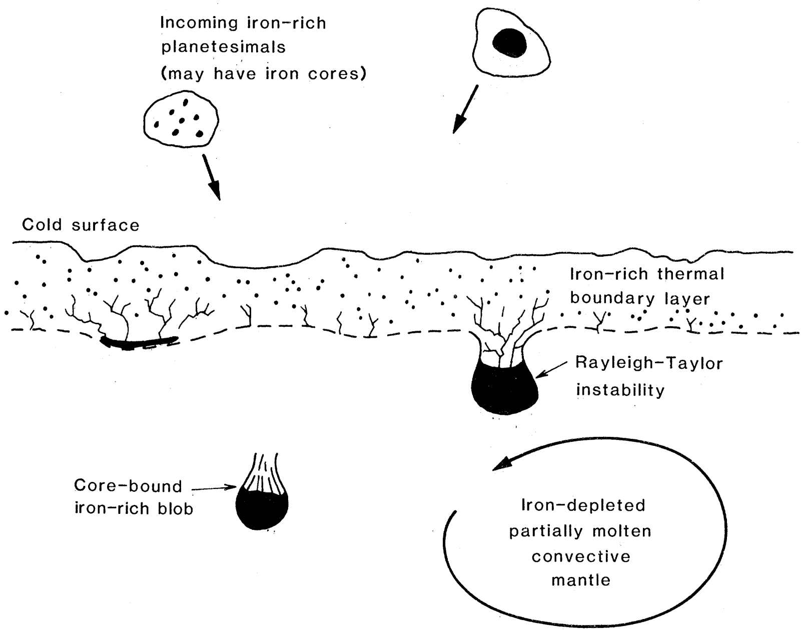
Why?

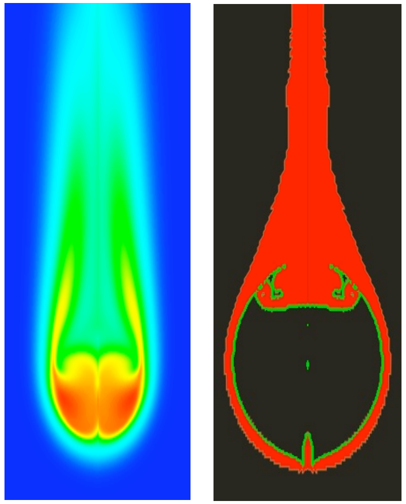
✓ Understand the dynamics

✓ Quantify core formation timing

✓ Fe-Si Chemical exchanges

✓ Core-mantle Energy Partitioning





How?

➪ Numerical modeling with a systematic approach

➪ Derive simple(r) semi-analytic / fully analytic scaling laws

Spherical axisymmetric model setup

4 conservation equations (stagYY)

Mass

$$\nabla . U = 0$$

Composition

$$\frac{DC}{Dt} = 0$$

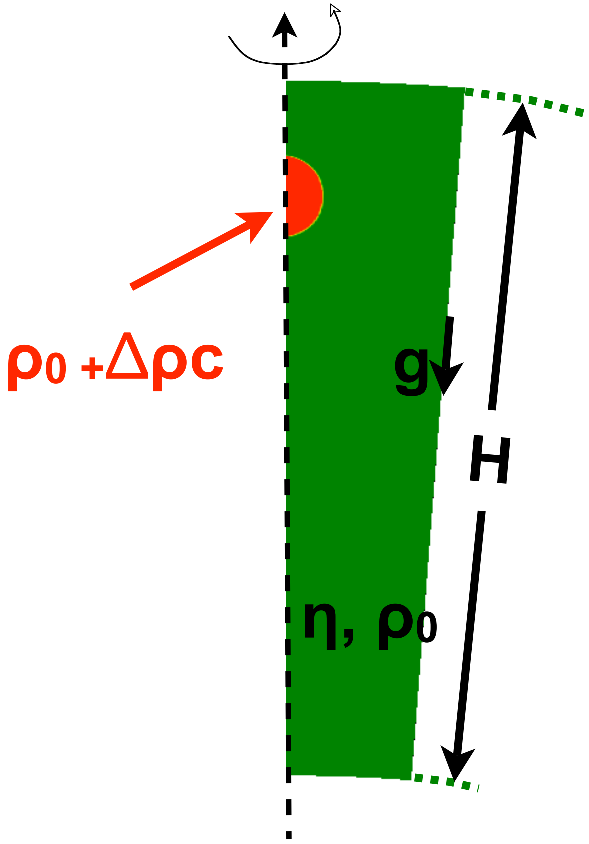
Momentum

$$\nabla p - \nabla . (\eta \dot{\varepsilon}) + Ra(\boxed{T} - BC)\vec{e_r} = 0$$

small

Energy

$$\frac{DT}{Dt} = \nabla^2 T + \frac{Di}{Ra}\sigma : \dot{\varepsilon} \boxed{- DiTU_r}$$



Inertia neglected

3 dimensionless numbers

$$Ra = \frac{\rho_0 \alpha g \Delta T H^3}{\eta_0 \kappa}$$

$$Di = \frac{\alpha g H}{Cp} \quad B = \frac{\Delta \rho_c}{\rho_0 \alpha \Delta T}$$ >>1

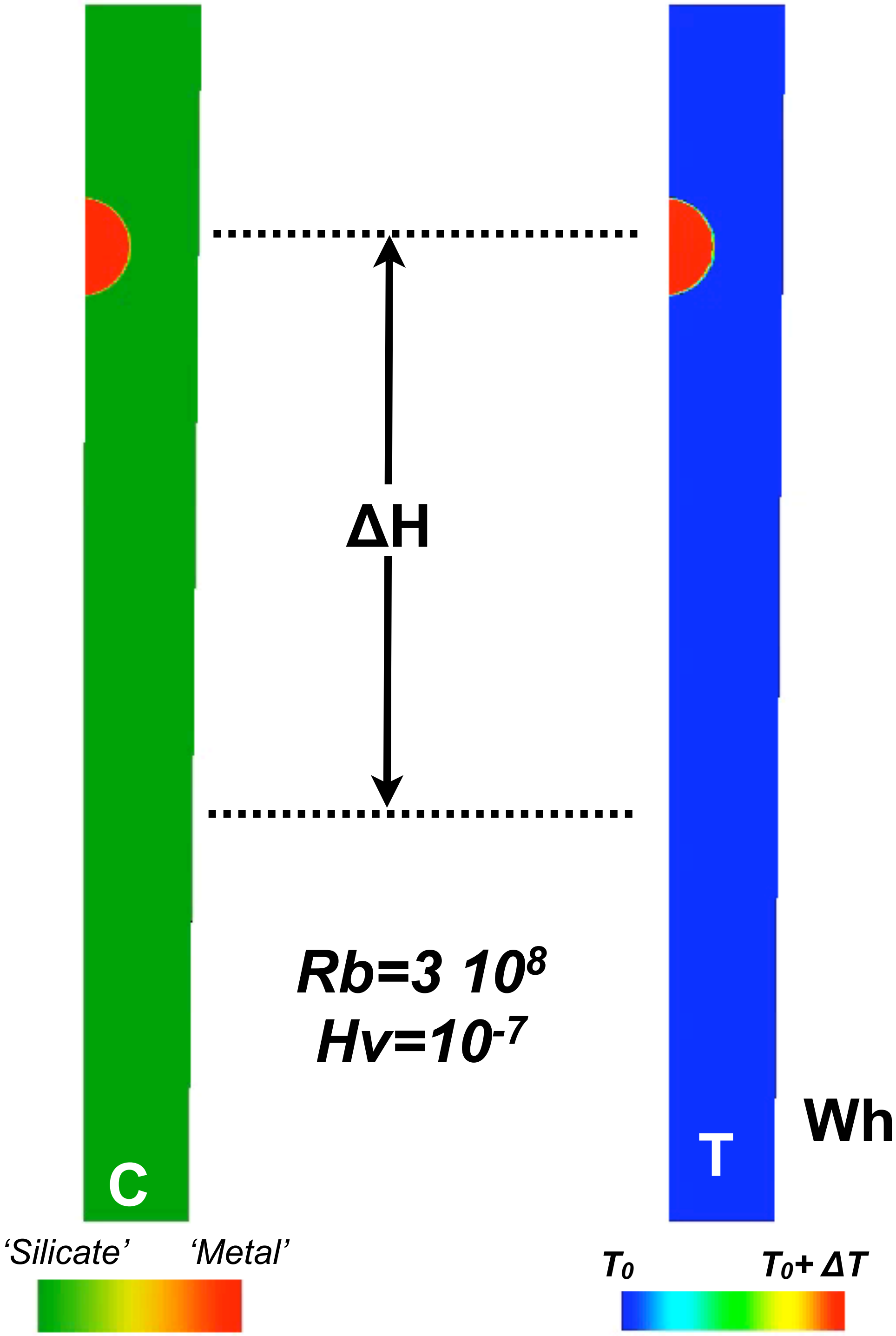
Important quantities

$$Rb = Ra\ B = \frac{\Delta \rho_c g H^3}{\eta_0 \kappa}$$ [10^5-10^{12}]

$$H_v = \frac{Di}{Ra}$$ [0-10^{-12}]

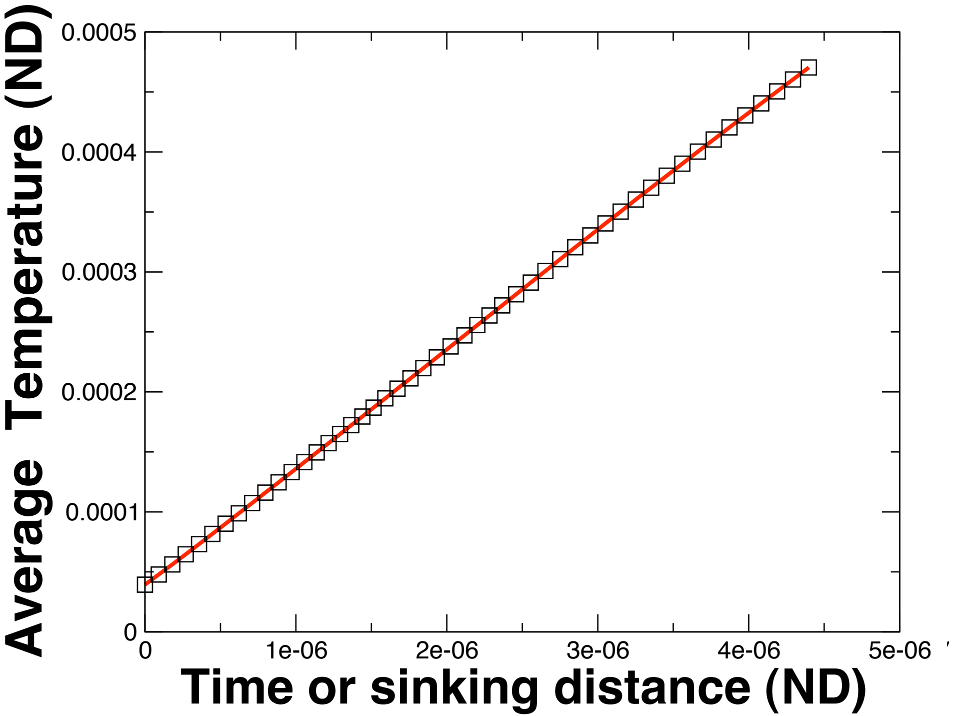
$$\eta = f(T, melt, C, \sigma)$$

Results: typical evolution



$Rb=3\ 10^8$
$Hv=10^{-7}$

Global temperature increase
⇨Conversion of potential (E_p) to thermal (E_{th}) energy via viscous heating

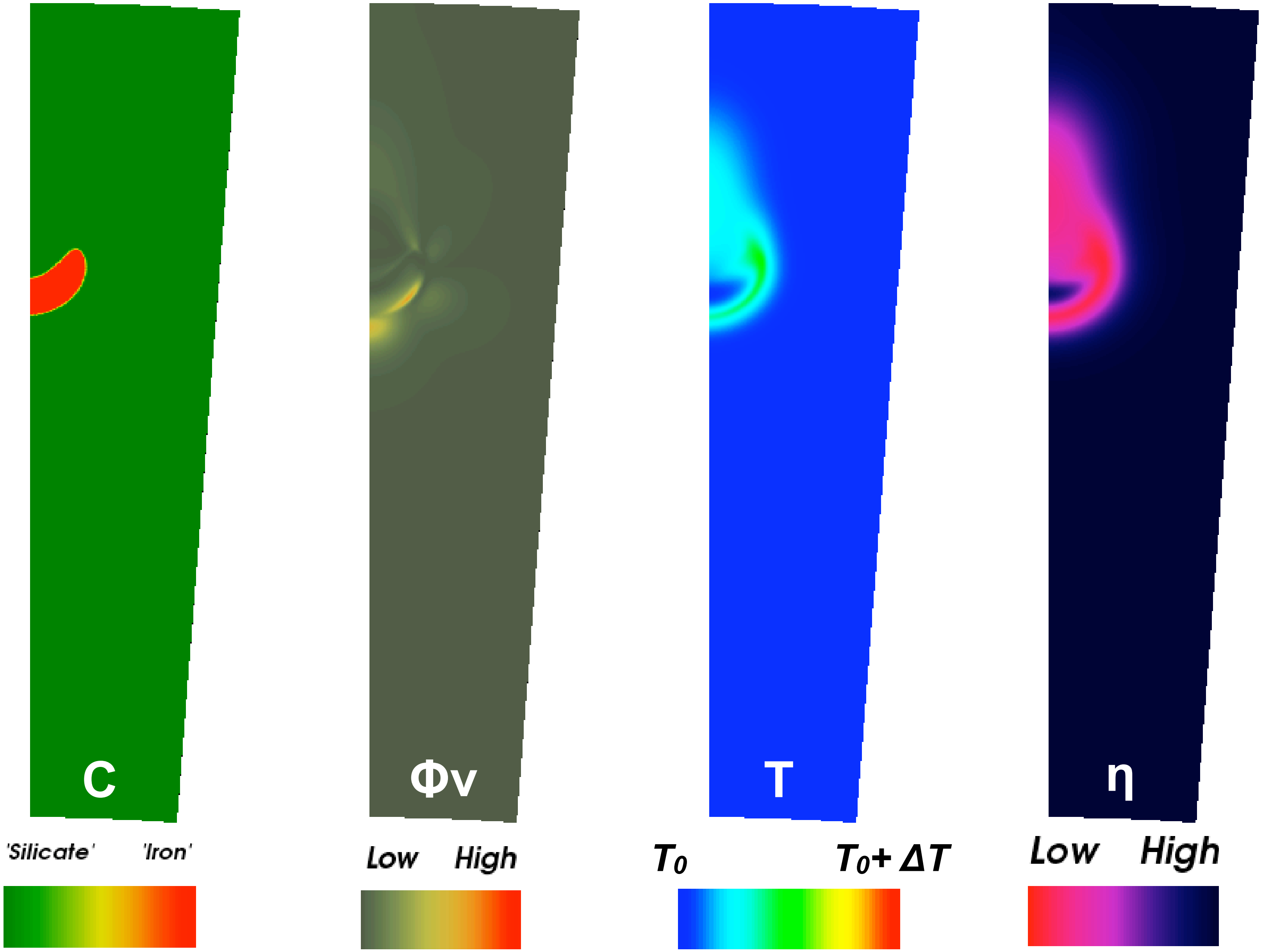


$$\delta T \sim \Delta H$$

Where does viscous heating take place?

Results: typical evolution

Rb=3 10^8
Hv=10^{-7}
Newtonian η

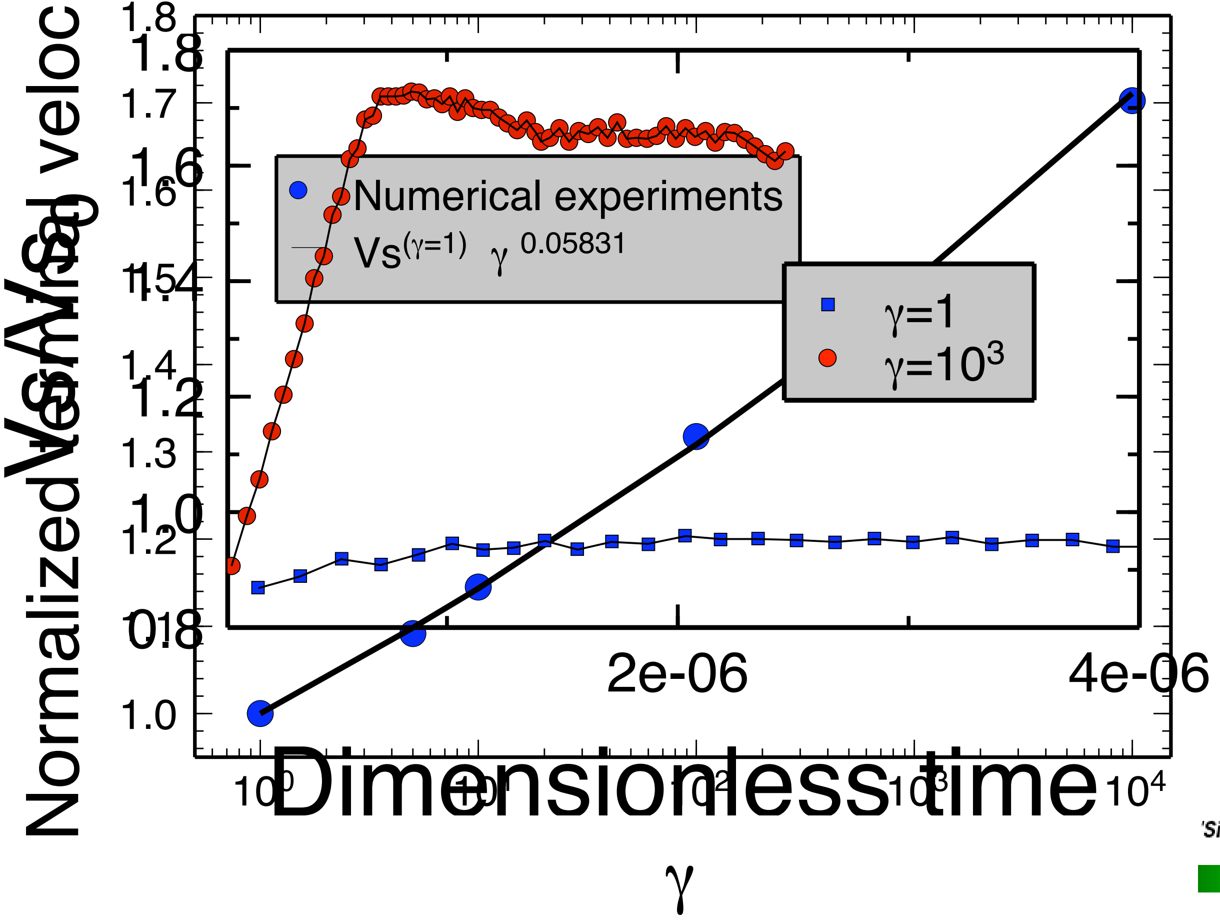


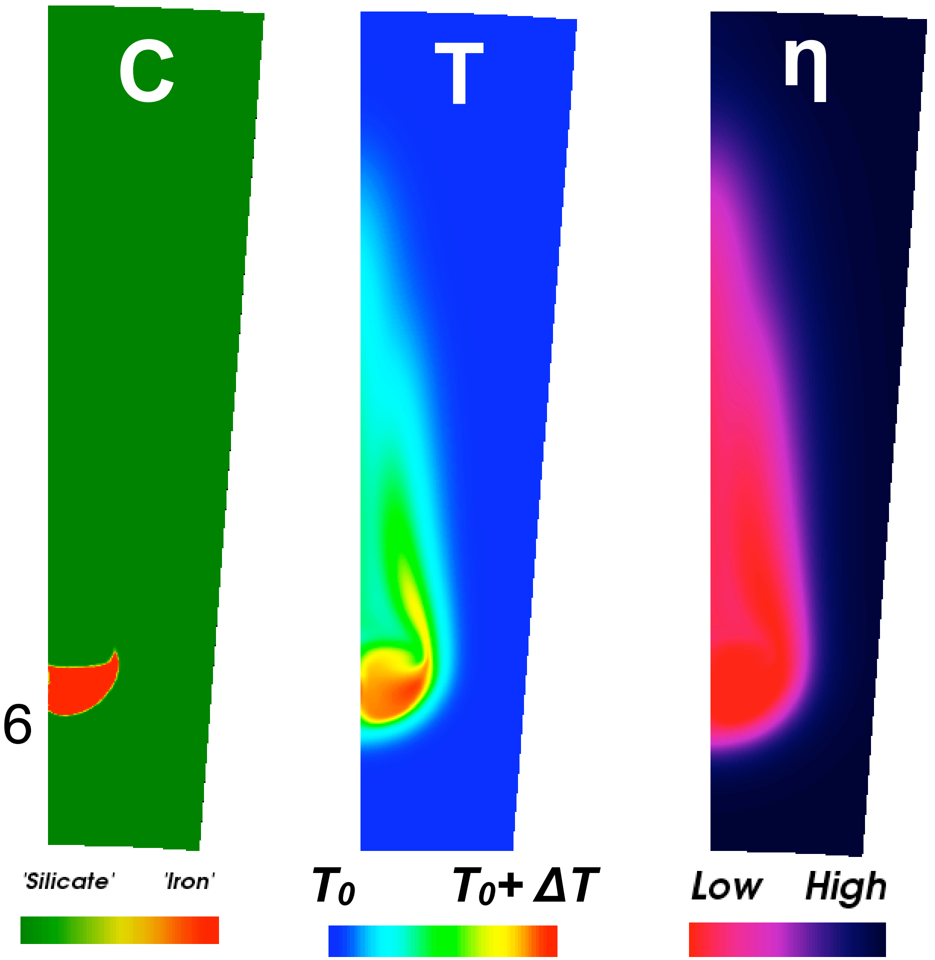
Φ_v max: interface
⇨ Tmax @ interface

ΔT ~ 1-1000K ⇨ melting
Timing < 10 Myrs

Results:influence of temperature dependent η

$$\eta = \eta_0 \exp[-ln(\gamma)T]$$

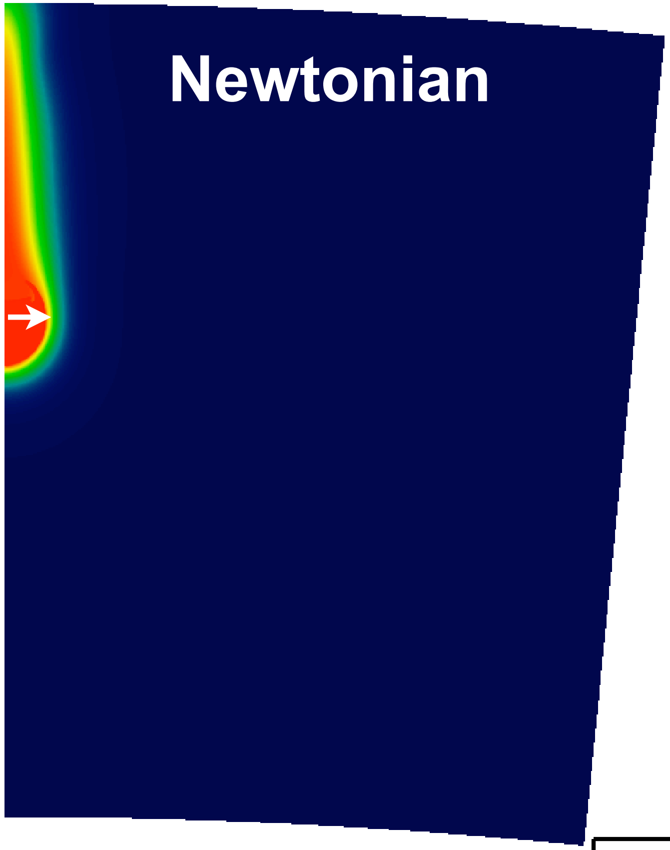




Diapir's tail: HighT Low η
➪ Upper/lower hemisphere asymmetry
➪ Diapir's shape goes from spherical to ~ hemispherical cup

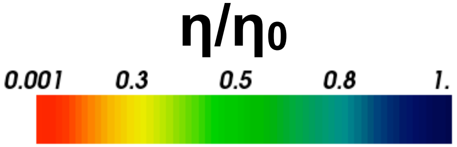
η variation localized
➪ weak influence on Vs

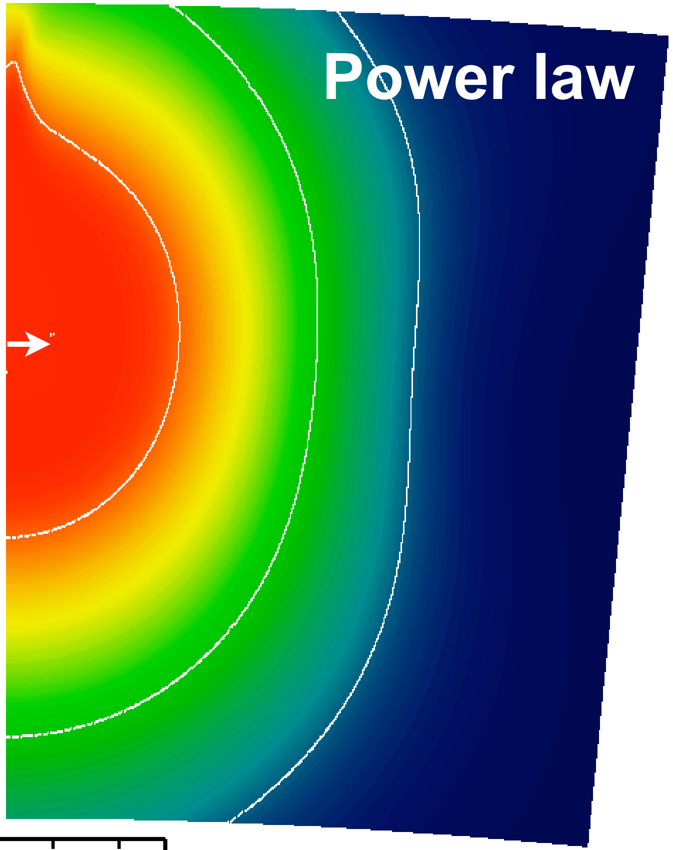
Results: Newtonian vs. Power law rheologies



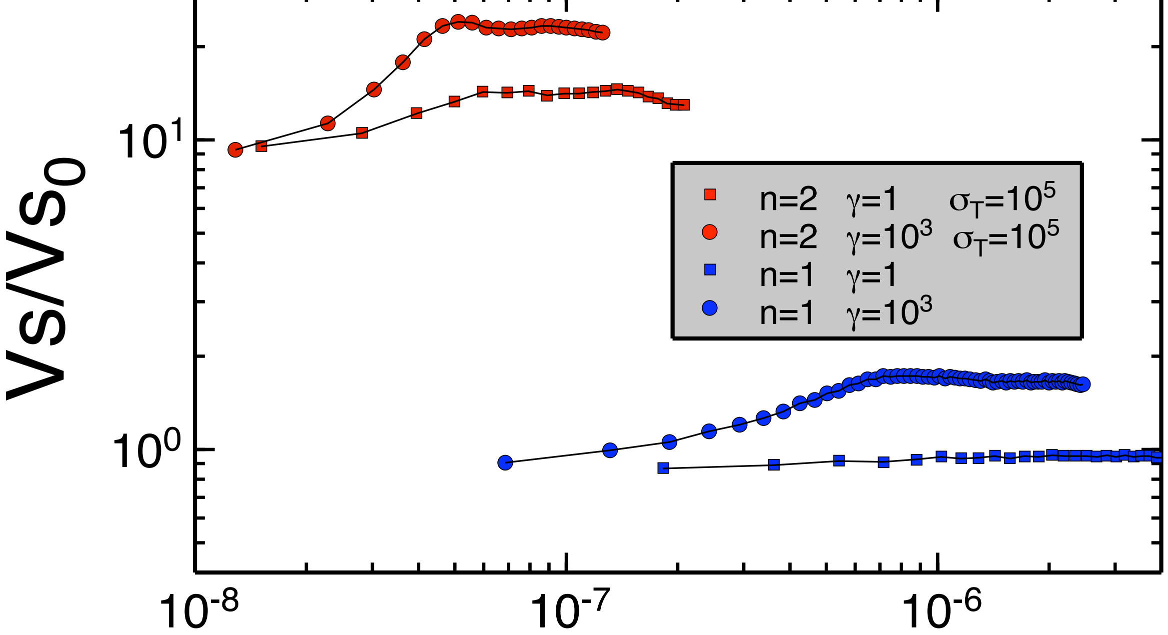
$Rb=3\ 10^8$

$Hv=10^{-7}$



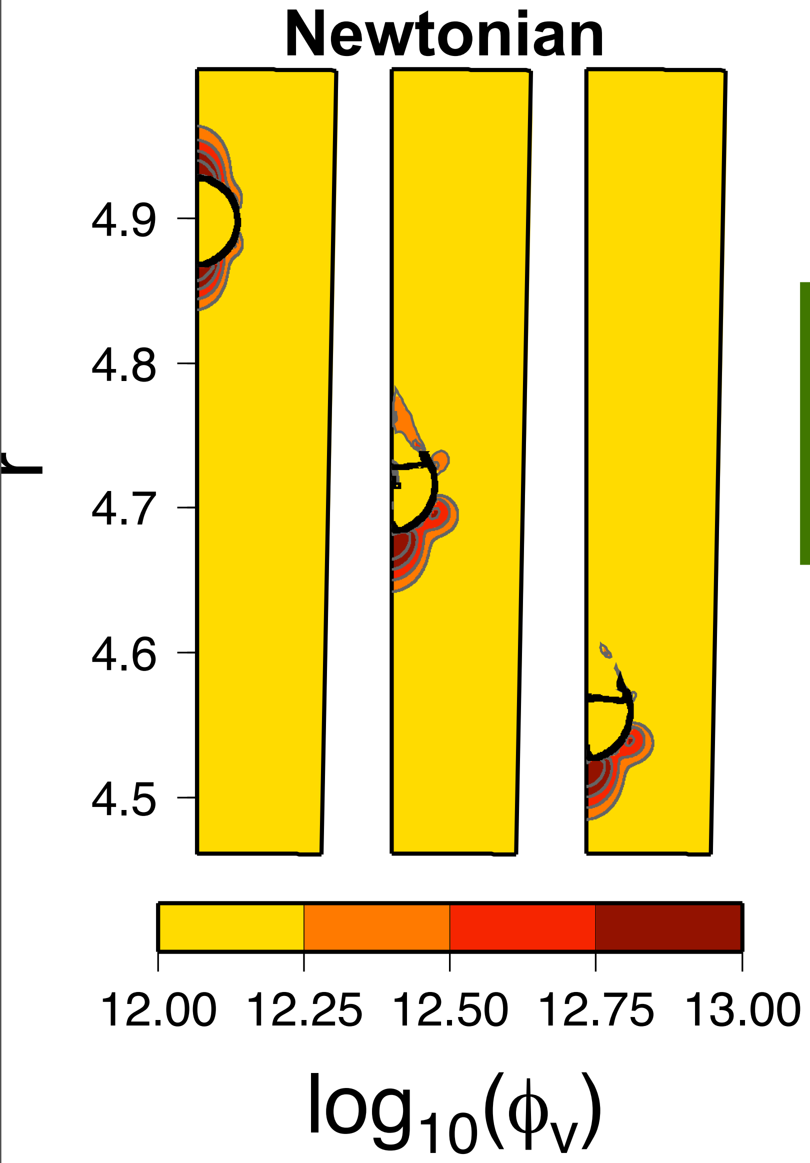


Viscosity decrease limited to a narrow zone



Viscosity decrease zone extends up to several radii

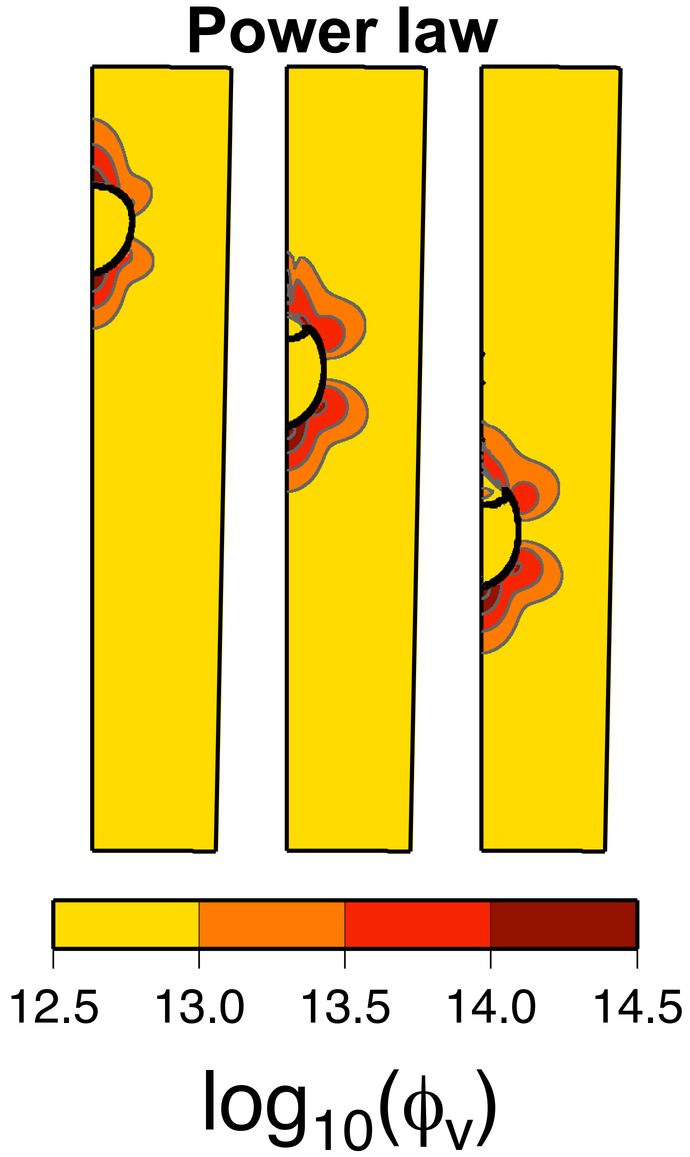
Results: Newtonian vs. Power law rheologies


Newtonian

$Rb = 3\ 10^8$

$Hv = 10^{-7}$

Viscous heating mainly located at the interface diapir-mantle (poles)

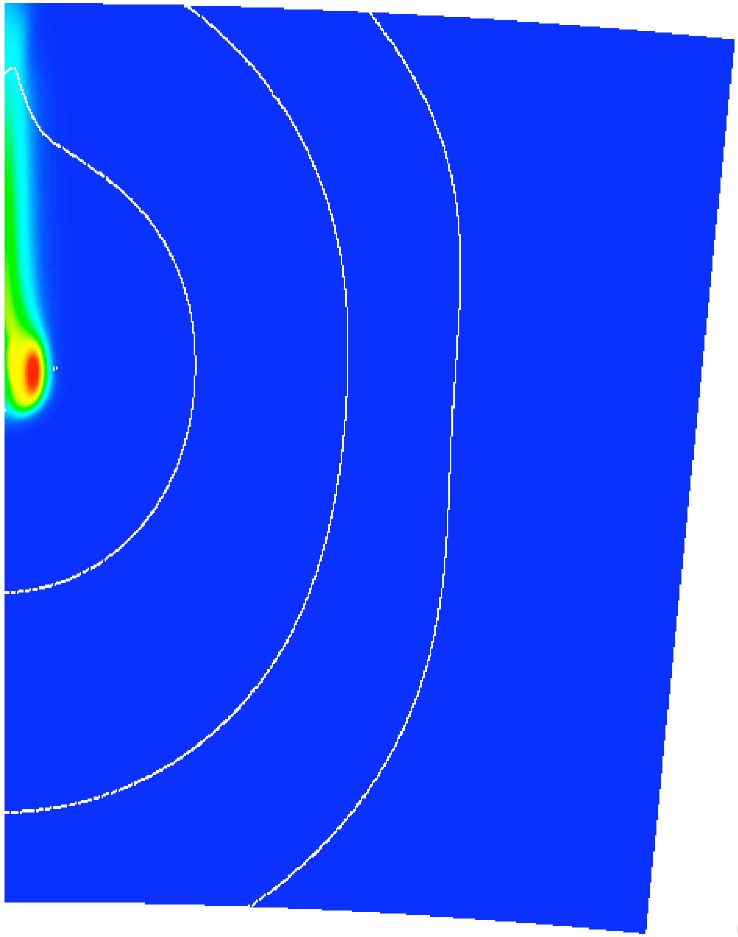

Power law

Viscosity decrease limited to a narrow, high deformation zone

Viscosity decrease zone extends up to several radii

Results: Newtonian vs. Power law rheologies

Newtonian



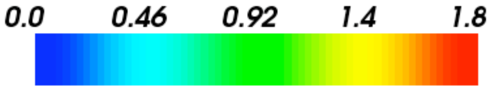
Viscosity decrease limited to a narrow, high deformation zone

$Rb=3\ 10^8$

$Hv=10^{-7}$

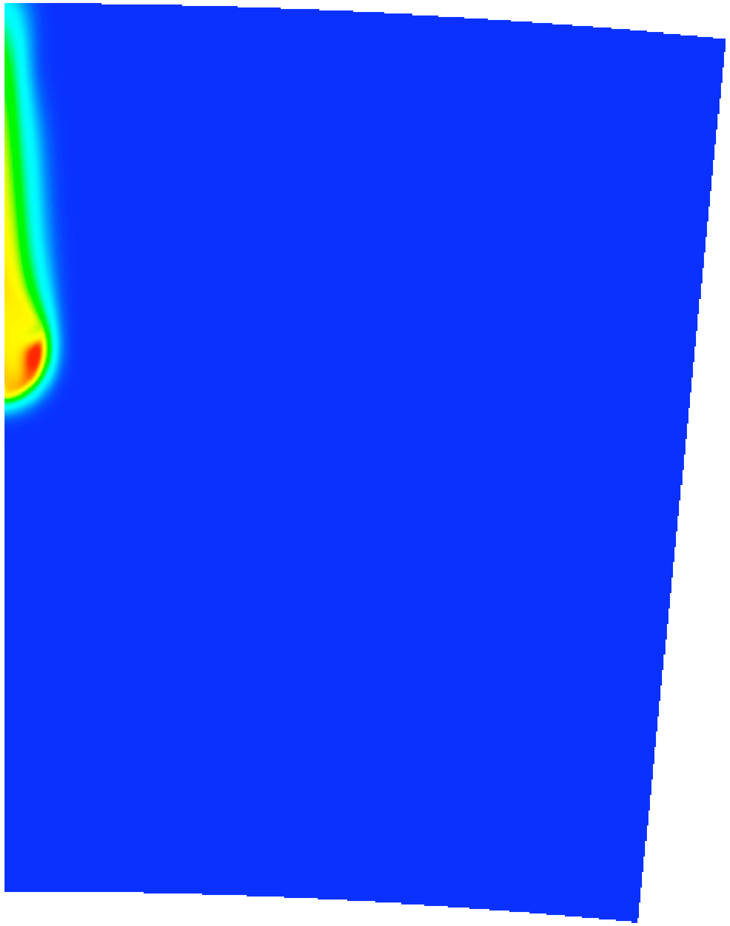
Viscous heating mainly located at the interface diapir-mantle (poles)

Similar temperature distribution



T/ΔT

Power law

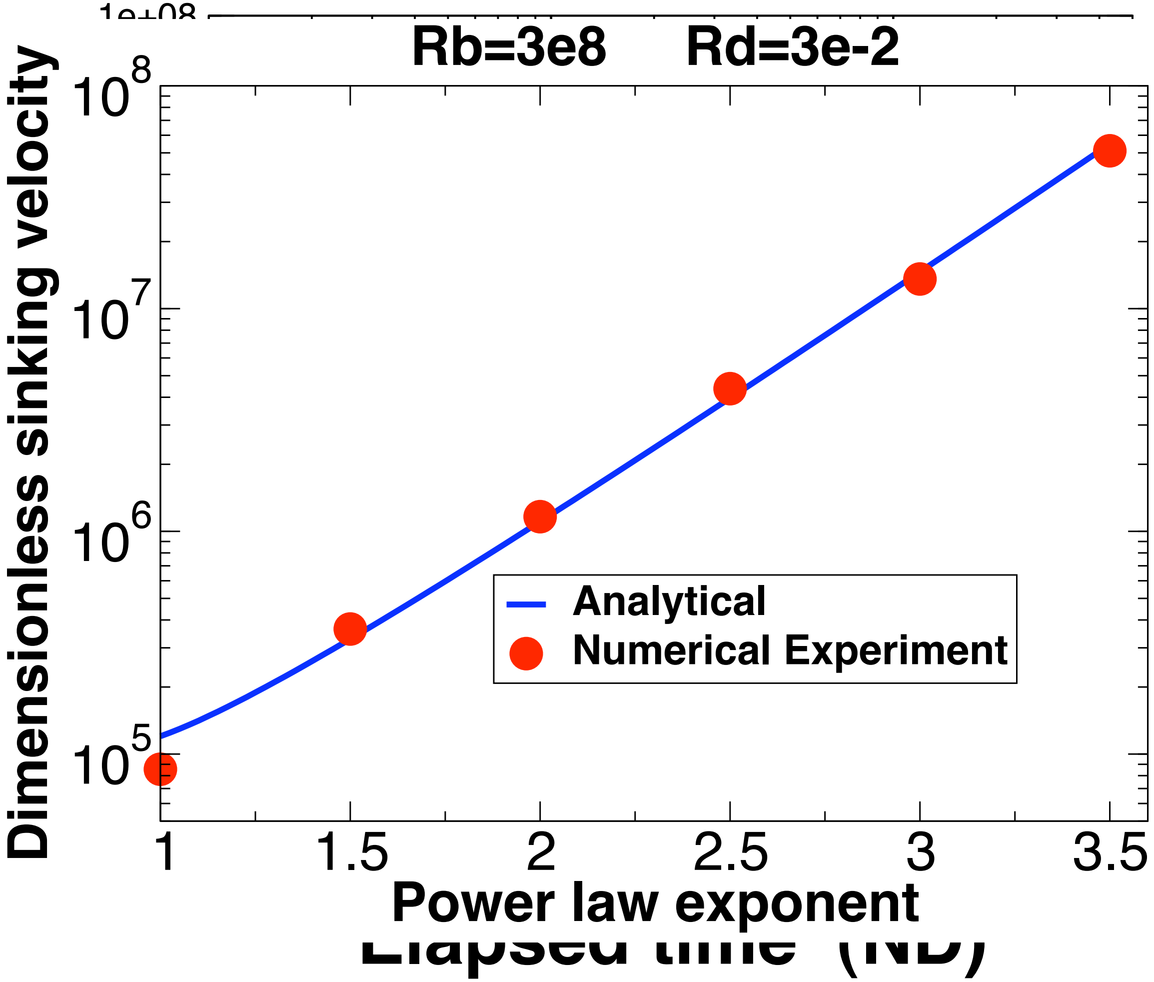


Viscosity decrease zone extends up to several radii

Scaling law for Diapir sinking velocity

Terminal velocity Vs quickly reached in all experiments

➪ Viscous drag ~ Buoyancy



Modified Stokes velocity:
(e.g. [*Weinberg ,1992*])

$$V_s = \frac{1}{3}\frac{\Delta\rho g {R_d}^2}{\eta_e} A(\gamma)$$

Diffusion creep dominant $\sigma_c<\sigma_T$

$$\eta_e = \eta_0$$

Dislocation creep dominant $\sigma_c \geq \sigma_T$

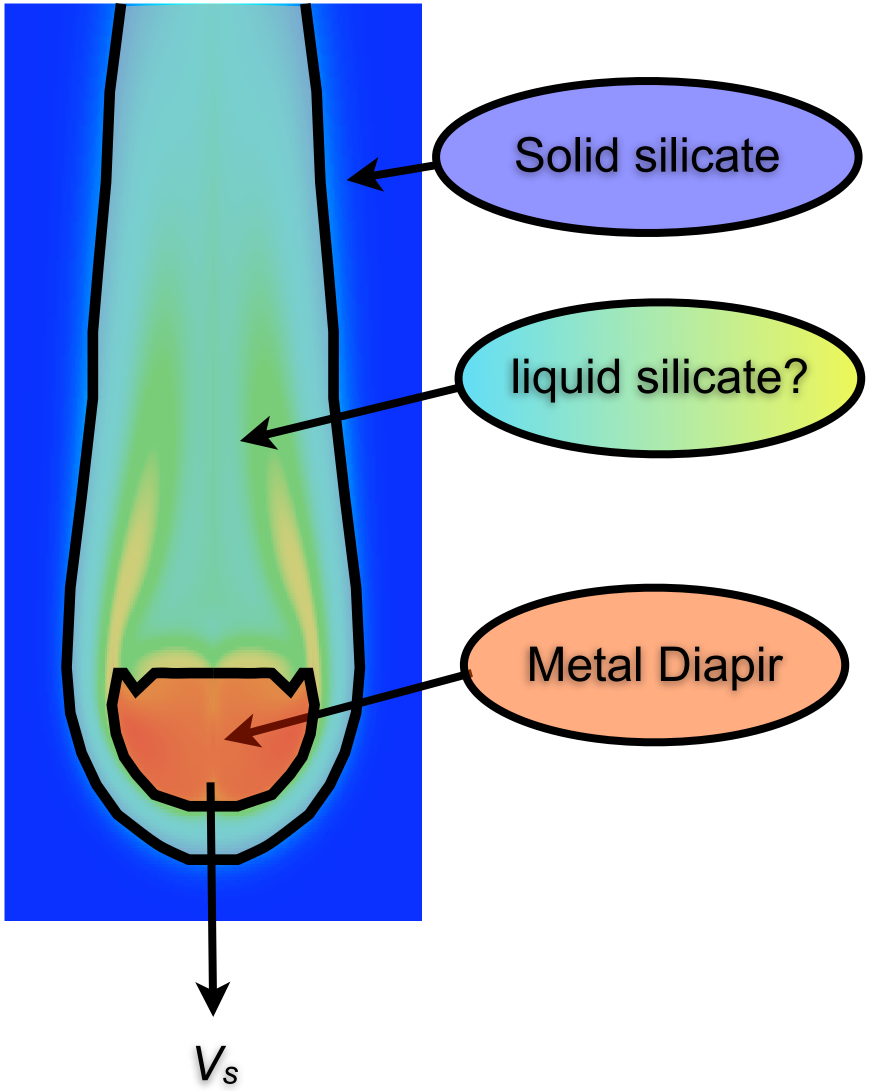
$$\eta_e = \eta_0 \left(\frac{\sigma_c}{\sigma_T}\right)^{1-n} f(n)$$

Describes well the results of our experiments

Melting & Equilibration

Melting and Fe-Si Equilibration

Melt leads to higher κ_c ⇨ Faster chemical exchanges



Under which conditions is silicate melt produced?

- ✓ m-km sized diapir
- ✓ Mostly solid silicate context

Semi-analytical model: simplified equations

Assumptions

Spherical diapir

$\Delta\rho c >> \Delta\rho_T$

T effect on η is localized

Characteristic scales

Velocity:Vs & Length:Rd

Momentum

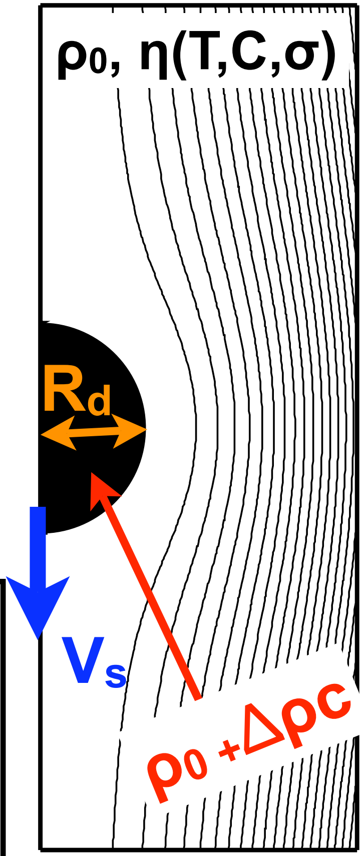
$$\nabla p - \nabla.(\eta\dot{\varepsilon}) + C\vec{e_r} = 0$$

➪ Flow approximated analytically *[hadamard,1911]*

➪ Momentum Eq. parameter free

Energy (2 parameters)

$$\frac{DT}{Dt} = \frac{1}{Pe_T}\nabla^2 T + \Pi_v\ \sigma : \dot{\varepsilon}$$



Governing quantities

$$Pe_T = \frac{V_s R_d}{\kappa_T}$$

Advection/Diffusion velocities

$$\Pi_v = \frac{V_s \eta_0}{R_d \Delta T \rho_0 Cp}$$

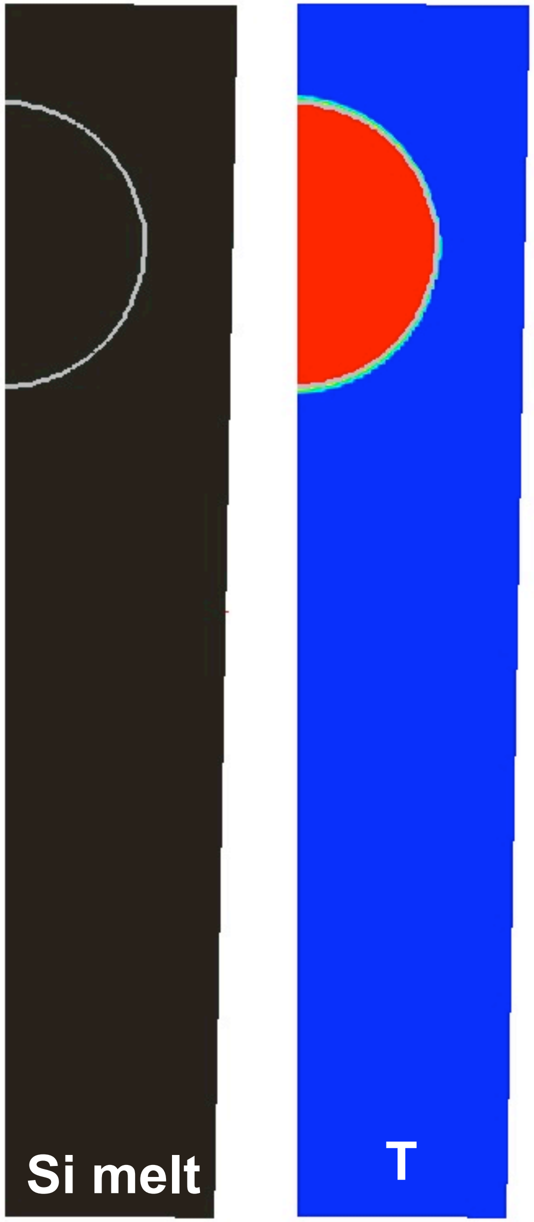
Efficiency of viscous heating

Only one equation to solve (instead of 4) with 2 parameters

Numerical expts. vs. semi-analytical model

Numerical Experiments

4 conservation equations



Semi-analytical model

$$\frac{DT}{Dt} = \frac{1}{Pe_T} \nabla^2 T + \Pi_v \; \sigma : \dot{\varepsilon}$$

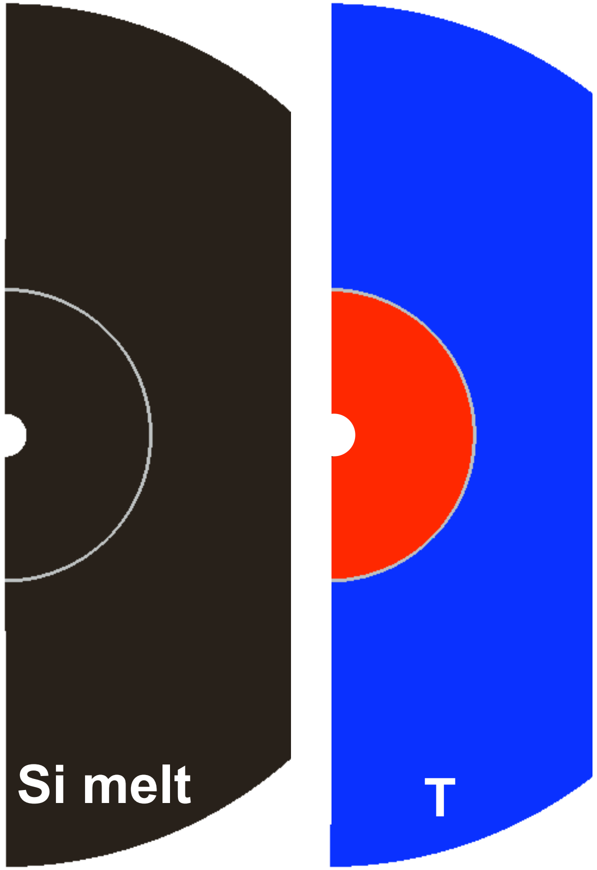
Pe_T=865 Πv=0.7

Sinking distance: 4 R_d

Silicate melt surrounding the Fe diapir is produced

Good agreement

Approximations made for the Semi-Analytical model are reasonable



Lower bound condition for silicate melt ?

$DT/Dt > 0$ ⇨ $F = \Pi_v \, Pe_T > 1$

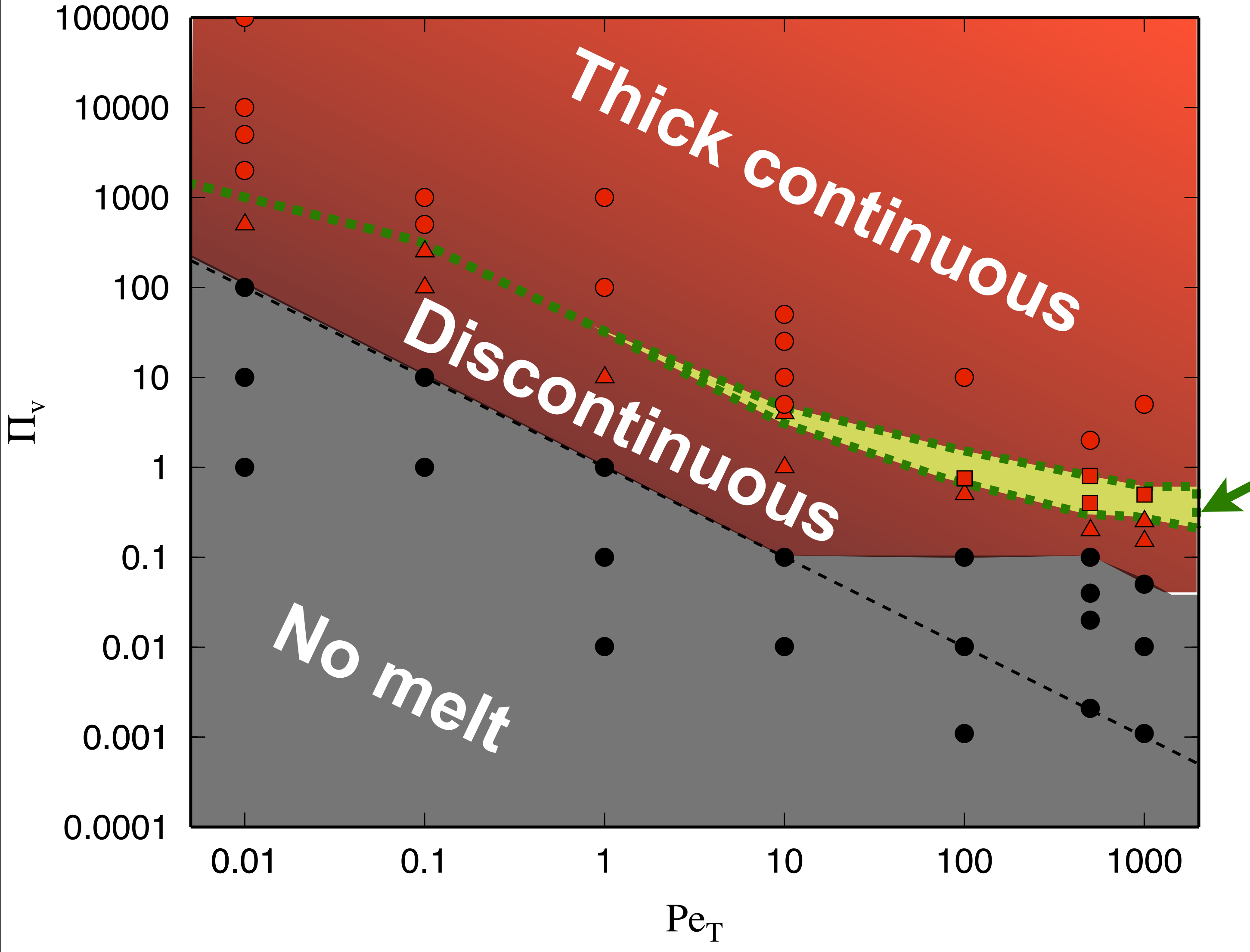
Viscous dissipation > Diffusion

Melt geometry: Which value for F?

Lower bound for silicate melt generation: $F = \Pi_v \, Pe_T > 1$

Upper bound for F?

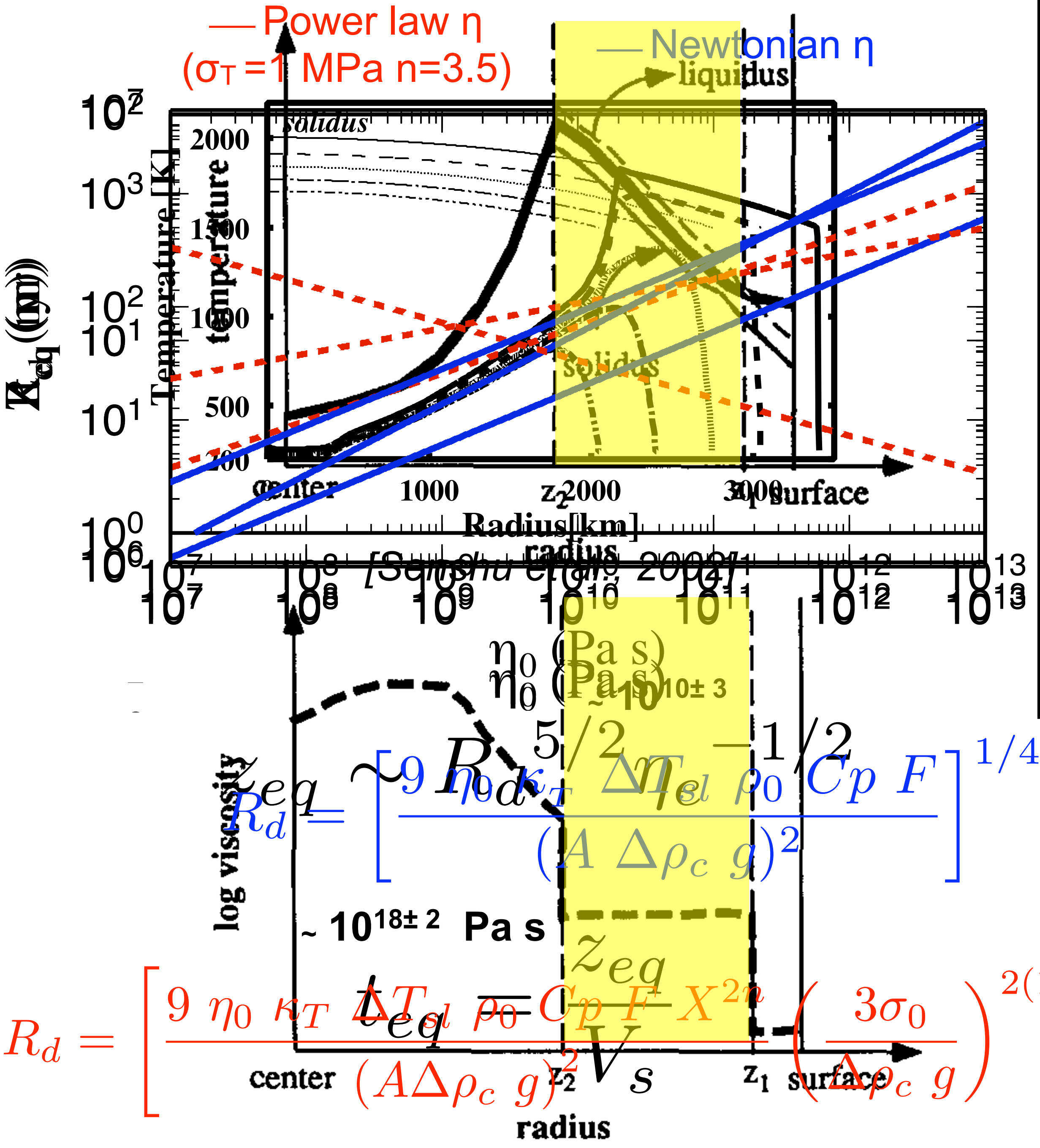
1. Continuous melt layer
2. Thin ~ δ_{CBL} - 0.1 R_d



Thin & continuous

➪ F~10-100

Conditions (R_d?) for metal silicate equilibration



[Senshu et al., 2002]

$$R_d = \left[\frac{9\,\eta_0\,\kappa_T\,\Delta T_{el}\,\rho_0\,C_p\,F}{(A\,\Delta\rho_c\,g)^2}\right]^{1/4}$$

$$z_{eq} \sim R_d^{5/2}\,\eta_0^{-1/2}$$

$$t_{eq} = \frac{z_{eq}}{V_S}$$

$$R_d = \left[\frac{9\,\eta_0\,\kappa_T\,\Delta T_{el}\,\rho_0\,C_p\,F\,X^{2n}}{(A\Delta\rho_c\,g)^2}\left(\frac{3\sigma_0}{\Delta\rho_c\,g}\right)^{2(1-n)}\right]^{\frac{1}{2(n+1)}}$$

[Karato & Murthy, 1997]

Constraints

1. Silicate melt : $\pi_v\,Pe_T \sim 10$
   ➪ $R_d, Z_{eq}, t_{eq} = f(\eta_0)$

2. $\eta_0 \sim 10^{10\pm 3}$ Pa s
   [Karato & Murthy, 1997]

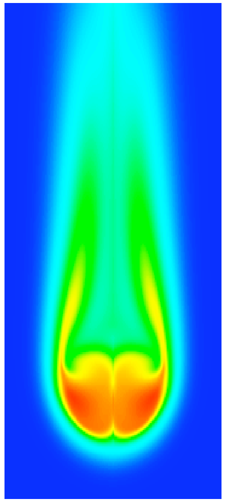
3. Timing < 100 Myrs
   [Kleine et al. , 2004]

R_d~ 1-100m

z_{eq}~ 1000 km

t_{eq}~ 1-10^3 years

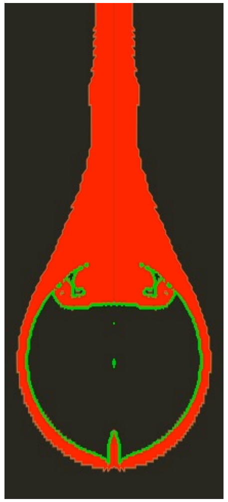
Conclusions I



Study the dynamics of sinking iron diapir surrounded by viscous silicate material

➪ Derived general scaling laws for Vs

➪ Derived general scaling laws for T/melting



Viscous dissipation important at the interface

➪ Produces higher temperatures in the vicinity of the diapir

➪ Silicate melt surrounding the diapir (if $\pi_v \, Pe_T > 1$)

➪ Favors Fe-Si equilibration during the descent of the diapirs

Negative diapirism is a plausible scenario for core formation

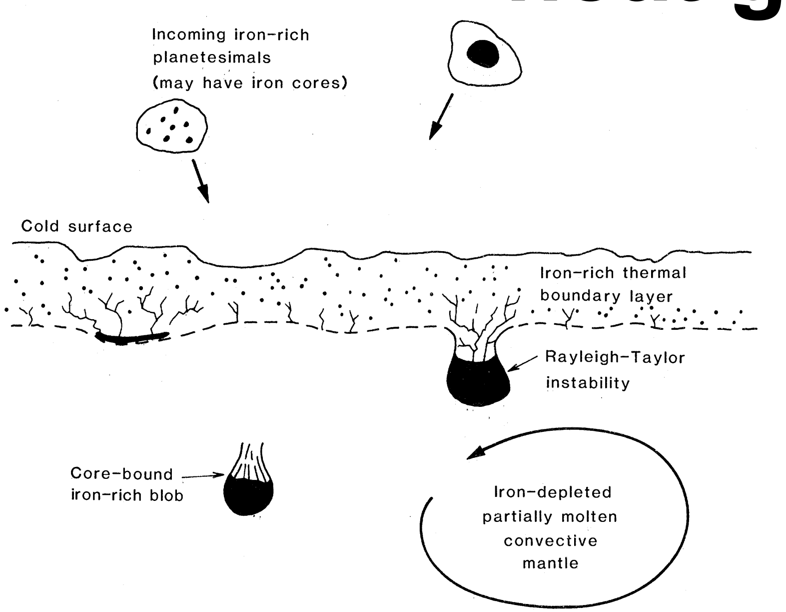
With R_d ~1-100 m , (for $\eta_0 \sim 10^{10\pm 3}$ Pa s) it satisfies:

➪ Hf/W constraints (timing of core formation < 100 Myrs)

➪ Fe-Si equilibration

Heat Distribution

Core formation by negative diapirism and heat generation



[Stevenson,1981]

$$\Delta E_p = \Delta \rho_c V_d \, g \Delta H$$

$$\Delta E_{th} = \rho C p V \delta T$$

$$\Delta E_p \sim \Delta E_{th}$$

Gravitational heating

For Mars: $\Delta E_p \sim 10^{29}$ Joules

For Earth: $\Delta E_p \sim 10^{31}$ Joules

- ⇨ $\Delta T \sim$ 100 K for the whole planet
- ⇨ $\Delta T \sim$ 1000 K for iron core only

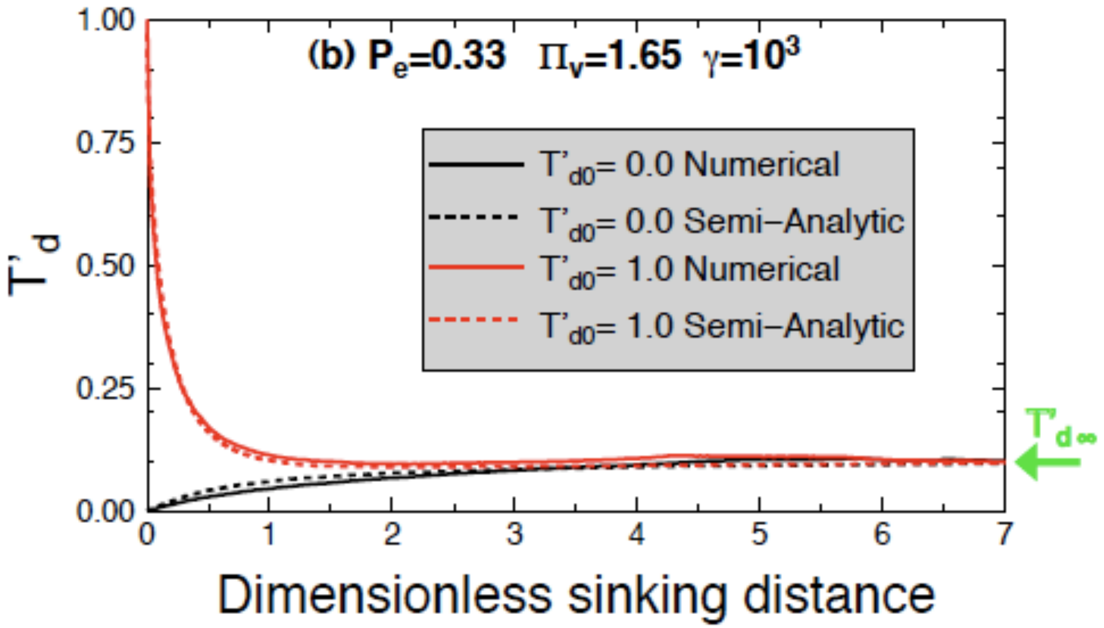
How is it distributed inside a planet?

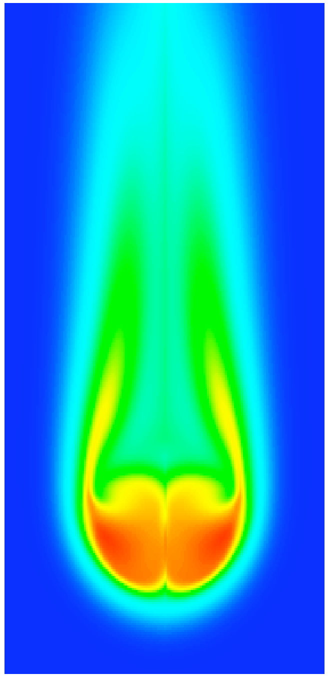
How does heat partitions between the diapirs and their surrounding as they sink?

T_d(t, π_v , Pe_T , rheology)?

$$\frac{DT}{Dt} = \frac{1}{Pe_T}\nabla^2 T + \Pi_v\ \sigma : \dot{\varepsilon}$$

Easy & fast to solve for 1 diapir



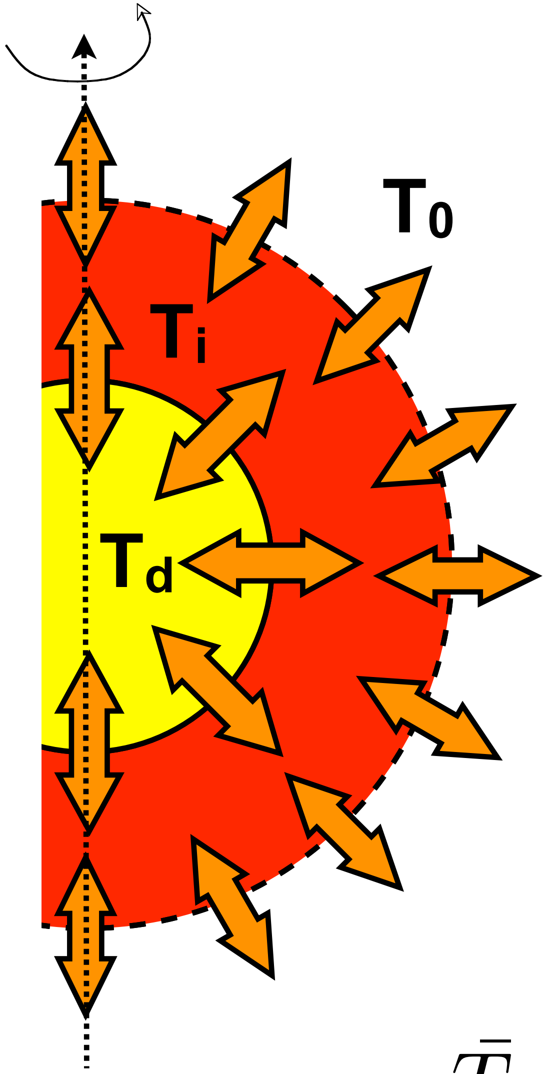


Very challenging for 10^6 diapirs

Additional simplification is required
➪ Scaling laws for T_d (t)

Seeking for T$_d$(t) : Analytical modeling

$$\frac{DT}{Dt} = \frac{1}{Pe_T}\nabla^2 T + \Pi_v \ \sigma : \dot{\varepsilon}$$



"Sphere-Shell" model

$$\boxed{\frac{d\bar{T}_d}{dt} = -C_d(\bar{T}_d - \bar{T}_i)}$$

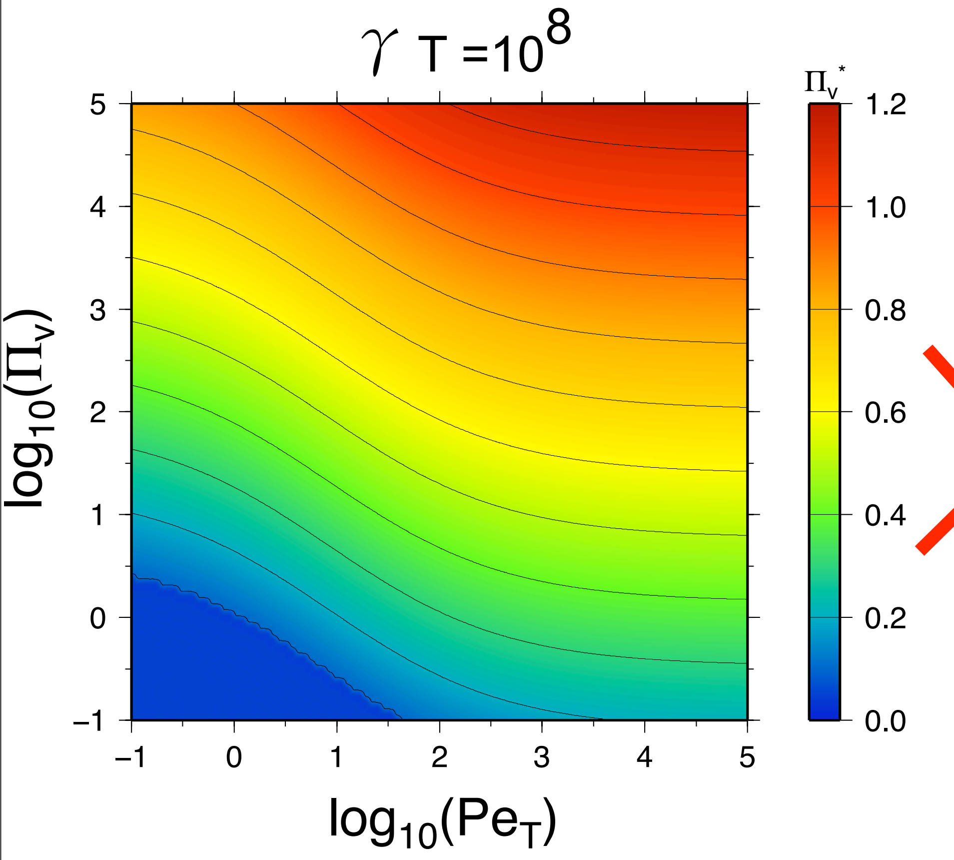
$$\boxed{\frac{d\bar{T}_i}{dt} \cong -C_i(2\bar{T}_i - \bar{T}_d) + \delta\bar{T}_i^v}$$

... Finally

$$\bar{T}_d = \Pi_v^* + (\bar{T}_d^0 - \Pi_v^*)\ \exp\left(-\frac{1}{Pe_T^*}\ t\right)$$

“Sphere-Shell” analytical-empirical model

$$\bar{T}_d = \Pi_v^* + (\bar{T}_d^0 - \Pi_v^*)\,\exp\left(-\frac{1}{Pe_T^*}\,t\right)$$ with $$Pe_T^* \sim Pe_T^{1/2}$$ $$\Pi_v^* = \lim_{t\to\infty} T_d$$



How to determine π_v*?

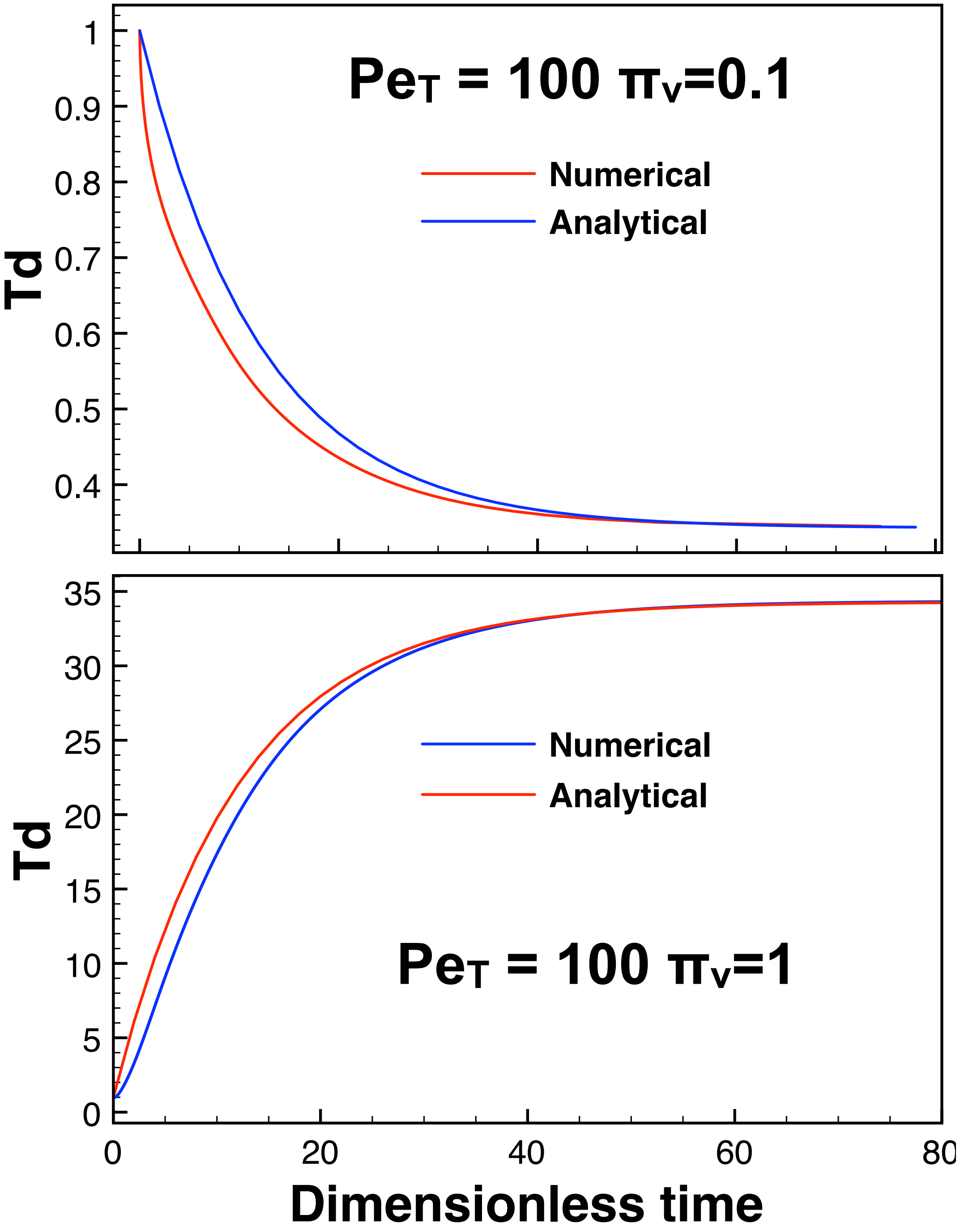
$$\cancel{\frac{\partial T}{\partial t}} + \boxed{\mathbf{U}.\nabla T = \frac{1}{Pe_T}\nabla^2 T + \Pi_v \sigma : \dot{\varepsilon}}$$

For various Pe_T, Π_v , γ T

Parameterization: $\Pi_v^* = f_1(Pe_T, \gamma_T) + f_2(\gamma_T)\ln(\Pi_v)$

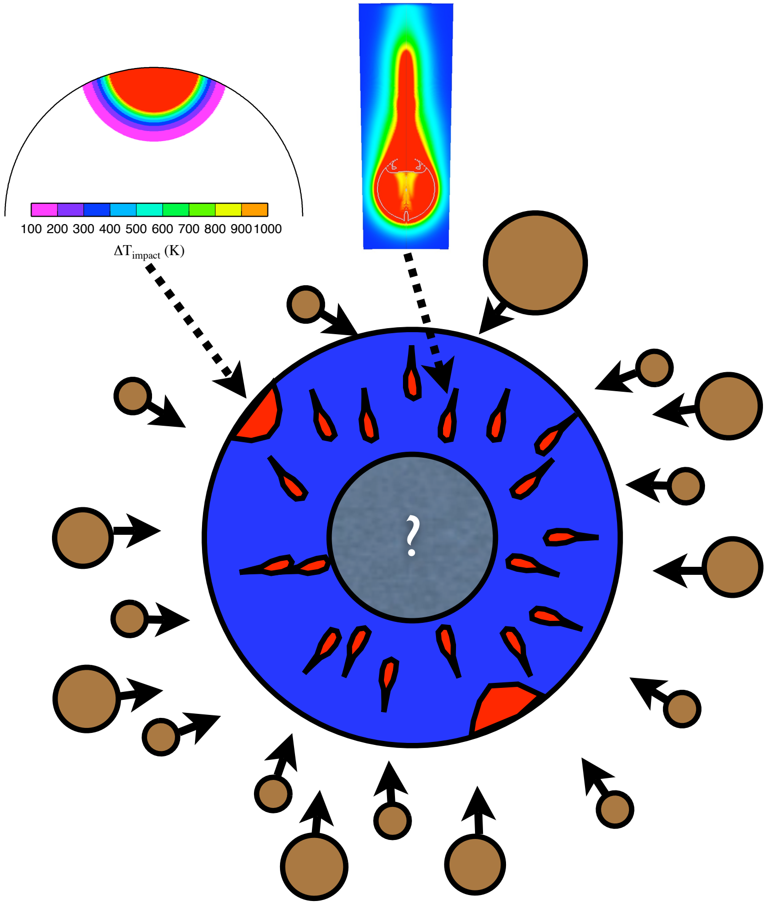
⇨We have everything we need!

Analytical vs. Numerical results



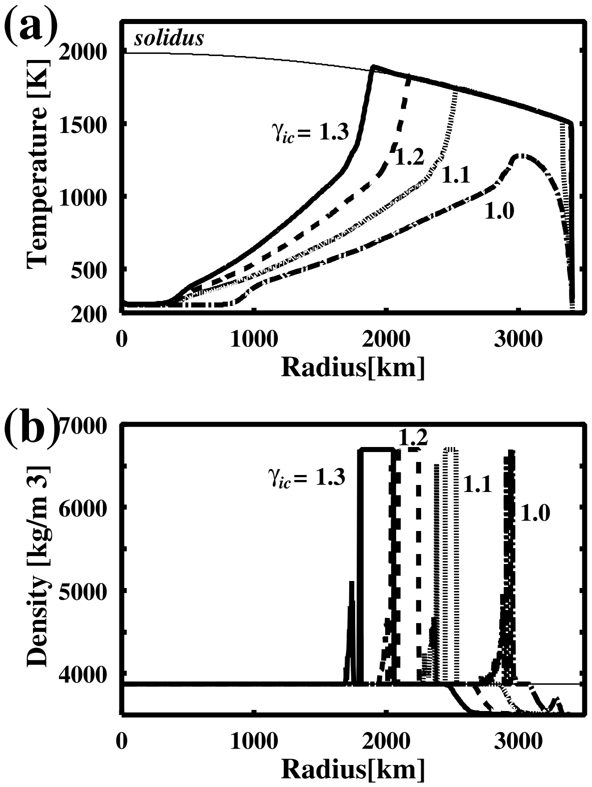
$T_d(t)$ can be described analytically in good agreement with the numerical results

Heat partitioning during core formation?



Parameterized models

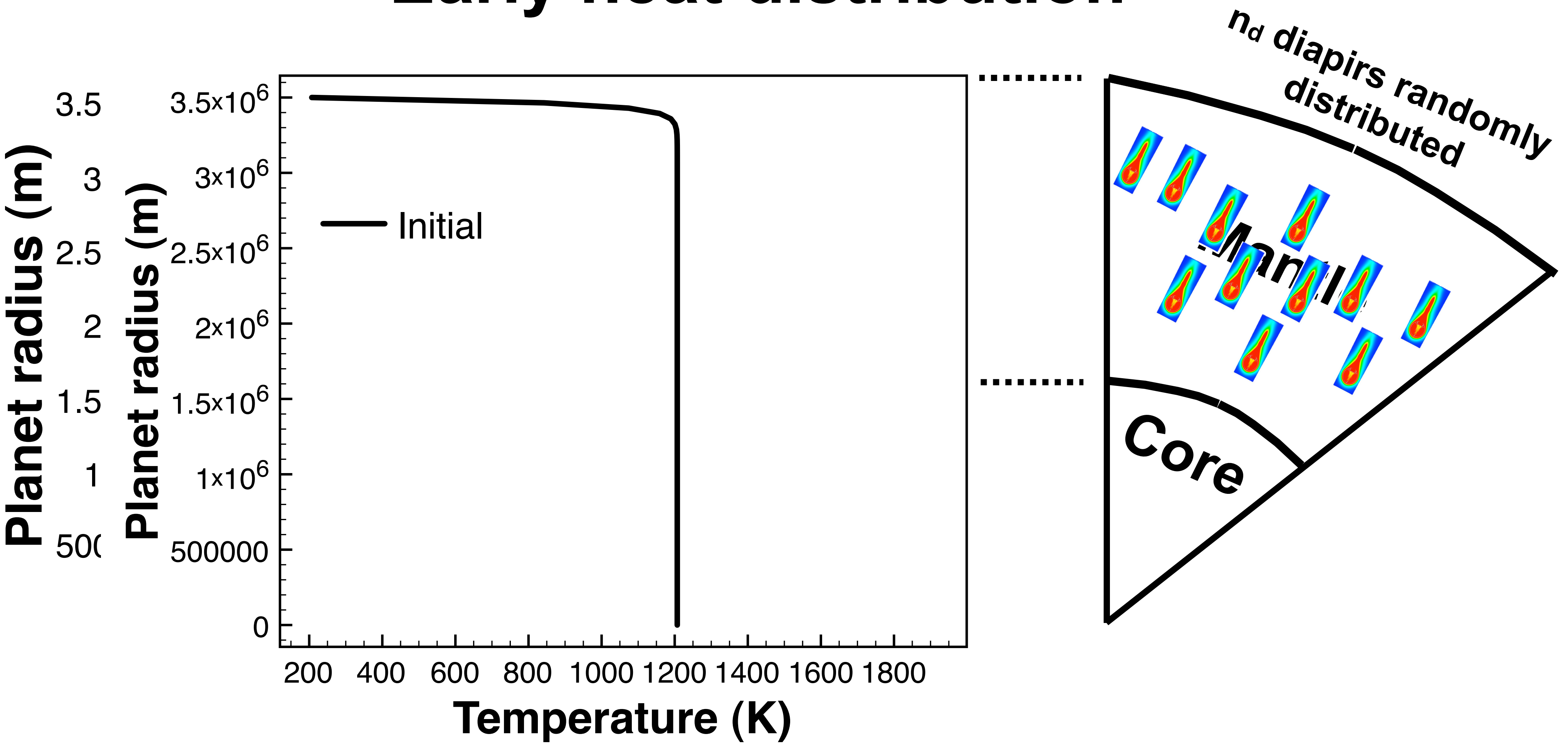
➪ Accretion
➪ Impact heating
➪ Isostatic readjustment
➪ Differentiation
➪ Convection
➪ Viscous heating
➪ Diffusion
➪ Radioactive heating ...



[Senshu et al., 2002]

Heat partitioning too simplified ?
Thermal equilibrium assumed [senshu et al., 2002]

Early heat distribution



Gravitational Heat partitioning

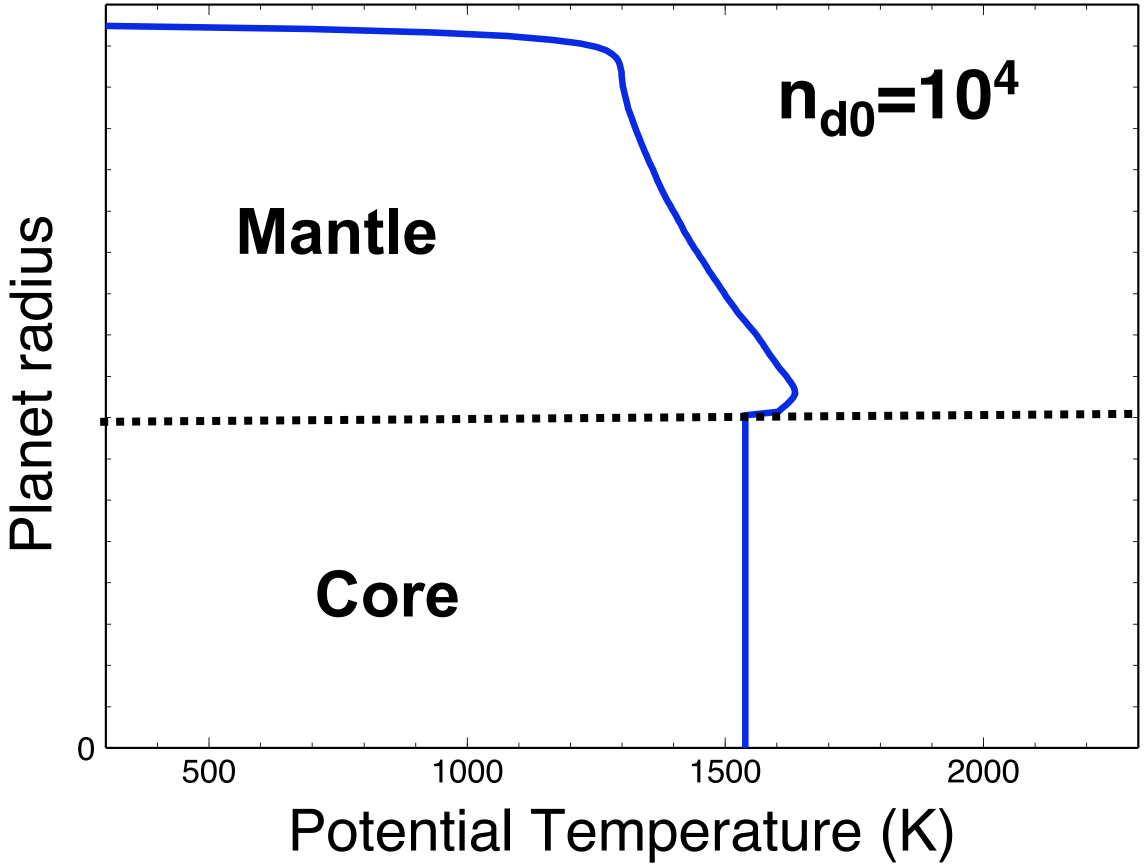
$$F_h = \frac{E_p^{\mathrm{Fe}}}{E_p^{\mathrm{total}}}$$ Fixed

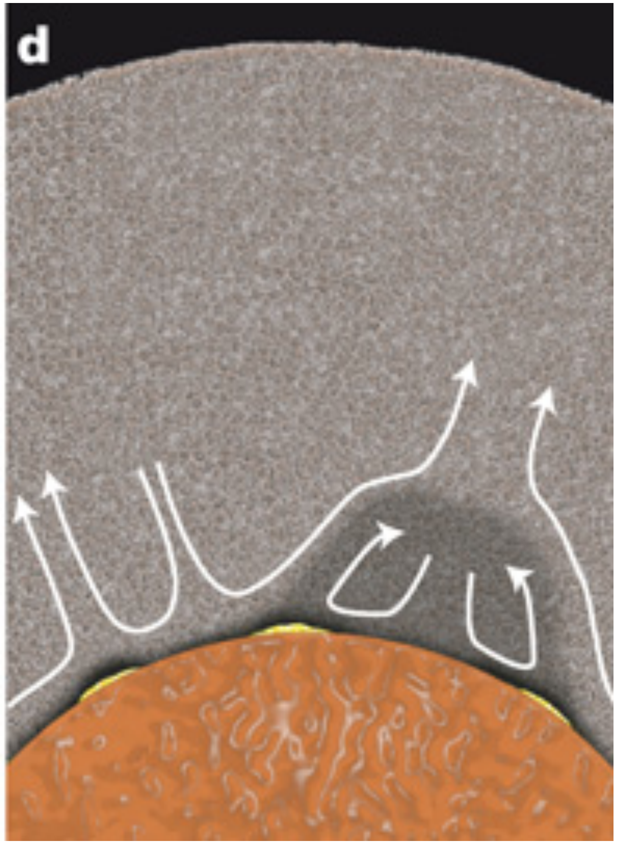
...Or

$$\bar{T}_d = \Pi_v^* + (\bar{T}_d^0 - \Pi_v^*)\ \exp\left(-\frac{1}{Pe_T^*}\ t\right)$$

Simple parameterizations (fixed Fh) unrealistic

Early heat distribution



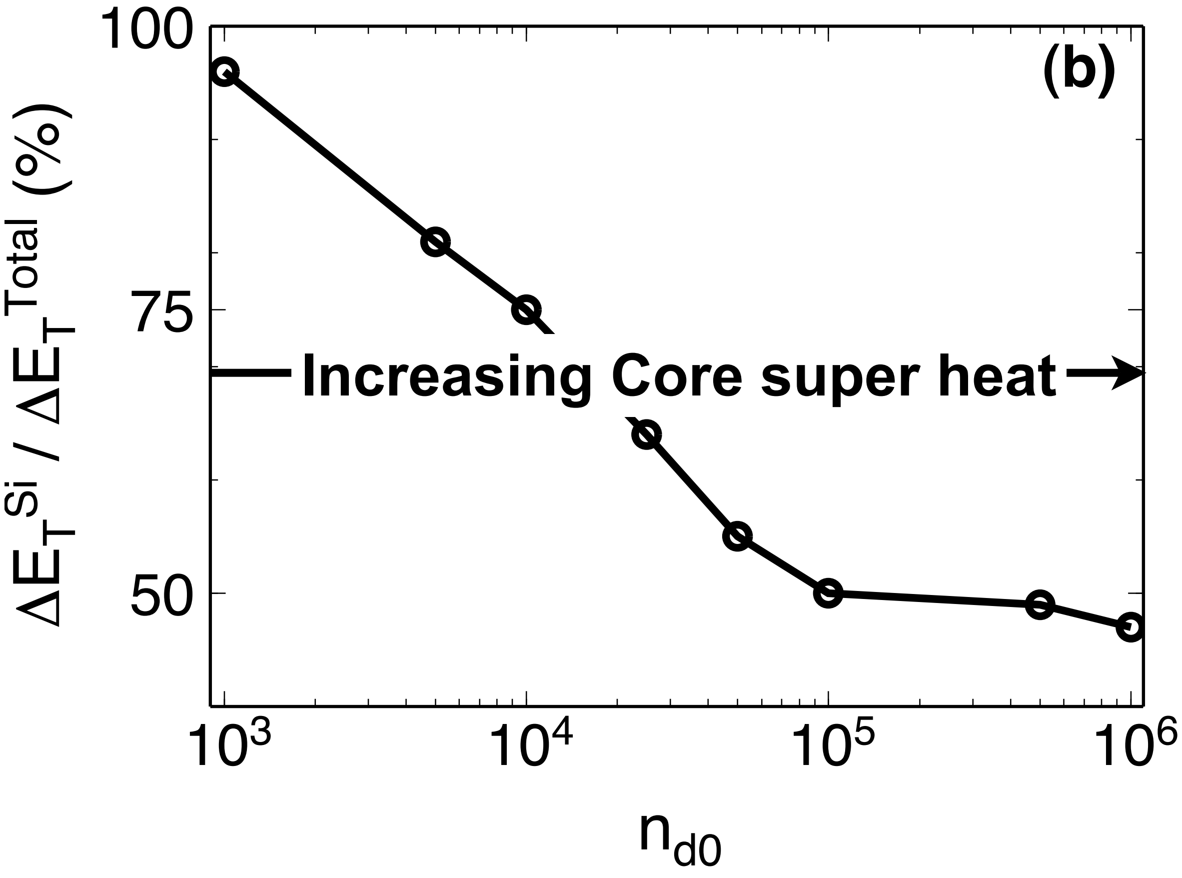


[Labrosse et al., 2007]

Favors a hotter lowermost mantle with melting (BMO?)

Heat partitioning and diapir size distribution

$$\bar{T}_d = \Pi_v^* + (\bar{T}_d^0 - \Pi_v^*)\ \exp\left(-\frac{1}{Pe_T^*}\ t\right)$$



Decreasing diapir radius →

Heat partitioning strongly depends on the size distribution of planetesimals

Smaller diapirs (larger n_{d0})
➪ lower P_{eT}
➪ faster heat exchanges
➪ higher core super heat

Can explain the presence (or absence) and sustainability of geodynamo on Earth or Mars

Conclusions II

✓ We derived scaling laws to determine heat partitioning between sinking iron diapir and their surroundings

✓ Our scaling laws can be used to model the thermo-chemical evolution of a growing planet

✓ Simple parameterizations for heat partitioning during core formation are unrealistic

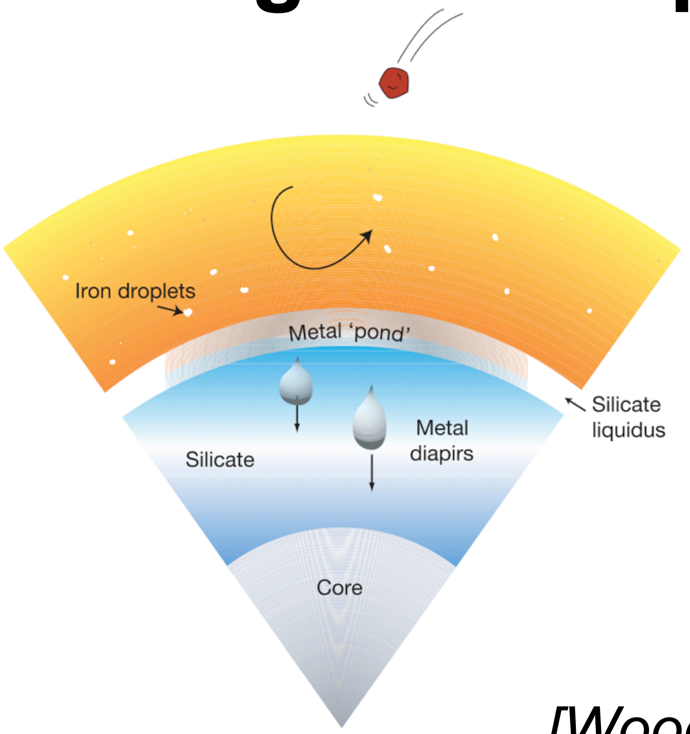
✓ Negative diapirism favor higher LM temperatures

✓ Small diapirs lead to higher core superheat

Now what?

Negative Diapirism in Magma Oceans

Negative diapirism in a magma oceans



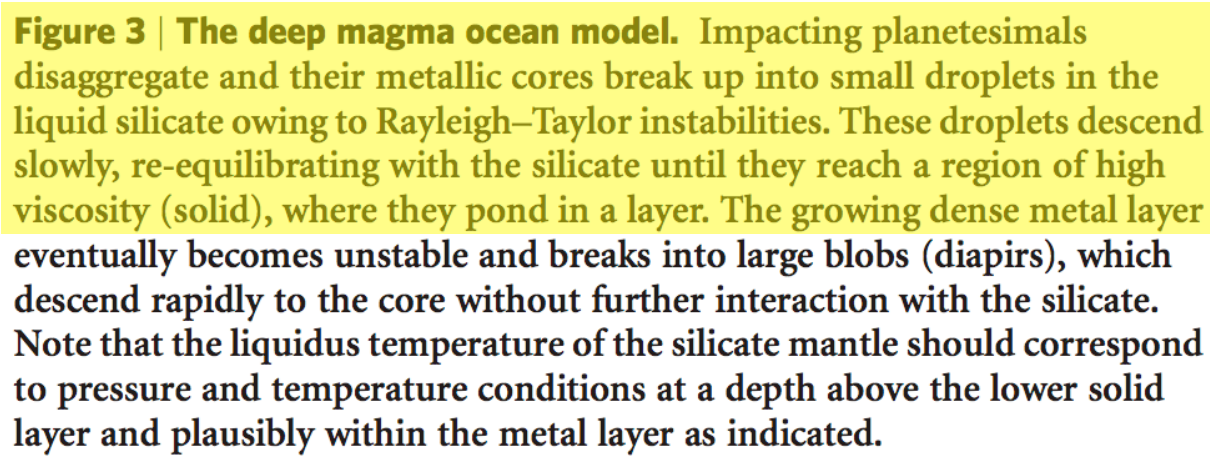


Figure 3 | The deep magma ocean model. Impacting planetesimals disaggregate and their metallic cores break up into small droplets in the liquid silicate owing to Rayleigh–Taylor instabilities. These droplets descend slowly, re-equilibrating with the silicate until they reach a region of high viscosity (solid), where they pond in a layer. The growing dense metal layer eventually becomes unstable and breaks into large blobs (diapirs), which descend rapidly to the core without further interaction with the silicate. Note that the liquidus temperature of the silicate mantle should correspond to pressure and temperature conditions at a depth above the lower solid layer and plausibly within the metal layer as indicated.

[Wood et al., 2006]

Motivations

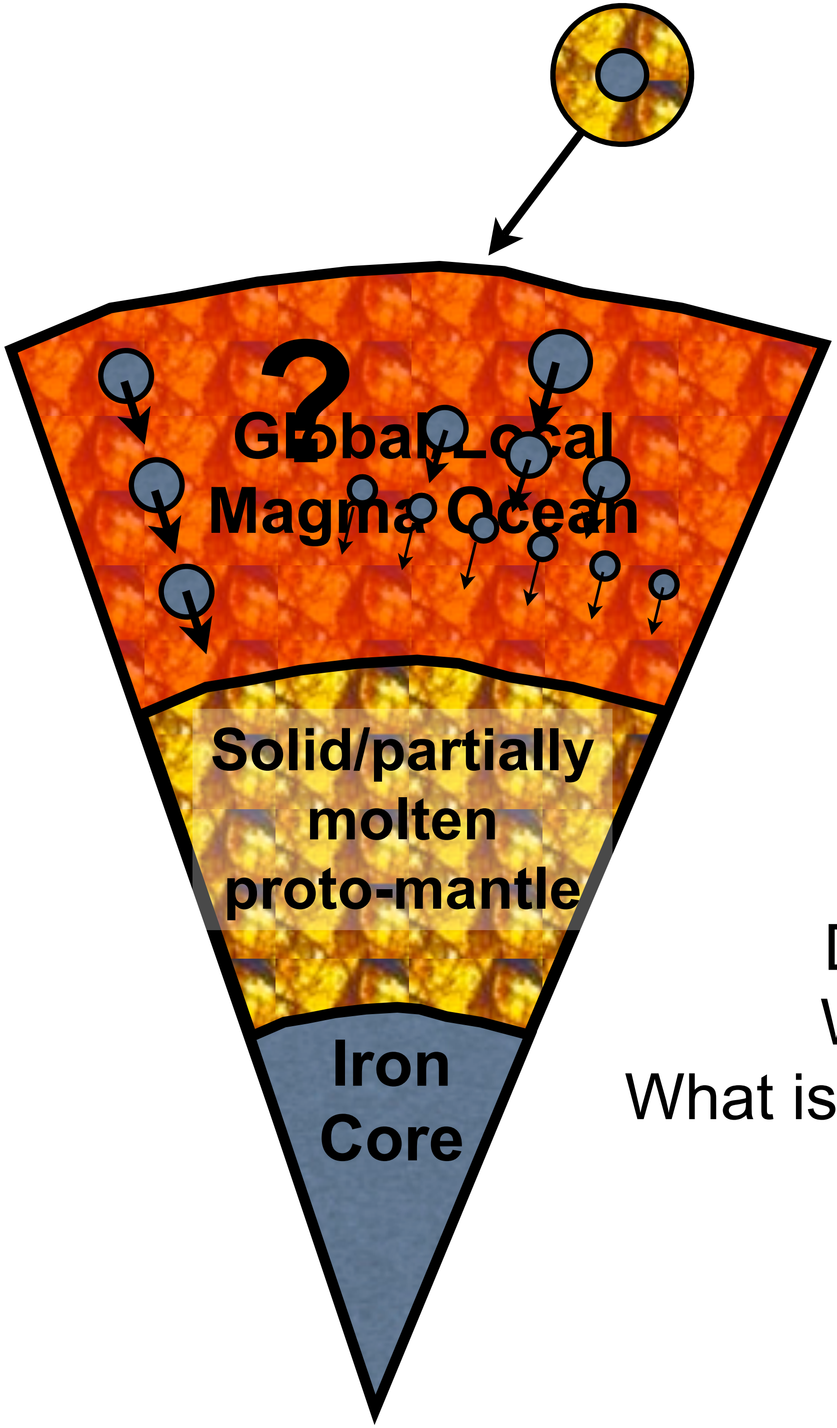
✓ Terrestrial planets probably experienced magma ocean stages during their formation

✓ Differentiated impactors may have hit growing planets

➪ what happened to the iron cores?

iron cores fragmentation/breakup into cm-sized droplets often **assumed**

Negative diapirism in a magma oceans



Differences with previous experiments

✓ Very low viscosities~ 10^{-2} Pa s
➪ Inertia effect cannot be neglected
✓ Shearing instabilities
➪ diapirs fragmentation
✓ Smaller scales involved
➪ surface tension might be important

Questions

Do diapirs fragmentate as they sink?
What is the timing for fragmentation?
What is the timing for complete Fe-Si segregation?
Conditions for Fe-Si equilibration?

Modeling Diapir Fragmentation

Important ingredients

- ✓ Inertia
- ✓ Surface tension
- ✓ Sharp and strong variations in material properties
- ✓ Variable scales to resolve accurately

Mass: $\nabla.\mathbf{U} = 0$

Momentum: ${P_r}^{-1} D_t \mathbf{U} = -\nabla p + \nabla.\tau_{ij} + R_a(T - BC)\mathbf{e}_z + \Pi_\sigma \xi \nabla C$

Composition (Fe-Si): $D_t C = 0$

Composition (Minor components: Ni, Co...): $D_t C_i = \kappa_i \nabla^2 C_i$

Energy: $D_t T = \nabla^2 T + H_v \phi_v - Di\ (T - T_0)\ U_z$

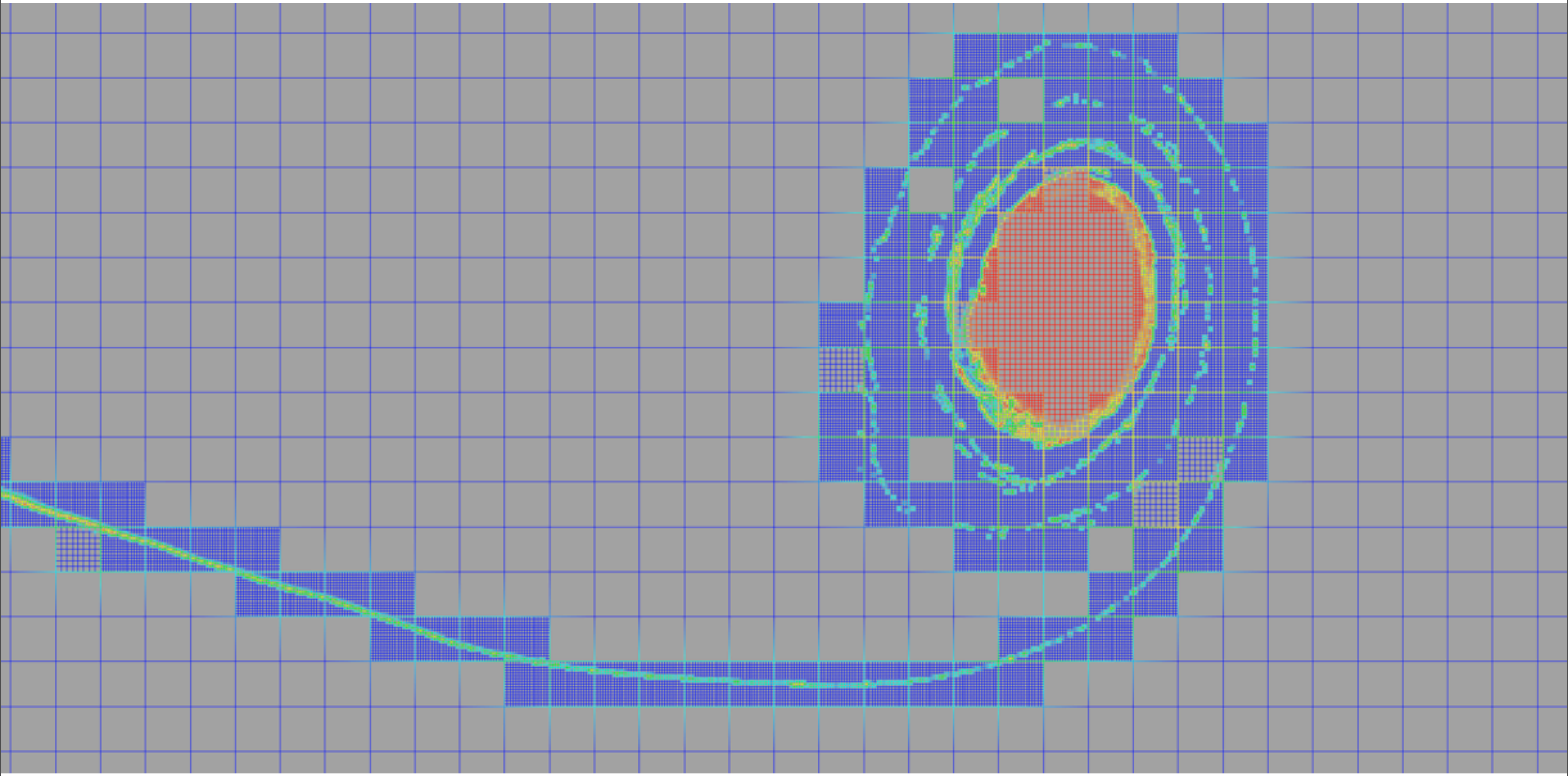
Governing parameters

$$B = \frac{\Delta\rho_c}{\rho_0 \alpha \Delta T} \qquad Di = \frac{\alpha g L}{C_p} \qquad P_r = \frac{\eta_0}{\rho_0 \kappa_T} \qquad \frac{\kappa_c^i}{\kappa_T}$$

$$Ra = \frac{\rho_0 g \alpha \Delta T L^3}{\eta_0 \kappa_T} \qquad H_v = \frac{Di}{R_a} \qquad \Pi_\sigma = \frac{\sigma L}{\eta_0 \kappa_T}$$

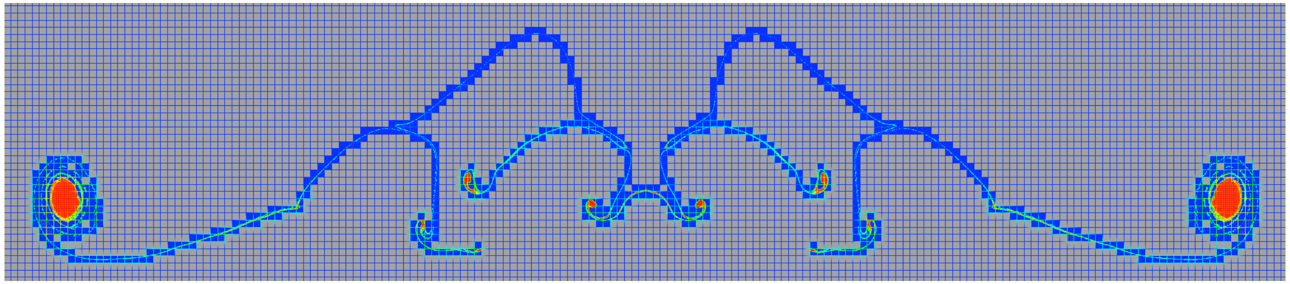
➪ **Solved with PIC, FV formulation +** Adaptive Mesh Refinement**: STREAMV**

Preliminary results



Fragmentation occurs
The diapir separates several smaller bodies
Cascade mechanism (self-similar?)

(Preliminary) ‘Conclusions’

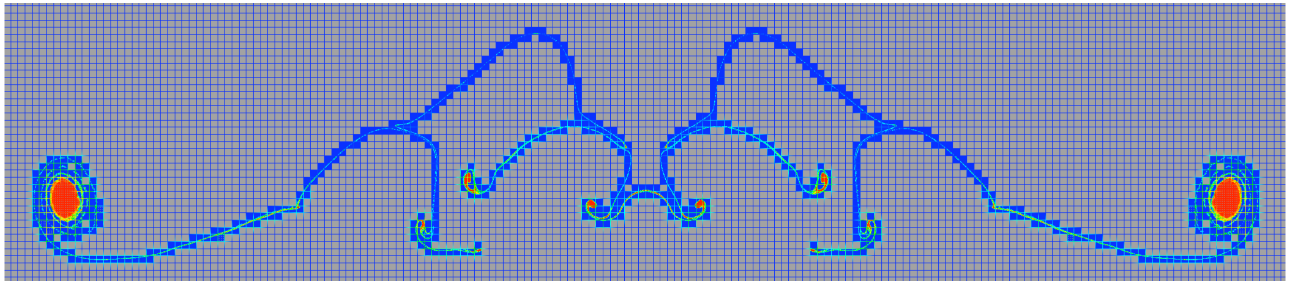


✓ Need to perform systematic runs : Pr, Rb, π_σ …

✓ Determine the conditions for fragmentation

✓ Characterize the dynamics of fragmentation (Self-similar? Size distribution of the new generations?)

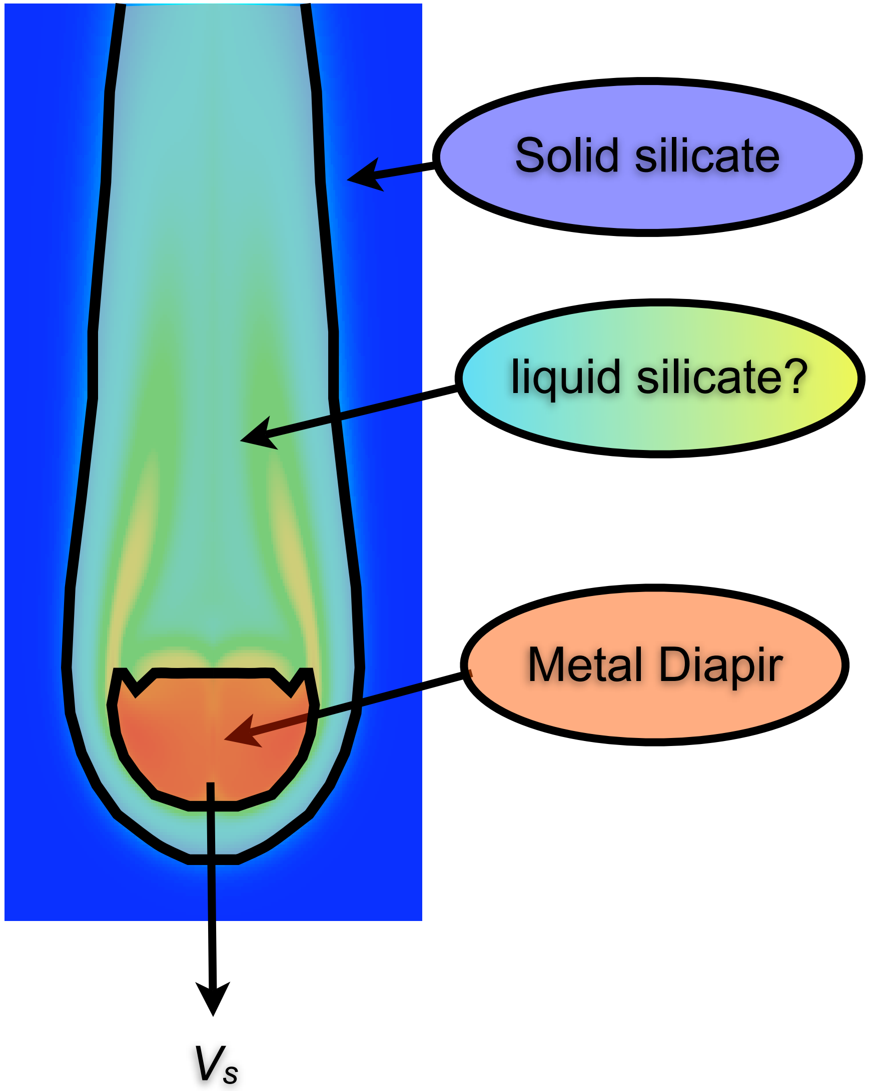
✓ Derive scalings!



Thank you

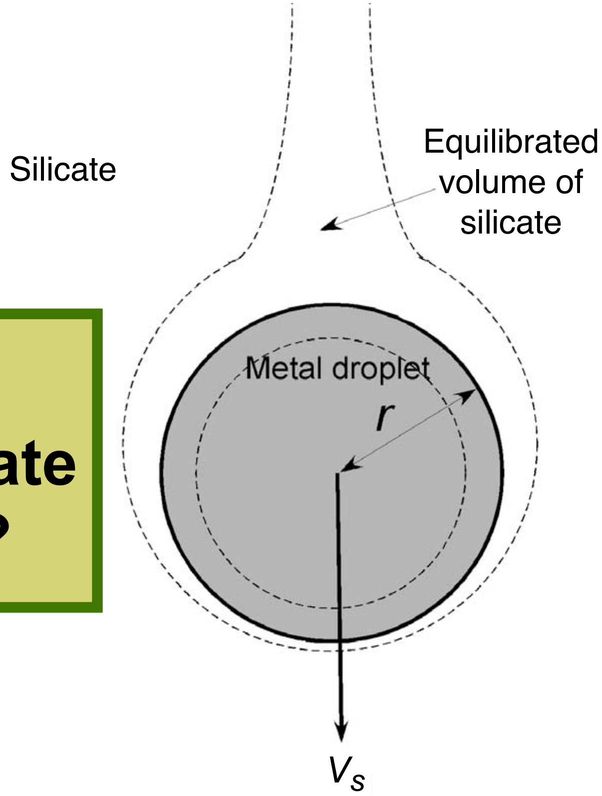
Melting and Fe-Si Equilibration

Melt leads to higher κ_c ➪ Faster chemical exchanges



- ✓ m-km sized diapir
- ✓ Mostly solid silicate context

Under which conditions is silicate melt produced?



From [Rubie et al, 2003]

- ✓ cm sized droplet
- ✓ magma ocean context