Damped Frank-Kamenetskii Approximation for Temperature- and Pressure-Dependent Rheologies and Consequences for the Simulation of Super-Earths

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The viscosity of a silicate mantle is strongly dependent on temperature and viscosity and can be described by the Arrhenius law [Karato and Wu 1993]. Assuming realistic values of the activation energy and volume, viscosity variations in a terrestrial mantle amount to values of about 10^35 Pas or larger. Most numerical codes, however, can only deal with much smaller viscosity contrasts. A common method to reduce the viscosity contrast is to use the Frank-Kamenetskii approximation [Frank-Kamenetskii 1969]. Applying this approximation for purely temperature-dependent viscosity, the results are similar to those using the Arrhenius law when the convection is in the steady-state stagnant lid regime [Solomatov and Moresi 1996; Plesa et al. 2010].

Here, we compare the Frank-Kamenetskii approximation to the Arrhenius law for both temperature- and pressure-dependent viscosity. For this, we have derived new Frank-Kamenetskii parameters that also include the pressure-dependence of the viscosity and differ from previous approximations [Christensen 1984, Hansen and Yuen 2000]. We show that using these parameters, the depth-dependence of the approximated viscosity is comparable to the more realistic Arrhenius viscosity. It is even possible to model a stagnant lower mantle that may form above the core-mantle boundary in case of high activation volume or high pressure like for massive Super-Earths [Noack et al. 2010].

Nevertheless, for high surface temperatures like for Venus this approximation does not represent the mantle flow as it is derived by the Arrhenius law. In these cases no stagnant lid forms with the classical Frank-Kamanetskii approximation because the linearized viscosity results in a viscosity contrast of less than ~10^5. For a viscosity contrast lower than ~10^5 the convection regime changes to the transitional or mobile regime [Solomatov and Moresi 1996]. To overcome this problem, we have further derived a new approximation, which we call the Damped Frank-Kamenetskii approximation. This is a mixture between the classical law and a second-order approximation controlled by a damping parameter. The second-order approximation is not a linearization of the exponential viscosity term like for the classical Frank-Kamenetskii approximation, but a quadratic approximation with a higher accuracy. The new method leads to a stagnant lid in all cases treated in our investigations. Note that for planets with low surface temperatures, this new approximation can be used as well with a damping parameter of zero that yields the standard Frank-Kamenetskii approximation.

Our studies suggest that the classical Frank-Kamenetskii approximation with the here derived parameters can be used to simulate the mantle dynamics in Super-Earths even with a high pressure-dependence of the viscosity if the viscosity contrast is above ~ 10^5 and a stagnant lid forms on top the convecting mantle. In case the constraint of a stagnant lid is not satisfied, the new Damped Frank-Kamenetskii approximation should be used instead.

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