Thermo-mechanical modelling with a free surface: the sticky air approach

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Numerical thermo-mechanical models of the crust or lithosphere often need to take into account the free surface in order capture the formation of dynamic topography. Dynamic topography may be important as an observable quantity of dynamic processes such as rifting, as a feed back into the dynamics of the model, e.g. for subduction dynamics, as a significant contribution to the geoid; and as an important contribution to the total topography

The boundary condition of a free surface in a dynamic model is zero traction at the (deformed) surface. In Lagrangian approaches such as Finite Elements this condition is easily fulfilled, however care has to be taken, to ensure that the mesh is advected with the displacement or flow. In Eulerian approches usually regular grids are used. Thus precise tracking of the free surface is necessacy. Two alternative approaches are usually used:

a) Free slip, no vertical movement at top of the model (*McKenzie, 1974*): The normal stress σ_{zz} at the surface due to the flow is taken to obtain the topography h using the first term of a Taylor series expansion, i.e. $\sigma_{zz} = - \rho\ g\ h$ (ρ - density, g – gravitational acceleration). This works well if h << wavelength and if the slopes remain small. As a disadvantage also the pressure is needed, which may cause numerical resolution problems for large viscosity contrasts. Another drawback is the assumption of instantaneous isostatic adjustment, which may cause spurious effects for time-dependent processes.

b) Sticky air layer: In this approach the model box is vertically enlarged by a thin layer of low viscous material of density zero (*Schmeling et al., 2008*). To properly resolve topography variations on an isostatic relaxation time scale, a condition for the thickness h_{st} and the viscosity η_{st} of the sticky air layer has to be fulfilled, namely C << 1 with

$$C = \frac{3}{8\pi}\left(\frac{L}{h_{st}}\right)^3 \frac{\eta}{\eta_{st}} \qquad (1)$$

where L = width of the model, η = characteristic model viscosity. In the long term isostatic limit C may be replaced by

$$C = \frac{1}{16}\frac{\Delta\rho}{\rho}\left(\frac{h_{model}}{h_{st}}\right)^3 \frac{\eta}{\eta_{st}} \qquad (2)$$

with h_{model} = model thickness. In both cases, usually the time step has to be below the isostatic relaxation time. Problems encountered by this approach include: Entrainment of sticky air in subduction modelling, thermal convection within the sticky air layer, numerical erosion at the model – sticky air interface.

Comparison of the different approaches show: In rift models viscous bending of the lithosphere with a few 100 m flank uplift is absent (approach a) or present (b). Sticky air captures isostatic uplift on a kyear scale, which is absent for the stress derived topography. In conclusion, fully free surface cases and sticky air cases agree well if the viscosity and thickness of the sticky air layer fullfil the above mentioned condition with C given by (2).

References:

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