RECENT DEVELOPMENTS IN RAYLEIGH-BÉNARD CONVECTION

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ABSTRACT: This review summarizes results for Rayleigh-Bénard convection which have been obtained over the past decade or so. It concentrates on convection in compressed gases and gas mixtures with Prandtl numbers near one and smaller. In addition to the classical problem of a horizontal stationary fluid layer heated from below, it also covers briefly convection in such a layer with rotation about a vertical axis, with inclination, and with modulation of the vertical acceleration.

CONTENTS

The whole review (70 pages) can be obtained as a pdf-file upon request (werner.pesch@uni-bayreuth.de)

1 INTRODUCTION

Fluid motion driven by thermal gradients (thermal convection) is a common and important phenomenon in nature. Convection is a major feature of the dynamics of the oceans, the atmosphere, and the interior of stars and planets (Busse, 1978, 1989; Getling, 1998). It is also important in numerous industrial processes. For many years, the quest for the understanding of convective flows has motivated numerous experimental and theoretical studies.

In spatially extended systems convection usually occurs when a sufficiently steep temperature gradient is applied across a fluid layer. The spatial variation of a convection structure often is referred to as a *pattern*. The nature of such convection patterns is at the center of this review. Pattern formation is determined by nonlinear aspects of the system under study. For this reason the elucidation of pattern formation is a challenging problem in condensed-matter physics as well as in fluid mechanics. Pattern formation is common also in many other spatially-extended nonlinear nonequilibrium systems in physics, chemistry, and biology (Manneville, 1990; Cross & Hohenberg, 1993). Patterns observed in diverse systems are often strikingly similar, and their understanding in terms of general, unifying concepts has long been a main direction of research (Cross & Hohenberg, 1993; Newell et al, 1993).

Many fundamental aspects of patterns and their instabilities have been studied intensively over the past three decades in the context of Rayleigh-Bénard convection (RBC). In a traditional RBC experiment a horizontal fluid layer of height d is confined between two thermally well conducting, parallel plates. When the difference $\Delta T = T_b - T_t$ between the bottom-plate temperature T_b and the top-plate temperature T_t exceeds a critical value ΔT_c, the conductive motionless state is unstable and convection sets in. The simplest pattern which can occur is that of straight, parallel convection rolls with a horizontal wavelength $\lambda \approx 2d$ (wave number $q \approx \pi/d$). Such rolls can be found near onset; however, as the dimensionless distance $\epsilon \equiv \Delta T/\Delta T_c - 1$ increases, the patterns often become progressively more complicated, and thus also more interesting.

Rayleigh-Bénard convection is perhaps the most thoroughly investigated and understood pattern-forming system. The experimental setup is simple in principle and the basic physical mechanism (buoyancy vs. dissipation) well understood. For the standard description in terms of the Oberbeck-Boussinesq equations, Eqs. ?? and ?? below, only two nondimensionalized control parameters are sufficient (Busse, 1978, 1989). The first is the Rayleigh number $R \equiv \alpha g \Delta T d^3/(\nu\kappa)$ with α the thermal expansion coefficient, κ the thermal diffusivity, ν the kinematic viscosity, and g the acceleration of gravity. Convection starts (under ideal conditions) at the critical value $R_c = 1708$. The second parameter is the Prandtl number $\sigma \equiv \nu/\kappa$, which can be viewed as the ratio of the vertical thermal diffusion time $t_v = d^2/\kappa$ to the vertical viscous relaxation time $t_\nu = d^2/\nu$. It measures the relative importance of the nonlinear terms in the Boussinesq equations, namely those terms describing temperature and momentum advection.

There are several recent reviews of Rayleigh-Bénard convection[1]. In the present one we will focus on new developments during the last decade or so. Let us, however, briefly outline some of the seminal earlier results which were of major importance to the later work. Quite early it was established theoretically that the stable pattern in an infinitely extended layer of a Boussinesq fluid close to onset (see Sect. ?? below) consists of straight, parallel rolls of wavenumber q (Schlüter et al, 1965). Further above onset, the stability regimes of these rolls in the $R-q$ space as functions of σ (the "Busse balloon") are well understood owing to the impressive work by Busse and coworkers (Busse, 1978, 1989). The value of σ varies widely for different experimental fluids, from $\mathcal{O}(10^{-2})$ for liquid metals to values near one for gases and for liquid helium, to the range from 2 to 12 for water, and into the 1000's for silicone oil (see de Bruyn et al, 1996, and references therein). Although R_c, the critical wavevector $q_c = 3.117$, and the patterns in the close vicinity of onset ($R \simeq R_c$) are independent of σ, this does not apply to the subsequent bifurcations which occur with increasing R. For example one finds an oscillatory secondary instability at medium and small Prandtl numbers, in contrast to a stationary bimodal ("knot") bifurcation at large σ (Busse, 1978, 1989). Not too far

[1] Busse (1989), Croquette (1989a,b), Cross & Hohenberg (1993), de Bruyn et al (1996), and Getling (1991, 1998).

from threshold the Busse balloon was found to agree well with the experiments for large σ (water) (Busse & Whitehead, 1971) and reasonably well for gases with $\sigma \simeq 1$ (Croquette, 1989a).

Although ideal periodic patterns can be created in experiments, "natural" convective patterns, particularly when they form in the presence of lateral walls, typically are disordered and develop persistent spatio-temporal dynamics as ϵ increases. Snapshots of such patterns are characterized by local roll patches with grain boundaries and point defects (dislocations) [2]. These spatio-temporal chaotic patterns are irregular in time and in the horizontal plane; but they maintain a relatively simple structure in the vertical direction. They should be contrasted with fully developed turbulence (Frisch, 1995), at very large R, which is disordered in three spatial dimensions and not the topic of this review.

For the description of nonuniform patterns not too far from threshold various reductions of the original hydrodynamical equations have proven to be useful (Manneville, 1990; Cross & Hohenberg, 1993; Newell et al, 1993). One particularly important theoretical result, which motivated much of the work during the last decade, was that for a fluid of $\sigma \simeq 1$ roll curvature induces slowly-varying long-range pressure gradients (Siggia & Zippelius, 1981) that drive a so called "mean flow" which in turn couples back to the roll curvature. In subsequent experimental work [3] these ideas have found their convincing confirmation. Model equations which generalize the so called Swift-Hohenberg equations (SH–equations) (Swift & Hohenberg, 1977) were developed. They allow the isotropic description of the pattern-formation processes in the presence of mean flow (Manneville, 1983; Greenside & Coughran, 1984). Although the SH–equations can not be derived systematically from the Boussinesq equations, they capture much of the observed physical behavior and have now become a general tool

[2] see Croquette (1989b), Manneville (1990), Busse et al (1992), Newell et al (1993), Cross & Hohenberg (1994), Xi et al (1997), and Busse & Clever (1998).

[3] See, for instance, Croquette et al (1986b); Croquette (1989a); Daviaud & Pocheau (1989); Hu et al (1995a); and Pocheau & Daviaud (1997)

to investigate not only RBC, but also other pattern-forming systems (Cross & Hohenberg, 1993).

References

Ahlers G. 1998. Experiments on spatio-temporal chaos. *Physica A* 249(1-4): 18–26

Ahlers G, Cannell DS, Steinberg V. 1985a. Time dependence of flow patterns near the convective threshold in a cylindrical container. *Phys. Rev. Lett.* 54(13): 1373–1376

Ahlers G, Cross MC, Hohenberg PC, Safran S. 1981. The amplitude equation near the convective threshold: application to time-dependent heating experiments. *J. Fluid Mech.* 110: 297

Ahlers G, Hohenberg PC, Lücke M. 1984. Externally modulated Rayleigh-Bénard convection: experiment and theory. *Phys. Rev. Lett* 53(1): 48–51

Ahlers G, Hohenberg PC, Lücke M. 1985b. Thermal convection under external modulation of the driving force. I. the Lorenz model. *Phys. Rev. A* 32(6): 3493–3518

Aranson I, Assenheimer M, Steinberg V, Tsimring LS. 1997. Large-scale flow and spiral core instability in Rayleigh-Bénard convection. *Phys. Rev. E* 55(5): R4877–4880

Ashwin P. 1998. Chaotic intermittency of patterns in symmetric systems. Tech. Rep. 98/29, University of Surrey, Centre for Interdisciplianry Nonlinear Mathematics, Guildford GU2 5XH

Assenheimer M. 1994. *Rayleigh-Bénard convection in the vicinity of the thermodynamical critical point*. Ph.D. thesis, Weizmann Institute of Science, Rehovot, Israel

Assenheimer M, Steinberg V. 1993. Rayleigh-Bénard convection near the gas-liquid critical point. *Phys. Rev. Lett* 70(25): 3888–3891

Assenheimer M, Steinberg V. 1994. Transition between spiral and target states in Rayleigh-Bénard convection. *Nature* 367(6461): 345–347

Assenheimer M, Steinberg V. 1996. Observation of coexisting upflow and downflow hexagons in Boussinesq Rayleigh-Bénard convection. *Phys. Rev. Lett.* 76(5): 756–759

Auer M. 1993. *Dreidimensionale Konvektionsströmungen in ebenen Schichten und im Spalt zwischen rotierenden Zylindern: schwach nichtlineare Untersuchungen in der Nähe multipler Verzweigungspunkte*. Ph.D. thesis, Universität Bayreuth

Auer M, Busse FH, Clever RM. 1995. Three-dimensional convection driven by centrifugal buoyancy. *J. Fluid Mech.* 301: 371–382

Babcock KL, Ahlers G, Cannell DS. 1994. Noise amplification in open Taylor-Couette flow. *Phys. Rev. E* 50(5): 3670–3692

Bajaj KMS, Cannell DS, Ahlers G. 1997. Competition between spiral-defect chaos and rolls in Rayleigh-Bénard convection. *Phys. Rev. E* 55(5): R4869–4872

Bajaj KMS, Jun L, Naberhuis B, Ahlers G. 1998. Square patterns in Rayleigh-Bénard convection with rotation about a vertical axis. *Phys. Rev. Lett* 81(4): 806–809

Berry MV, Bodenschatz E. 1999. Caustics, multiply reconstructed by Talbot interference. *Journal of Modern Optics* 46(2): 349–365

Bestehorn M, Fantz M, Friedrich R, Haken H, et al. 1992. Spiral patterns in thermal convection. *Zeitschrift fur Physik B* 88(1): 93–94

Bodenschatz E, Cannell DS, de Bruyn JR, Ecke R, et al. 1992a. Experiments on three systems with nonvariational aspects. *Physica D* 61(1-4): 77–93

Bodenschatz E, de Bruyn JR, Ahlers G, Cannell DS. 1991. Transitions between patterns in thermal convection. *Phys. Rev. Lett.* 67(22): 3078–3081

Bodenschatz E, Morris SW, de Bruyn J, Cannell DS, et al. 1993. Gallery of fluid motion. *Phys. Fluids A* 5(9): S1–11

Bodenschatz E, Morris SW, de Bruyn JR, Cannell DS, et al. 1992b. Convection in gases at elevated pressures. In *Proceedings of the KIT International Workshop on the Physics of Pattern Formation in Complex Dissipative Systems*, ed. S Kai, World Scientific, Singapore, pp. 227–237

Bowman C, Ercolani N, Indik R, Newell AC, et al. 1998. Patterns, defects and integrability. *Physica D* 123(1-4): 474–492

Bowman C, Newell AC. 1998. Natural patterns and wavelets. *Rev. Mod. Phys.* 70(1): 289–301

Brazovskii SA. 1975. Phase transition of an isotropic system to an inhomogeneous state. *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki* 68(1): 175–185

Brown SN, Stewartson K. 1978. On finite amplitude Bénard convection in a cylindrical container. *Proceedings of the Royal Society of London* 360(1703): 455–469

Buell JC, Catton I. 1986a. Wavenumber selection in large-amplitude axisymmetric convection. *Phys. Fluids* 29(1): 23–30

Buell JC, Catton I. 1986b. Wavenumber selection in ramped Rayleigh-Bénard convection. *J. Fluid Mech.* 171: 477–494

Busse FH. 1967. The stability of finite amplitude cellular convection and its relation to an extremum principle. *J. Fluid Mech.* 30: 625–649

Busse FH. 1972. The oscillatory instability of convection rolls in a low Prandtl number fluid. *J. Fluid Mech.* 52: 97–112

Busse FH. 1978. Non-linear properties of thermal convection. *Rep. Prog. Phys.* 41: 1929–1967

Busse FH. 1989. Fundamentals of thermal convection. In *Mantle Convection: Plate Tectonics and Global Dynamics*, ed. WR Peltier, Gordon and Breach, Montreux, pp. 23–95

Busse FH, Clever RM. 1992. Three-dimensional convection in an inclined layer heated from below. *J. Eng. Math.* 26(1): 1–19

Busse FH, Clever RM. 1998. Mechanism of the onset of time-dpendence in thermal convection. In *Time-Dependent Nonlinear Convection*, ed. T P.A., Southampton: Computational Mechanics Publications, pp. 1 – 50

Busse FH, Hartung G, Jaletzky M, Sommermann G. 1998. Experiments on thermal convection in rotating systems motivated by planetary problems. *Dyn. Atm. Oceans* 27(1-4): 161–174

Busse FH, Kropp M, Zaks M. 1992. Spatio-temporal structures in phase-turbulent convection. *Physica D* 61(1-4): 94–105

Busse FH, Whitehead JA. 1971. Instabilities of convection rolls in a high Prandtl number fluid. *J. Fluid Mech.* 47: 305–320

Cakmur RV, Egolf DA, Plapp BB, Bodenschatz E. 1997a. Bistability and competition of spatiotemporal chaotic and fixed point attractors in Rayleigh-Bénard convection. *Phys. Rev. Lett* 79(10): 1853–1856

Cakmur RV, Egolf DA, Plapp BB, Bodenschatz E. 1997b. Transition from spatiotemporal chaos to ideal straight rolls in Rayleigh-Bénard convection. *http://xxx.lanl.gov/abs/patt-sol/* 9702003: 1–4

Chandrasekhar S. 1961. *Hydrodynamic and Hydromagnetic Stability*. Oxford: Oxford University Press

Chen Y, Pearlstein AJ. 1989. Stability of free-convection flows of variable-viscosity fluids in vertical and inclined slots. *J. Fluid Mech.* 198: 513–541

Ciliberto S, Coullet P, Lega J, Pampaloni E, et al. 1990. Defects in roll-hexagon competition. *Phys. Rev. Lett.* 65(19): 2370–2373

Ciliberto S, Pampaloni E, Perez-Garcia C. 1988. Competition between different symmetries in convective patterns. *Phys. Rev. Lett.* 61(10): 1198–1201

Ciliberto S, Pampaloni E, Perez-Garcia C. 1991. The role of defects in the transition between different symmetries in convective patterns. *J. Stat. Phys.* 64(5-6): 1045–1058

Clever RM. 1973. Finite amplitude longitudinal convection rolls in an inclined layer. *Trans. ASME C* 95(3): 407–408

Clever RM, Busse FH. 1974. Transition to time-dependent convection. *J. Fluid Mech.* 65: 625–645

Clever RM, Busse FH. 1977. Instabilities of longitudinal convection rolls in an inclined layer. *J. Fluid Mech.* 81: 107–125

Clever RM, Busse FH. 1979. Nonlinear properties of convection rolls in a horizontal layer rotating about a vertical axis. *J. Fluid Mech.* 94: 609–627

Clever RM, Busse FH. 1996. Hexagonal convection cells under conditions of vertical symmetry. *Phys. Rev. E* 53(3): R2037–2040

Clune T, Knobloch E. 1993. Pattern selection in rotating convection with experimental boundary conditions. *Phys. Rev. E* 47(4): 2536–2550

Croquette V. 1989a. Convective pattern dynamics at low Prandtl number: Part I. *Contemp. Phys.* 30(2): 113–133

Croquette V. 1989b. Convective pattern dynamics at low Prandtl number:Part II. *Contemp. Phys.* 30(3): 153–171

Croquette V, Le Gal P, Pocheau A. 1986a. Spatial features of the transition to chaos in an extended system. *Phys. Scr.* T13: 135–138

Croquette V, Le Gal P, Pocheau A, Guglielmetti R. 1986b. Large-scale flow characterization in a Rayleigh-Bénard convective pattern. *Europhys. Lett.* 1(8): 393–399

Croquette V, Mory M, Schosseler F. 1983. Rayleigh-Bénard convective structures in a cylindrical container. *J. Phys. France* 44(3): 293–301

Croquette V, Pocheau A. 1984. Wavenumber selection in Rayleigh-Bénard convective structure. In *Cellular Structures* *in Instabilities*, eds. JE Wesfreid, S Zaleski, Springer-Verlag, Berlin, vol. 210 of *Lecture Notes in Physics*, pp. 104–126

Croquette V, Williams H. 1989a. Nonlinear competition between waves on convective rolls. *Phys. Rev. A* 39(5): 2765–2768

Croquette V, Williams H. 1989b. Nonlinear waves of the oscillatory instability on finite convective rolls. *Physica D* 37(1-3): 300–314

Cross M. 1996. Theoretical modelling of spiral chaos in Rayleigh-Bénard convection. *Physica D* 97(1-3): 65–80

Cross MC. 1982. Ingredients of a theory of convective textures close to onset. *Phys. Rev. A* 25(2): 1065–1076

Cross MC. 1983. Phase dynamics of convective rolls. *Phys. Rev. A* 27(1): 490–498

Cross MC, Hohenberg PC. 1993. Pattern formation out of equilibrium. *Rev. Mod. Phys.* 65(3): 851–1112

Cross MC, Hohenberg PC. 1994. Spatiotemporal chaos. *Science* 263(5153): 1569–1570

Cross MC, Hohenberg PC, Lücke M. 1983. Forcing of convection due to time-dependent heating near threshold. *J. Fluid Mech.* 136: 269–276

Cross MC, Meiron D, Tu Y. 1994. Chaotic domains: a numerical investigation. *Chaos* 4(4): 607–619

Cross MC, Tu Y. 1995. Defect dynamics for spiral chaos in Rayleigh-Bénard convection. *Phys. Rev. Lett* 75(5): 834–837

Daniels KE, Bodenschatz E, Pesch W, Brausch O. 1999. Pattern formation in inclined layer convection. In *Bull. Am. Phys. Soc.*, CENT99: QC28.01

Daviaud F, Pocheau A. 1989. Inhibition of phase turbulence close to onset of convection by permeable lateral boundary condition for the mean flow. *Europhys. Lett.* 9(7): 675–680

Davis SH. 1967. Convection in a box: linear theory. *J. Fluid Mech.* 30: 465–478 de Bruyn JR, Bodenschatz E, Morris SW, Trainoff SP, et al. 1996. Apparatus for the study of Rayleigh-Bénard convection in gases under pressure. *Rev. Sci. Inst.* 67(6): 2043–2067

Decker W. 1995. *Mathematische Methoden zur Beschreibung strukturbildender Systeme - eine kritische Analyse*. Ph.D. thesis, Universität Bayreuth, Germany

Decker W, Pesch W. 1994. Order parameter and amplitude equations for the Rayleigh-Bénard convection. *J. Phys. France* 4(3): 419–438

Decker W, Pesch W, Weber A. 1994. Spiral defect chaos in Rayleigh-Bénard convection. *Phys. Rev. Lett.* 73(5): 648–651

Donnelly RJ, Reif F, Suhl H. 1962. Enhancement of hydrodynamic stability by modulation. *Phys. Rev. Lett.* 9(9): 363 – 365

Ecke RE, Hu Y. 1997. Spiral defect chaos in Rayleigh-Bénard convection: defect population statistics. *Physica A* 239(1-3): 174–188

Ecke RE, Hu Y, Mainieri R, Ahlers G. 1995. Excitation of spirals and chiral symmetry breaking in Rayleigh-Bénard convection. *Science* 269(5231): 1704–1707

Egolf DA, Melnikov IV, Bodenschatz E. 1998. Importance of local pattern properties in spiral defect chaos. *Phys. Rev. Lett.* 80(15): 3228–3231

Ercolani NM, Indik R, Newell AC, T. P. 1999. The geometry of the phase diffusion equation. *J. Nonl. Science* In print

Fantz M, Friedrich R, Bestehorn M, Haken H. 1992. Pattern formation in rotating Bénard convection. *Physica D* 61(1-4): 147–154

Fisher ME. 1968. Renormalization of critical exponents by hidden variables. *Phys. Rev.* 176(1): 257–272

Frisch U. 1995. *Turbulence*. Oxford: Cambridge University Press

Fujimura K, Kelly RE. 1993a. Mixed mode convection in an inclined slot. *J. Fluid Mech.* 246: 545–568

Fujimura K, Kelly RE. 1993b. Mixed mode convection in an inclined slot containing air. In *Bifurcation Phenomena and Chaos in Thermal Convection, ASME HTD-***214**, pp. 73–83

Fujimura K, Kelly RE. 1995. Interaction between longitudinal convection rolls and transverse waves in unstably stratified plane Poiseuille flow. *Phys. Fluids* 7(1): 68–79

Gershuni GZ, Zhukhovitskii EM. 1969. Stability of plane-parallel convective motion with respect to spatial perturbations. *Prikl. Mat. i Mekh.* 33: 855–860

Getling AV. 1991. Formation of spatial structures in Rayleigh-Bénard convection. *Uspekhi Fizicheskii Nauk* 161(9): 1–80

Getling AV. 1998. *Rayleigh-Bénard Convection*. Singapore: World Scientific

Gollub JP. 1994. Spirals and chaos. *Nature* 367: 318

Graham A. 1933. Shear patterns in an unstable layer of air. *Philos. Trans. R. Soc. A* 232(714): 285 – 296

Greenside HS, Coughran Jr WM. 1984. Nonlinear pattern formation near the onset of Rayleigh-Bénard convection. *Phys. Rev. A* 30(1): 398–428

Haken H. 1978. *Synergetics*. Springer

Hart JE. 1971a. Stability of the flow in a differentially heated inclined box. *J. Fluid Mech.* 47: 547–576

Hart JE. 1971b. Transition to a wavy vortex regime in convective flow between inclined plates. *J. Fluid Mech.* 48: 265–271

Heutmaker MS, Fraenkel PN, Gollub JP. 1985. Convection patterns: time evolution of the wave-vector field. *Phys. Rev. Lett.* 54(13): 1369–1372

Heutmaker MS, Gollub JP. 1986. Wavenumber distributions and time dependence in Rayleigh-Bénard convection patterns. *Physica D* 23D(1-3): 230–234

Heutmaker MS, Gollub JP. 1987. Wave-vector field of convective flow-patterns. *Phys. Rev. A* 35(1): 242–260

Hohenberg PC, Swift JB. 1992. Effects of additive noise at the onset of Rayleigh-Bénard convection. *Phys. Rev. A* 46(8): 4773–4785

Hu Y, Ecke R, Ahlers G. 1993. Convection near threshold for Prandtl numbers near 1. *Phys. Rev. E* 48(6): 4399–4413

Hu Y, Ecke R, Ahlers G. 1995a. Convection for Prandtl numbers near 1: dynamics of textured patterns. *Phys. Rev. E* 51(4): 3263–3279

Hu Y, Ecke RE, Ahlers G. 1994. Behavior of focus patterns in low Prandtl number convection. *Phys. Rev. Lett.* 72(14): 2191–2194

Hu Y, Ecke RE, Ahlers G. 1995b. Time and length scales in rotating Rayleigh-Bénard convection. *Phys. Rev. Lett* 74(25): 5040–5043

Hu Y, Ecke RE, Ahlers G. 1995c. Transition to spiral-defect chaos in low Prandtl number convection. *Phys. Rev. Lett* 74(3): 391–394

Hu Y, Ecke RE, Ahlers G. 1997. Convection under rotation for Prandtl numbers near 1: linear stability, wave-number selection, and pattern dynamics. *Phys. Rev. E* 55(6): 6928–6949

Hu Y, Pesch W, Ahlers G, Ecke RE. 1998. Convection under rotation for Prandtl numbers near 1: Küppers-Lortz instability. *Phys. Rev. E* 58(5): 5821–5833

Jeanjean S. 1997. Front propagation of spatio temporal chaos in Rayleigh-Bénard convection. Tech. rep., Cornell University, Laboratory of Atomic and Solid State Physics, 618 Clark Hall

Kelly RE. 1994. The onset and development of thermal convection in fully developed shear flows. *Adv. in Appl. Mech.* 31: 35

Kirchartz KR, Oertel Jr. H. 1988. Three-dimensional thermal cellular convection in rectangular boxes. *J. Fluid Mech.* 192: 249–286

Koschmieder EL. 1974. Bénard convection. In *Adv.Chem. Phys.*, eds. I Prigogine, SA Rice, Wiley-Interscience, Chichester, vol. 26, pp. 177–212

Koschmieder EL, Pallas SG. 1974. Heat transfer through a shallow, horizontal convecting fluid layer. *Int. J. Heat Mass Transfer* 17(9): 991–1002

Kramer L, Bodenschatz E, Pesch W. 1990. Interaction and dynamics of defects in anisotropic pattern- forming systems. *Phys. Rev. Lett.* 64(21): 2588

Kramer L, Pesch W. 1995. Convection instabilities in nematic liquid–crystals. *Ann. Rev. Fluid Mech.* 27: 515–541

Küppers G. 1970. The stability of steady finite amplitude convection in a rotating fluid layer. *Phys. Lett. A* 32(1): 7–8

Küppers G, Lortz D. 1969. Transition from laminar convection to thermal turbulence in a rotating fluid layer. *J. Fluid Mech.* 35: 609–620

Kurzweg UH. 1970. Stability of natural convection within an inclined channel. *Trans. ASME C* 92(1): 190–191

Landau LD, Lifshitz EM. 1958. *Statistical Physics*. London: Pergamon Press, 2nd ed.

Landau LD, Lifshitz EM. 1959. *Fluid Mechanics*. Reading MA: Addison-Wesley, 2nd ed.

Laveder D, Passot T, Ponty Y, Sulem PL. 1999. Effect of random noise on scaling laws of finite prandtl number rotating convcection near threshold. *Phys. Rev. E.* 59

Li X, Xi HW, Gunton JD. 1998. Nature of roll to spiral-defect-chaos transition. *Phys. Rev. E* 57(2): 1705–1716

Li XJ, Xi HW, Gunton JD. 1996. Dynamical properties of multiarmed global spirals in Rayleigh-Bénard convection. *Phys. Rev. E* 54(4): R3105–R3108

Liu J, Ahlers G. 1996. Spiral-defect chaos in Rayleigh-Bénard convection with small Prandtl numbers. *Phys. Rev. Lett* 77(15): 3126–3129

Liu J, Ahlers G. 1997. Rayleigh-Bénard convection in binary-gas mixtures: thermophysical properties and the onset of convection. *Phys. Rev. E* 55(6): 6950–6968

Manneville P. 1983. A two-dimensional model for three-dimensional convective patterns in wide containers. *J. Phys. France* 44(7): 759–765

Manneville P. 1990. *Dissipative structures and weak turbulence.* San Diego: Academic Press

Manneville P, Piquemal JM. 1983. Zigzag instability and axisymmetric rolls in Rayleigh-Bénard convection: the effects of curvature. *Phys. Rev. A* 28(3): 1774–1790

Meyer CW, Cannell DS, Ahlers G. 1992. Hexagonal and roll flow patterns in temporally modulated Rayleigh-Bénard convection. *Phys. Rev. A* 45(12): 8583–8604

Meyer CW, Cannell DS, Ahlers G, Swift JB, et al. 1988. Pattern competition in temporally modulated Rayleigh-Bénard convection. *Phys. Rev. Lett.* 61(8): 947–950

Millan-Rodriguez J, Bestehorn M, Perez-Garcia C, Friedrich R, et al. 1995. Defect motion in rotating fluids. *Phys. Rev. Lett.* 74(4): 530–533

Morris SW, Bodenschatz E, Cannell DS, Ahlers G. 1993. Spiral defect chaos in large aspect ratio Rayleigh-Bénard convection. *Phys. Rev. Lett* 71(13): 2026–2029

Morris SW, Bodenschatz E, Cannell DS, Ahlers G. 1996. The spatio-temporal structure of spiral-defect chaos. *Physica D* 97(1-3): 164–179

Neufeld M, Friedrich R. 1994. Model equations for quasi-two-dimensional pattern forming systems. *Ann. Phys.* 19(6): 721–733

Neufeld M, Friedrich R, Haken H. 1993. Order parameter equation and model equation for high Prandtl number. Rayleigh-Bérnard convection in a rotating large aspect ratio system. *Z. Phys. B* 92(2): 243–256

Newell AC, Passot T. 1992. Instabilities of dislocations in fluid patterns. *Phys. Rev. Lett.* 68(12): 1846–1849

Newell AC, Passot T, Bowman C, Ercolani N, et al. 1996. Defects are weak and self-dual solutions of the Cross-Newell phase diffusion equation for natural patterns. *Physica D* 97(1-3): 185–205

Newell AC, Passot T, Lega J. 1993. Order parameter equations for patterns. *Annu. Rev. Fluid Mech.* 25: 399–453

Newell AC, Passot T, Souli M. 1990. The phase diffusion and mean drift equations for convection at finite Rayleigh numbers in large containers. *J. Fluid Mech.* 220: 187–252

Newell AC, Passot T, Souli M. 1991. Focus instability in axisymmetric Rayleigh-Bénard convection. *Euro. J. Mech. B* 10(2): 151–157

Newell AC, Whitehead JA. 1969. Finite bandwidth, finite amplitude convection. *J. Fluid Mech.* 38: 279–303

Niemela JJ, Donnelly RJ. 1986. Direct transition to turbulence in rotating Bénard convection. *Phys. Rev. Lett.* 57(20): 2524–2527

Ning L, Ahlers G, Cannell DS. 1990. Wave-number selection and traveling vortex waves in spatially ramped taylor-couette flow. *Phys. Rev. Lett.* 64(11): 1235–1238

Pampaloni E, Perez-Garcia C, Albavetti L, Ciliberto S. 1992. Transition from hexagons to rolls in convection in fluids under non-Boussinesq conditions. *J. Fluid Mech.* 234: 393–416

Perez-Garcia C, Pampaloni E, Ciliberto S. 1990. Finite-size effects in the transition from hexagons to rolls in convective systems. *Europhys. Lett.* 12(1): 51–55

Pesch W. 1996. Complex spatiotemporal convection patterns. *Chaos* 6(3): 348–357

Pesch W, Kramer L. 1995. General mathematcal description of pattern forming instabilities. In *Pattern formation in liquid crystals*, eds. A Buka, L Kramer, Springer, New York, pp. 70–90

Pismen LM, Nepomnyashchy AA. 1993. Structure of dislocations in the hexagonal pattern. *Europhys. Lett.* 24(6): 461–465

Plapp BB, Bodenschatz E. 1996. Core dynamics of multi-armed spirals in Rayleigh-Bénard convection. *Phys. Scr.* T67: 111–116

Plapp BB, Egolf DA, Bodenschatz E, Pesch W. 1998. Dynamics and selection of giant spirals in Rayleigh-Bénard convection. *Phys. Rev. Lett.* 81(24): 5334–5337

Plapp P. 1997. *Spiral-pattern formation in Rayleigh-Bénard convection*. Ph.D. thesis, Cornell University, Ithaca NY

Pocheau A, Daviaud F. 1997. Sensitivity of convective structures to mean flow boundary conditions: a correlation between symmetry and dynamics. *Phys. Rev. E* 55(1): 353–373

Pomeau Y, Manneville P. 1979. Stability and fluctuations of a spatially periodic convective flow. *J. Phys. France Lett.* 40(23): L609–612

Pomeau Y, Zaleski S, Manneville P. 1985. Axisymmetric cellular structures revisited. *Z. Ang. Math. Phys.* 36(3): 367–394

Ponty Y, Passot T, Sulem PL. 1997. Pattern dynamics in rotating convection at finite Prandtl number. *Phys. Rev. E* 56(4): 4162–4178

Quentin G, Rehberg I. 1995. Direct measurement of hydrodynamic fluctuations in a binary mixture. *Phys. Rev. Lett.* 74(9): 1578–1581

Rabinovich MI, Tsimring LS. 1994. Dynamics of dislocations in hexagonal patterns. *Phys. Rev. E* 49(1): R35–38

Rasenat S, Steinberg V, Rehberg I. 1990. Experimental studies of defect dynamics and interaction in electrohydrodynamic convection. *Phys. Rev. A* 42(10): 5998–6008

Rehberg I, Rasenat S, de la Torre Juarez M, Schöpf W, et al. 1991. Thermally induced hydrodynamic fluctuations below the onset of electroconvection. *Phys. Rev. Lett.* 67(5): 596–599

Riecke H, Paap HG. 1987. Perfect wave-number selection and drifting patterns in ramped taylor vortex flow. *Phys. Rev. Lett.* 59(22): 2570–2573

Rogers JL, Schatz MF, Swift JB, Bougie J. 1998. Rayleigh-Bénard experiment under modulated acceleration. In *Bull. Am. Phys. Soc.*, DFD98: BI.03

Ruth DW. 1980. On the transition to transverse rolls in inclined infinite fluid layers-steady solutions. *Int.J. Heat Mass Trans.* 23(5): 733–737

Ruth DW, Raithby GD, Hollands KGT. 1980. On the secondary instability in inclined air layers. *J. Fluid Mech.* 96: 481–492

Schlüter A, Lortz D, Busse F. 1965. On the stability of steady finite amplitude convection. *J. Fluid Mech.* 23: 129–144

Schmitz R, Zimmermann W. 1996. Spatially periodic modulated Rayleigh-Bénard convection. *Phys. Rev. E* 53(6): 5993–6011

Schöpf W, Rehberg I. 1994. The influence of thermal noise on the onset of travelling- wave convection in binary fluid mixtures: an experimental investigation. *J. Fluid Mech.* 271: 235–265

Segel LA. 1969. Distant side-walls cause slow amplitude modulation of cellular convection. *J. Fluid Mech.* 38: 203–224

Sensoy BA, Greenside HS. 1999. Pattern formation and dynamics of a convecting fluid in an annular domain. In *Bull. Am. Phys. Soc.*, CENT99: QC06.02

Shadid JN, Goldstein RJ. 1990. Visualization of longitudinal convection roll instabilities in an inclined enclosure heated from below. *J. Fluid Mech.* 215: 61–84

Siggia ED, Zippelius A. 1981. Pattern selection in Rayleigh-Bénard convection near threshold. *Phys. Rev. Lett* 47(12): 835–838

Steinberg V, Ahlers G, Cannell DS. 1985. Pattern formation and wave-number selection by Rayleigh- Bénard convection in a cylindrical container. *Phys. Scr.* T9: 97–110

Swift J, Hohenberg PC. 1977. Hydrodynamic fluctuations at the convective instability. *Phys. Rev. A* 15(1): 319–328

Swift JB, Hohenberg PC, Ahlers G. 1991. Stochastic Landau equation with time-dependent drift. *Phys. Rev. A* 43(12): 6572–6580

Tesauro G, Cross MC. 1986. Climbing of dislocations in nonequilibrium patterns. *Phys. Rev. A* 34(2): 1363–1379

Tschammer A. 1997. *Nichtlineare Aspekte der Rayleigh-Bénard Konvektion in isotropen und anisotropen Fluiden*. Ph.D. thesis, Universität Bayreuth (FRG)

Tsimring LS. 1995. Penta-hepta defect motion in hexagonal patterns. *Phys. Rev. Lett.* 74(21): 4201–4204

Tsimring LS. 1996. Dynamics of penta-hepta defects in hexagonal patterns. *Physica D* 89(3-4): 368–380

Tu Y, Cross. 1992a. Chaotic domain structure in rotating convection. *Phys. Rev. Lett* 69(17): 2215–2219

Tu Y, Cross MC. 1992b. Chaotic domain structure in rotating convection. *Phys. Rev. Lett.* 69(17): 2515–2518

van Beijeren H, Cohen EGD. 1988. The effects of thermal noise in a Rayleigh-Bénard cell near its first convective instability. *J. Stat. Phys* 53(1-2): 77–94

Wiener RJ, Snyder GL, Prange MP, Frediani D, et al. 1997. Period-doubling cascade to chaotic phase dynamics in Taylor vortex flow with hourglass geometry. *Phys. Rev. E* 55(5): 5489 – 5497

Wu M, Ahlers G, Cannell DS. 1995. Thermally induced fluctuations below the onset of Rayleigh- Bénard convection. *Phys. Rev. Lett.* 75(9): 1743–1746

Xi HW, Gunton JD. 1995. Spatiotemporal chaos in a model of Rayleigh-Bénard convection. *Phys. Rev. E* 52(5): 4963–4975

Xi HW, Gunton JD, Viñals J. 1993. Spiral defect chaos in a model of Rayleigh-Bénard convection. *Phys. Rev. Lett* 71(13): 2030–2033

Xi HW, Li XJ, Gunton JD. 1997. Direct transition to spatiotemporal chaos in low Prandtl number fluids. *Phys. Rev. Lett.* 78(6): 1046–1049

Xi HW, Viñals J, Gunton JD. 1992. Numerical study of pattern formation following a convective instability in non-Boussinesq fluids. *Phys. Rev. A* 46(8): R4483–4486

Zhong F, Ecke R. 1992. Pattern dynamics and heat transfer in rotating Rayleigh- Bénard convection. *Chaos* 2(2): 163–171

Zhong F, Ecke R, Steinberg V. 1991. Rotating Rayleigh-Bénard convection: Küppers-Lortz transition. *Physica D* 51(1-3): 596–607

Zoldi SM, Liu J, Bajaj KMS, Greenside HS, et al. 1998. Extensive scaling and nonuniformity of the Karhunen-Loeve decomposition for the spiral-defect chaos state. *Phys. Rev. E* 58(6): R6903–6906