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Comment on "On the threshold characteristics of the flexoelectric domains arising in a homogeneous electric field: The case of anisotropic elasticity" by Y.G. Marinov and H.P. Hinov

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Abstract. It is demonstrated that the key findings of the paper by Y.G. Marinov and H.P. Hinov, Eur. Phys. J. E **31**, 179-189 (2010) are in direct conflict with the general physical background of flexoelectric domains. This is caused by a methodological error in the theoretical analysis of the paper.

Flexoelectric domains (flexodomains) which appear in experiments as stripe patterns have been first described by Bobylev&Pikin [1]. They are observed in a planarly aligned nematic layer (parallel to the $x-y$ plane) of thickness d, when an electric potential, U, larger than a threshold U_c is applied along the z-direction. Flexodomains are characterized by a specific spatial variation of the locally preferred axis of the uniaxial nematics, i.e., of the director $\mathbf{n}$: the planar basic director configuration $\mathbf{n_0} = \hat{\mathbf{x}}$ is modified by a distortion $\delta\mathbf{n} = (0, \delta n_y, \delta n_z)$ in the form of splay ($\delta n_z \neq 0$) and twist ($\delta n_y \neq 0$). The director distortion $\delta\mathbf{n}$ depends on z and is in addition spatially periodic along the y-axis with wavenumber q.

The existence of flexodomains requires a balance of the dielectric, elastic and flexo torques on the director [2]. The first is determined by two dielectric permittivities $\epsilon_\parallel(\epsilon_\perp)$, for an electric field parallel (perpendicular) to $\mathbf{n}$. Obviously this torque tends to stabilize (destabilize) the planar basic state when the dielectric anisotropy $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$ is negative (positive). The strength of the elastic torques, stabilizing the planar configuration, is measured by two positive elastic constants, namely by k_{11} and k_{22} in the presence of splay and twist distortions, respectively. The parameterization $k_{11,22} = k_{av}(1 \pm \delta k)$ with $k_{av} = (k_{11} + k_{22})/2$ and $-1 < \delta k < 1$ is convenient. Nonzero flexo torques, which are destabilizing in the present case, necessitate a splay distortion ($\delta n_z \neq 0$) accompanied with a certain amount of twist ($\delta n_y \neq 0$). Their strength is determined by the parameter combination $\delta e = |e_1 - e_3|$ of the flexo-electric coefficients e_1, e_3.

The calculation of the threshold U_c and the critical wavenumber $q = q_c$ at $U = U_c$ of flexodomains in [1] addresses only the special case of isotropic elasticity ($\delta k = 0$). In the paper of Marinov&Hinov [3] (MH) the general case of anisotropic elasticity ($\delta k \neq 0$) has been investigated. Their analysis starts from the familiar system of two coupled linear differential equations for the director perturbations δn_y, δn_z (see Eqs. (6) in MH), which result directly from the minimization of the orientational free energy density of nematics (see, e.g., Eq. (5) in MH) [4]. As a central result of their analysis (see Eq. (34) in MH), the authors claim that flexodomains can only exist, if the elastic constants are restricted to the interval $1/3 < k_{22}/k_{11} < 3$ (corresponding to $|\delta k| < 1/2$ in our notation). On the grounds of general considerations it will be demonstrated below, that such a restriction has no physical basis and points thus to a crucial error in the paper.

To understand the generic features of the existence regime of flexodomains in the $(\epsilon_a, \delta k)$ plane (see Fig. 1) it is most useful to consider at first the case $\delta e = 0$ where only the dielectric and the elastic torques compete. For $\epsilon_a < 0$ (below the dot-dashed abscissa in Fig. 1) the system remains for arbitrary U in the planar basic state, which is stabilized by both torques. For any $\epsilon_a > 0$, however, the destabilizing dielectric torque will eventually overcome the elastic torques with increasing voltage U: in the interval $-1 < \delta k < \delta k_c$ with $\delta k_c \approx 0.535$ ($k_{22}/k_{11} \approx 0.303$), i.e., to the left of the dotted vertical line ST in Fig. 1, the standard splay Freedericksz transition takes place at $U_F = \pi\sqrt{k_{av}(1 + \delta k)/(\epsilon_0\epsilon_a)}$. The resulting distortion δn_z of the planar state does not vary in the $x - y$ plane, while $\delta n_y \equiv 0$. In contrast, in the interval $\delta k_c < \delta k < 1$ (on the right of the line ST and above the dot-dashed line $\epsilon_a = 0$), the destabilization of the basic state happens in form of the splay-twist Freedericksz transition first described by Lonberg&Meyer [5]. In perfect analogy to flexodomains the resulting director distortions show nonzero $\delta n_y, \delta n_z$ contributions, which are periodic along the y-direction. The determination of the transition voltage $U_{ST} < U_F$ and of $\delta k_c \approx 0.535$ quoted above, requires the numerical solution of a transcendental equation given in [5].

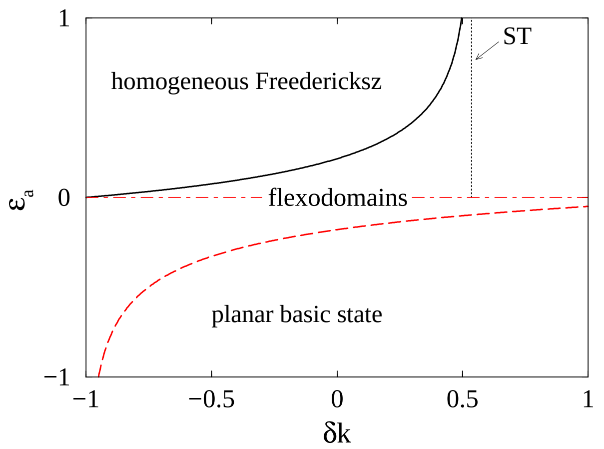

Fig. 1. Schematic illustration of the upper and lower limit curves, $\epsilon_a^u(\delta k)$ (solid) and $\epsilon_a^l(\delta k)$ (dashed), respectively of the existence range of flexodomains in the $(\epsilon_a, \delta k)$ plane (arbitrary units for ϵ_a). $\epsilon_a^u(\delta k)$ diverges at $\delta k \approx 0.535$ (dotted line ST), while $\epsilon_a^l(\delta k) \to -\infty$ for $\delta k \to -1$ (for details see text). In the absence of flexo torques ($\delta e = 0$) the planar state exists everywhere below the dot-dashed line $\epsilon_a = 0$. For $\epsilon_a > 0$ the homogeneous Freedericksz-state is replaced by the splay-twist Freedericksz-state to the right of the line ST (dotted).

In the case of finite δe the destabilizing flexo torque enhances in general the tendency towards spatially periodic splay-twist director variations, i.e., towards flexodomains. Thus an "upper" transition curve $\epsilon_a = \epsilon_a^u(\delta k)$ must exist in the whole interval $-1 < \delta k < \delta k_c$, at which the homogeneous Freedericksz configurations with $q_c = 0$ transform continuously into the flexodomains with finite q_c for $\epsilon_a < \epsilon_a^u(\delta k)$. For $\delta k > \delta k_c$ and $\epsilon_a > 0$ the flexo torques yield only a minor modification of the periodic state already present for $\delta e = 0$.

For $\epsilon_a < 0$ the flexo torques may also overcome the stabilizing dielectric and elastic torques allowing for flexodomains in this regime as well. As is well known, the strength of the flexo torque increases linearly with the wavenumber q of the flexodomains. Thus when decreasing ϵ_a their critical wavenumber q_c has to become larger in order to over-compensate the increasing stabilizing effect of the dielectric torque ($\propto |\epsilon_a|$). Eventually q_c and also U_c diverge at the "lower" transition line $\epsilon_a^l(\delta k)$ separating the flexodomains and the planar basic state. Note, that, as indicated in Fig. 1, both functions, $\epsilon_a^{u,l}(\delta k)$, have to decrease strictly with decreasing δk. Regarding $\epsilon_a^u(\delta k)$, where a finite δn_z is already provided by the dielectric torque, the increase of k_{22} hinders more and more the necessary twist distortion (δn_y) of the director. In contrast, for $\epsilon_a^l(\delta k)$, which diverges at $\delta k \to -1$ (i.e., at $k_{11} \to 0$), the more easy generation of the finite splay distortion is decisive.

In summary, the existence regime of the periodic splay-twist distortions for $\delta e = 0$, $\epsilon_a > 0$ and $\delta k > \delta k_c$ in the upper right corner of Fig. 1 is bound to expand for $\delta e \neq 0$ into the "dumbell"-like existence regime of flexodomains intervening between the homogeneous Freedericksz state and the basic state for $-1 < \delta k < \delta k_c$ in Fig. 1. The critical wavenumber q_c increases monotonically from 0 to ∞ when moving with ϵ_a from the upper to the lower transition line. In the limit $\delta k \to \delta k_c$ from below the line ST is approached by $\epsilon_a^u(\delta k)$ in the limit $\epsilon_a \to \infty$. Note, that the general scenario described above is consistent with the special case of $\delta k = 0$ where one finds the analytical results $\epsilon_a^u(0) = -\epsilon_a^l(0) = \delta e^2/(\epsilon_0 k_{av})$ [1].

The phase diagram for the flexodomains shown in Fig. 1, which has been constructed from basic physical arguments, is in strong disagreement with the results of MH. Besides the unphysical limitation $|\delta k| < 1/2$ (Eq. (34) in MH) for the existence of flexodomains, their upper limit function $\epsilon_a^u(\delta k)$ (according to Eq. (35) in MH) is strictly increasing with decreasing δk until it diverges at $\delta k = -1/2$. This is in distinct contrast to Fig. 1 where $\epsilon_a^u(\delta k)$ approaches zero with $\delta k \to -1$.

A closer look at MH shows that their analysis suffers in general from a basic mathematical error. Instead of starting with the standard text-book ansatz $\delta n_y(z), \delta n_z(z) \propto \exp(\lambda z)$ to analyze their Eqs. (6), they have applied a sequence of matrix manipulations to "diagonalize" them. This method, which has been also applied by the authors in other papers (see, e.g., [6]), has been first proposed in [7]. Unfortunately, in all these publications it has been overlooked, that the procedure (starting at Eq. (14) in MH) works only for constant matrices while these depend in fact on z in the present case. As a consequence, the expressions for the critical voltage and the wavenumber (Eqs. (26), (28) in MH) are incorrect, which thus applies to all the results presented in the paper.

Our general, qualitative considerations in this comment have been confirmed by the standard treatment of the underlying equations (Eqs. (6) of MH) in a recent publication [8]. Here one finds detailed discussions of U_c and q_c in dependence on the material parameters. In addition the application of an ac-voltage and the competition with patterns arising from the electrohydrodynamic instability have been discussed.

References

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