Thermal Convection in a Twisted Horizontal Magnetic Field

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Abstract

The onset of convection in a layer of an electrically conducting fluid heated from below is considered in the case when the layer is permeated by a horizontal magnetic field of strength B_0 the orientation of which varies sinusoidally with height. The critical value of the Rayleigh number for the onset of convection is derived as a function of the Chandrasekhar number Q. With increasing Q the height of the convection rolls decreases, while their horizontal wavelength slowly increases. Potential applications to the penumbral filaments of sunspots are briefly discussed.

1. Introduction

Convection in electrically conducting fluids driven by thermal buoyancy in the presence of an imposed magnetic field has been studied theoretically and experimentally for several decades. For an early review we refer to Chandrasekhar's (1961) book. Usually the case of an imposed homogeneous vertical magnetic field is treated which exerts a strongly inhibiting influence on convection. A most famous example are sunspots which appear dark because the heat transport by convection in the solar atmosphere is almost completely suppressed in the umbra of the spot by the emerging radial magnetic field.

Homogeneous horizontal magnetic fields exert a far lesser influence on convection. Two-dimensional convection rolls aligned with the magnetic field do not feel any effect at all and the critical value of the Rayleigh number R for the onset of convection is the same as in the non-magnetic case. Three-dimensional instabilities of such rolls are influenced by the homogeneous horizontal magnetic field , however, and so are three-dimensional forms of convection at higher values of R (Busse and Clever, 1983, 1989).

Another situation in which a horizontal magnetic field affects convection is that of a twisted magnetic field which changes its direction as function of the vertical coordinate. Such a field is accompanied by a current density. In the following we shall consider the case of a constant twist where the electric current is directed parallel to the magnetic field such that a static "force-free" configuration exists. The onset of convection in such a configuration is the topic of this paper. The same force-free configuration as adopted in the present paper has been used by Kuang and Roberts (1990, 1991) in their investigation of magnetic instabilities in a rotating fluid layer. Diffusivities other than the magnetic one have been neglected in their analysis. A comparison with the present work is thus not possible even where they had considered an unstable density stratification. Another analysis in which the influence of a varying orientation of a horizontal magnetic field had been included is the simulation by Cattaneo et al. (1990) of the Rayleigh-Taylor instability of a dense, magnetic field free gas overlaying a lighter gas penetrated by the horizontal field.

It has long been known that magnetic fields of sunspots exhibit a torsion which is equivalent to a twist in a local approximation. In the penumbra of large sunspots where the magnetic field becomes nearly horizontal, convection appears to assume the form of thin roll like structures called filaments. The small wavelength of these rolls is usually attributed to the influence of the vertical component of the magnetic field. In the following sections the additional effects that may be caused by a twist of the magnetic field will be explored in terms of our model.

After the mathematical formulation of the problem in section 2 we shall solve the linearized problem in section 3 in order to determine the critical value R_c of the Rayleigh number for onset of convection as function of the Chandrasekhar number Q. In section 4 an asymptotic approach is described which allows to solve the linear problem for very high values of Q. The confinement of the convection rolls to a sublayer of the fluid layer for high values of Q suggests that the onset of the first set of rolls will be followed by the onset of further sets of rolls with different orientations. This topic will be addressed briefly in section 5. An outlook on future work will be given in the concluding section 6.

2. Mathematical Formulation of the Problem

We consider a horizontal layer of height d of an electrically conducting fluid heated from below and cooled from above. T_1 and T_2 are the temperatures at the upper and lower boundaries of the layer. The fluid is permeated by a horizontally homogeneous magnetic field of the form

$$\boldsymbol{B}_0 = B_0(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z) \tag{1}$$

where $\boldsymbol{i}$ and $\boldsymbol{j}$ denote the unit vectors in the horizontal x- and y-directions of a cartesian system of coordinates and where the z-coordinate is opposite to the direction of gravity. Since the field (1) is force-free, i.e. $(\nabla\times\boldsymbol{B}_0)\times\boldsymbol{B}_0 = 0$, a motionless static solution of the problem exists.

Using d as length scale, d^2/ν as time scale where ν denotes the kinematic viscosity of the fluid, and $(T_2-T_1)P/R$ as scale of the deviation Θ of the temperature from its static distribution we obtain the equations of motion for the dimensionless velocity vector $\boldsymbol{u}$ and the heat equation for Θ in the following form

$$(\partial_t + \boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla\pi + \boldsymbol{k}\Theta + \nabla^2\boldsymbol{u} + \boldsymbol{B}\cdot\nabla\boldsymbol{B}\ , \tag{2a}$$

$$0 = \nabla\cdot\boldsymbol{u}\ , \tag{2b}$$

$$P(\partial_t + \boldsymbol{u}\cdot\nabla)\Theta = R\boldsymbol{k}\cdot\boldsymbol{u} + \nabla^2\Theta \tag{2c}$$

where $\boldsymbol{k}$ is the unit vector in the z-direction and the Prandtl number P and the Rayleigh number R are defined by

$$P = \frac{\nu}{\kappa}, \qquad R = \frac{\alpha(T_2-T_1)\breve{g}d^3}{\nu\kappa}. \tag{3}$$

Here κ and α denote the thermal diffusivity and the coefficient of thermal expansion of the fluid and $\breve{g}$ is the acceleration of gravity. The term $\nabla\pi$ inludes all terms that can be written as gradients including the part $\nabla|\boldsymbol{B}|^2/2$ of the Lorentz force. In order to treat the problem in its simplest physically realistic form the Boussinesq approximation has been assumed in which the density ρ is regarded as a constant except in connection with the gravity term where its dependence on the temperature has been taken into account.

The general representation for solenoidal vector fields in terms of poloidal and toroidal components can be used to write the dimensionless magnetic field in the form

$$\boldsymbol{B} = \frac{B_0 d}{\nu\sqrt{\rho\mu}}(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z + \frac{\nu}{\lambda}(\nabla\times(\nabla h\times\boldsymbol{k}) + \nabla g\times\boldsymbol{k}))\ . \tag{4}$$

where λ is the magnetic diffusivity which is defined as the inverse of the product of the electrical conductivity σ and the magnetic permeability μ. By taking the vertical components of the equation of induction,

$$\frac{\lambda}{\nu}\nabla^2\boldsymbol{B} = \partial_t\boldsymbol{B} - \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) \tag{5}$$

and of its curl we obtain the equations for the functions h and g

$$\nabla^2\Delta_2 h = (\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z)\cdot\nabla u_z, \tag{6a}$$

$$\nabla^2\Delta_2 g = -(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z)\cdot\nabla(\partial_y u_x - \partial_x u_y - \gamma u_z), \tag{6b}$$

Δ_2 denotes the horizontal Laplacian, $\Delta_2 = \partial^2_{xx} + \partial^2_{yy}$. All terms multiplied by ν/λ have been neglected since we shall consider only the limit $\nu \ll \lambda$ which is appropriate for liquid metals, but also for solar plasmas. The term $\boldsymbol{B}\cdot\nabla\boldsymbol{B}$ in equation (2a) can now be evaluated,

$$\boldsymbol{B}\cdot\nabla\boldsymbol{B} = Q[(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z)\cdot\nabla(\nabla\times(\nabla h\times\boldsymbol{k}) + \nabla g\times\boldsymbol{k}) + \Delta_2 h\gamma(\boldsymbol{i}\,\sin\gamma z + \boldsymbol{j}\,\cos\gamma z)] \tag{7}$$

where Q is the Chandrasekhar number,

$$Q = \frac{B_0^2 d^2}{\nu\rho\mu\lambda}. \tag{8}$$

In the following we shall first restrict the analysis to the linear problem of the onset of steady convection in which case the left hand sides of equations (2) can be neglected. Oscillatory onset of convection is possible in the presence of a magnetic field, but is not realized in the limit $\nu \ll \lambda$ when the time derivative in equation (5) can be dropped. As long as a steady onset of convection is presumed the linear analysis is actually independent of the magnetic Prandtl number, $Pm = \nu/\lambda$, just as it is independent of the ordinary Prandtl number P.

By taking the z-component of the double curl of equation (2a), i.e. by operating with $\boldsymbol{k}\cdot\nabla\times(\nabla\times ...)$ onto it, we find

$$\nabla^4 u_z + \Delta_2\Theta - Q(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z)\cdot\nabla(\nabla^2\Delta_2 h + \gamma^2\Delta_2 h) = 0. \tag{9}$$

With the help of equations (2c) and (6a) Θ and h can be eliminated from this equation,

$$\begin{aligned}\nabla^6 u_z - R\Delta_2 u_z - Q\Big([(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z)\cdot\nabla]^2(\nabla^2-\gamma^2)u_z -\\ 2\gamma^2[(\boldsymbol{i}\,\sin\gamma z + \boldsymbol{j}\,\cos\gamma z)\cdot\nabla]^2 u_z +\\ 4\gamma(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z)\cdot\nabla(\boldsymbol{i}\,\sin\gamma z + \boldsymbol{j}\,\cos\gamma z)\cdot\nabla\partial_z u_z\Big)\\ = -Q[\gamma^4(\boldsymbol{i}\,\cos\gamma z - \boldsymbol{j}\,\sin\gamma z)\cdot\nabla\Delta_2 h + 2\gamma^3(\boldsymbol{i}\,\sin\gamma z + \boldsymbol{j}\,\cos\gamma z)\cdot\nabla\Delta_2\partial_z h]\ .\end{aligned} \tag{10}$$

On the right hand side of this equation terms involving h are still left. We shall neglect these terms by making the assumption that the parameter γ is sufficiently small such that terms multiplied by γ^n with $n \geq 3$ are negligible in comparison with those with a lower power of n.

Equation (10) admits solutions of the form

$$u_z = f(z) exp\{ia\,(\boldsymbol{i}\,\sin\chi - \boldsymbol{j}\,\cos\chi) \cdot \boldsymbol{r}\} \tag{11}$$

where χ denotes the angle by which the axis of the convection roll described by (11) is turned away from the positive x-axis towards the negative y-axis. The Rayleigh number R for onset of convection will be minimized when convection sets in at a height z_0 such that the angle χ satisfies $\chi = \gamma z_0$. In this case the dominant term multiplied by Q in equation (10) vanishes for $z = z_0$. It does not vanish for $z \neq z_0$ and it is appropriate to use a Taylor expansion, $\cos\gamma z = \cos\gamma z_0 - (z - z_0)\gamma \sin\gamma z_0 + ...$, and likewise for $\sin\gamma z$. It must be expected that the lowest value of R is attained for $\chi = z_0 = 0$ where the constraining effect of the boundaries at $z = \pm 0.5$ is minimized. We thus obtain the ordinary differential equation for $f(z)$,

$$[(\frac{d^2}{dz^2} - a^2)^3 + Ra^2 + Q\gamma^2 a^2 (z^2(\frac{d^2}{dz^2} - a^2) + 2 + 4z\frac{d}{dz})]f(z) = 0 \tag{12}$$

where terms up to the order γ^2 have been kept. This reformulation of the problem offers two advantages:

1. Instead of two external parameters Q and γ only a single combination of them, $G \equiv \gamma^2 Q$, enters the formulation of the problem.

2. Since the poloidal potential h of the magnetic field has entered in the derivation of equation (12) only in the form $\nabla^2 \Delta_2 h$ the onset of convection is independent of the magnetic boundary conditions at $z = \pm 0.5$. This property continues to hold at finite amplitudes of steady two-dimensional solutions, but will be lost when their stability with respect to three-dimensional disturbances is analyzed (see, for example, Busse and Clever, 1983).

The validity of the assumption $z_0 = 0$ and of the neglect of terms with γ^n with $n \geq 3$ will be confirmed in section 5 through an independent analysis of the basic equations.

3. Numerical Results

In order to obtain solutions of equation (12) we use the shooting method. Because of the assumption of stress-free isothermal boundaries,

$$f = f'' = f'''' = 0 \qquad at \qquad z = \pm\frac{1}{2}, \tag{13}$$

three initial conditions at $z_0 = -0.5$ are fixed. Three independent solutions, $f^{(n)}(z), n = 1, 2, 3$, can thus be generated by the integration of equation (12) for given values of $R, a, G \equiv \gamma^2 Q$ with a fourth-order Runge-Kutta method. For this purpose the three additional initial conditions

$$\frac{d}{dz}f^{(n)} = \delta_{n1}, \qquad \frac{d^3}{dz^3}f^{(n)} = \delta_{n2}, \qquad \frac{d^5}{dz^5}f^{(n)} = \delta_{n3}, \qquad at \qquad z = -\frac{1}{2} \tag{14}$$

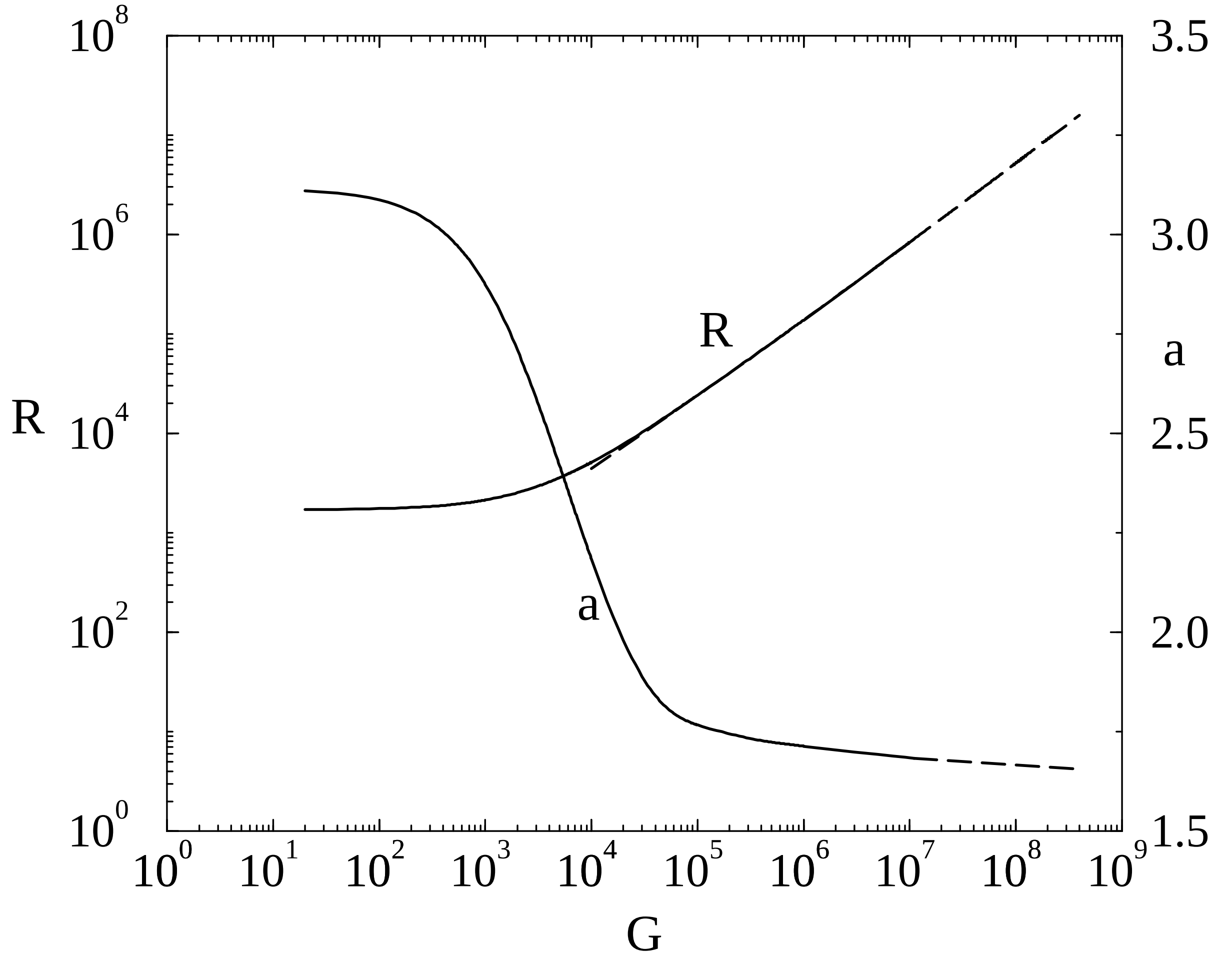

Figure 1: The critical Rayleigh number R_c and the critical wavenumber a_c as a function of G. The dashed lines represent the asymptotic result (22).

are used. Since the solutions with the lowest values of the Rayleigh number are always symmetric with respect to $z = 0$ we can restrict the integration to the interval $-0.5 \leq z \leq 0$ and demand that the conditions

$$f' = f''' = f''''' = 0 \qquad at \qquad z = 0 \tag{15}$$

are satisfied. These conditions yield a system of three homogeneous equations for the unknown coefficients A_n in the expression $f = \sum_{n=1}^{3} A_n f^{(n)}$. A necessary and sufficient condition for a non-trivial solution of the equations is that the determinant of 3×3 coefficient matrix vanishes. This will happen for discrete values of R which can be determined through a Newton-iteration procedure. Only the lowest value of $R(a, G)$ is of interest. The minimization of $R(a, G)$ as a function of the wavenumber a yields the critical wavenumber $a = a_c$ and the critical Rayleigh number, $R_c(G) = R(a_c, G)$. $R_c(G)$ and a_c have been plotted in figure 1. As expected R_c increases strongly with G, while a_c decreases. Plots of the solutions $f(z)$ for typical values of G are shown in figure 2. They demonstrate the increasing concentration with increasing G of the amplitude of convection in a narrow neighborhood around $z = 0$. In figure 3 the temperature distribution and the streamlines are shown for a particular case

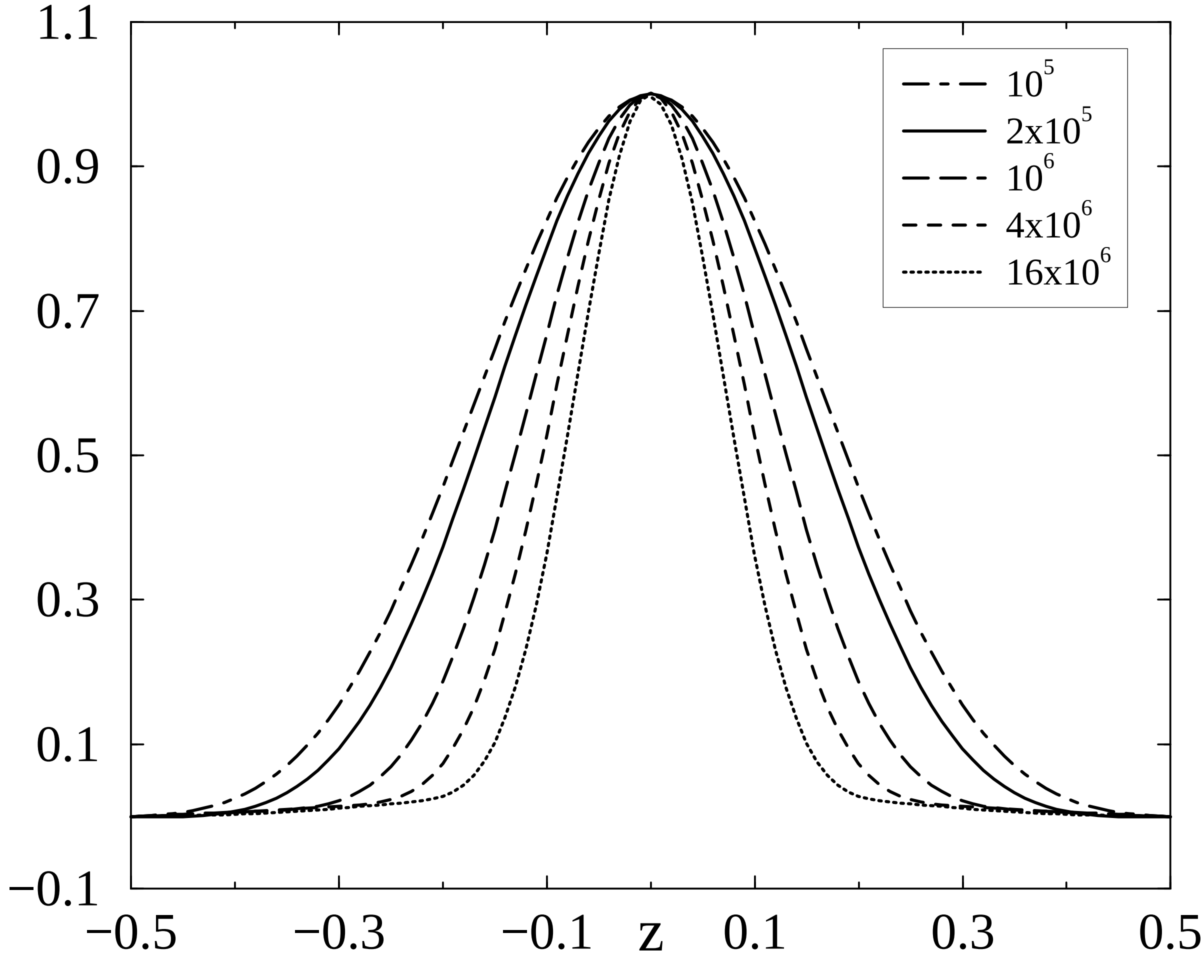

Figure 2: The function $f(z)$ for various values of G as indicated. The functions $f(z)$ have been normalized by $f(0) = 1$.

which demonstrates that the streamlines are squashed by the effects of the magnetic field, while the temperature distribution is much less affected.

The height δ to which convection is confined in the neighborhood of $z = 0$ decreases in proportion to $G^{-1/6}$ as G tends to infinity as is evident from the form of equation (12). Because of the concentration of the convection solution towards the midplane of the layer the conditions outside this neighborhood and the boundary conditions at $z = \pm 0.5$ have rather little influence for large G. Computations carried out for no-slip conditions instead of stress-free conditions at the boundaries, for example, yield the same results for large values of G.

4. An Asymptotic Approach

The fact that the critical wavenumber a_c assumes a relatively small value of the order unity as G tends to infinity suggests that several terms involving a^2 in equation (12) can be neglected in first approximation and that a perturbation approach based on an expansion in powers of a^2 may provide an asymptotically valid description of the solution. We are thus led to the

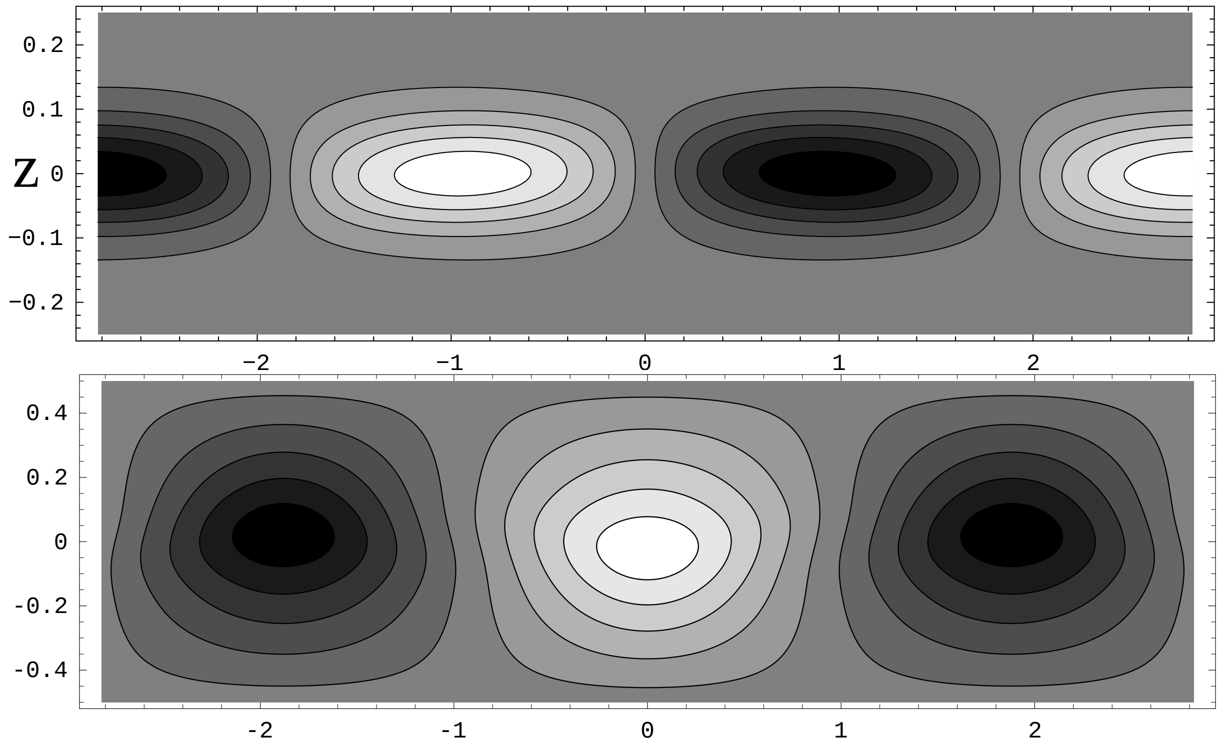

Figure 3: Streamlines and temperature Θ of the convection flow in the case $\gamma = 0.2, Q = 10^9$ at a value R 10% above R_c

following asymptotic analysis. Using the representation

$$Ra^2 \equiv r = r_0 + a^2 r_1 + ..., \qquad f(z) = f_0(z) + a^2 f_1(z) + ..., \tag{16}$$

we find that the functions $f_n(z)$ satisfy the following set of equations,

$$[\frac{d^6}{dz^6} + r_0 + \hat{g}(z^2 \frac{d^2}{dz^2} + 2 + 4z\frac{d}{dz})]f_0(z) = 0, \tag{17a}$$

$$[\frac{d^6}{dz^6} + r_0 + \hat{g}(z^2 \frac{d^2}{dz^2} + 2 + 4z\frac{d}{dz})]f_1(z) = [3\frac{d^4}{dz^4} + \hat{g}z^2 - r_1]f_0(z), \tag{17b}$$

$$...., \tag{17c}$$

where $\hat{g}$ is defined by $\hat{g} \equiv Ga^2$. In this formulation r and $\hat{g}$ are large parameters, $r \gg 1, \hat{g} \gg 1$, and convergence of expressions (16) can be expected as long as $a^{2n}r_n \ll a^{n-1}r_{n-1}$ etc. is satisfied for $n > 0$.

The unknown coefficient r_1 is determined by the solvability condition for equation (17b). This condition is obtained when the right hand side of equation (17b) is multiplied by the solution $f_0^*(z)$ of the adjoint homogeneous equation and integrated over the interval $-0.5 \leq z \leq 0$,

$$r_1\langle f_0(z)f_0^*(z)\rangle = \langle f_0^*(z)[3\frac{d^4}{dz^4} + \hat{g}z^2]f_0(z)\rangle, \tag{18}$$

where the angular brackets indicate the average over the interval $-0.5 \leq z \leq 0.5$. The function $f_0^*(z)$ is obtained as the solution of the equation

$$[\frac{d^6}{dz^6} + r_0 + \hat{g}z^2\frac{d^2}{dz^2}]f_0^*(z) = 0 \tag{19}$$

with the same boundary conditions as applied for f_0.

In order to determine the critical value R_c of the Rayleigh number $R \equiv r/a^2$ must be minimized with respect to a^2. As necessary condition we obtain

$$a^{-2}\frac{d}{da^2}r_0 - r_0 a^{-4} + \frac{d}{da^2}r_1 + ... = 0, \tag{20}$$

where the derivatives with respect to a^2 must be done at a fixed value of G. Since r_0 and r_1 depend on a^2 through their dependence on $\tilde{g}$ we find

$$a_c^2 = [r_0/\hat{g} - \frac{dr_0}{d\hat{g}}]/\frac{dr_1}{d\hat{g}} \tag{21}$$

where terms of higher order have been neglected. Expression (21) can be evaluated only numerically. For values of G of the order 10^8 we find $a_c^2 \simeq 2.65$. This value decreases slightly with increasing G. In good approximation the critical values R_c and a_c^2 of the Rayleigh number and of the square of the wavenumber obey the power law relationships

$$R_c \simeq 1.59 \cdot G^{0.81}, \qquad a_c^2 \simeq 41.1 \cdot G^{-0.15}. \tag{22}$$

5. Stability Analysis

The confinement of the convection rolls to a region with a height δ of the order $G^{-1/6}$ around $z = 0$ suggests that additional convection rolls with an appropriate orientation will set in within the nearly quiescent regions below and above the given rolls. This expectation is indeed confirmed when an stability analysis of finite amplitude rolls of the form (11) with $\chi = 0$ is carried out. After the representation

$$\boldsymbol{u} = \nabla \times (\nabla\Phi \times \boldsymbol{k}) + \nabla\Psi \times \boldsymbol{k} \equiv \boldsymbol{\delta}\Phi + \boldsymbol{\eta}\Psi \tag{23}$$

for the velocity vector has been introduced, the equations

$$\nabla^4\Delta_2\Phi + Q(\boldsymbol{i}\cos\gamma z - \boldsymbol{j}\sin\gamma z)\cdot\nabla(\nabla^2\Delta_2 h + \gamma^2\Delta_2 h) - \Delta_2\Theta = \boldsymbol{\delta}\cdot(\boldsymbol{v}\cdot\nabla\boldsymbol{v}) + \partial_t\nabla^2\Delta_2\Phi, \tag{24a}$$

$$\nabla^2\Delta_2\Psi + Q(\boldsymbol{i}\cos\gamma z - \boldsymbol{j}\sin\gamma z)\cdot\nabla(\Delta_2 g - \gamma\Delta_2 h) = \boldsymbol{\eta}\cdot(\boldsymbol{v}\cdot\nabla\boldsymbol{v}) + \partial_t\Delta_2\Psi, \tag{24b}$$

must be solved together with the heat equation,

$$(\nabla^2 - \partial_t)\Theta - R\Delta_2\Phi = P(\boldsymbol{\delta}\Phi + \boldsymbol{\eta}\Psi)\cdot\nabla\Theta, \tag{25}$$

and equations (6). Two-dimensional steady finite amplitude solutions of equations (24), (25) and (6) have been obtained with a Galerkin method. A more detailed description of this method has been given in the recent paper by Kurt et al. (2004) and can also be found in the papers by Busse and Clever (1983, 1989). While no-slip boundary conditions for the velocity field and vanishing potentials h and g have been used at $z = \pm 0.5$, the results are essentially independent of the boundary conditions once the parameter G is sufficiently large, just as has been found in the linear analysis described above. In fact, the results for R_c and a_c based on the Galerkin method agree almost exactly with those obtained with the shooting method of section 3 once a sufficiently small γ, say $\gamma = 0.1$, has been assumed. This confirms the validity of the neglect of terms of the order γ^3 in the derivation of equation (12).

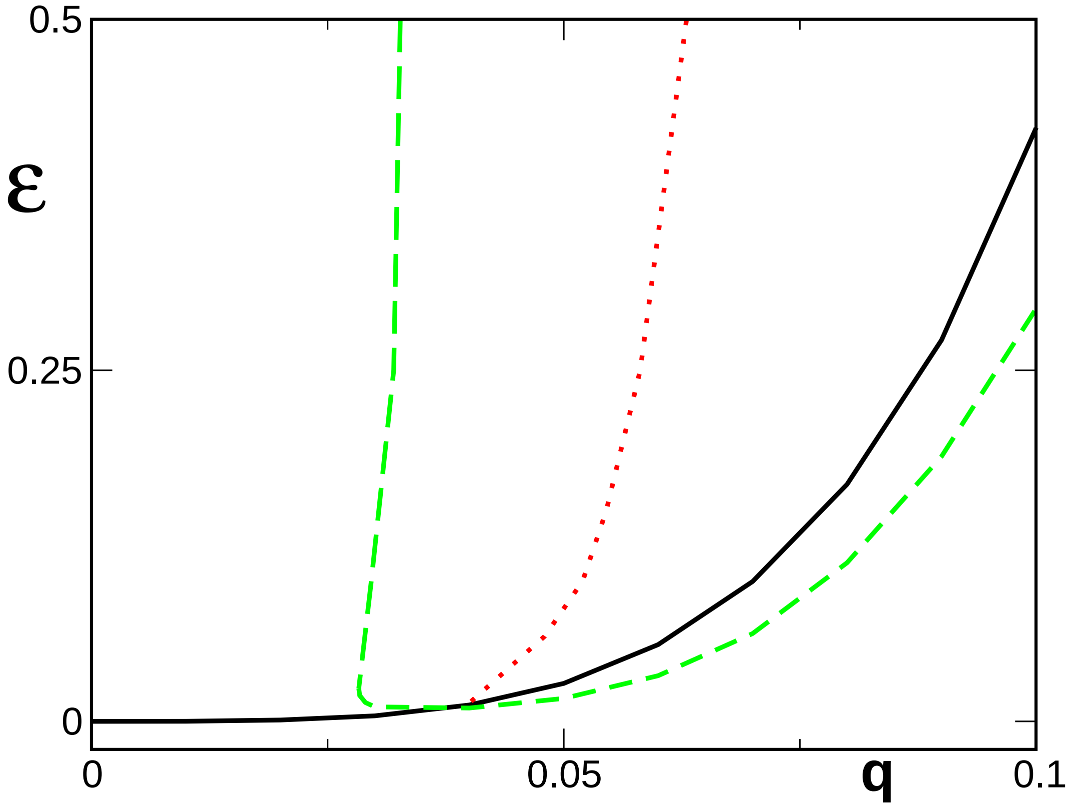

Figure 4: Rolls with the wavevector $(0, a_c)$ are unstable with respect to disturbance rolls with wavevectors $(\pm q, a_c)$ above the dashed curve and to the right of it on the left side of the figure. ϵ is defined by $\epsilon = (R - R_c)/R_c$. $R_c = 2492 \cdot 10^3$ corresponds to $G = 4 \cdot 10^7$. The neutral curve for the onset of convection rolls with the wavevector (q, a_c) is indicated by the solid line. The dotted line indicates the value of q for which the growth rate reaches its maximum.

Once a steady nonlinear solution has been obtained with a Newton-iteration procedure its stability can be investigated through the superposition of infinitesimal disturbances. For this purpose equations (24), (25) and (6) must be linearized in the disturbance amplitude. The linear homogeneous system of equations then represents an eigenvalue problem for the complex growthrate σ. Here we wish to describe results only in the particular, but representative case $Q = 10^9, P = 0.2, \gamma = 0.2$ corresponding to $R_c = 2492 \cdot 10^3, a_c = 1.67$. (Since γ is not quite as small as assumed above the value of R_c is smaller by 0.08% than the value obtained from the solution of equation (12).) In the stability analysis all growthrates with positive real parts have exhibited a vanishing imaginary part. In figure 4 growing disturbances are found above the dashed curve as a function of the $x-$component $q = a_c \tan\chi$ of the disturbance wavevector, (q, a_c). The reduced control parameter $\epsilon = (R - R_c)/R_c$ provides a measure of the amplitude of the basic roll solution with wavevector $(0, a_c)$. The fact that the neutral curve for the onset of convection rolls with the wavevector (q, a_c) lies above the dashed curve indicates that the presence of the finite amplitude rolls actually promotes the onset of rolls in the neighboring layers. The physical reason for this property is that the heat transport carried by the basic rolls causes a decrease of the absolute value of the mean temperature gradient around $z = 0$, while it is increased for larger values of $|z|$. The local Rayleigh number is thus amplified and causes the onset of rolls at values of R

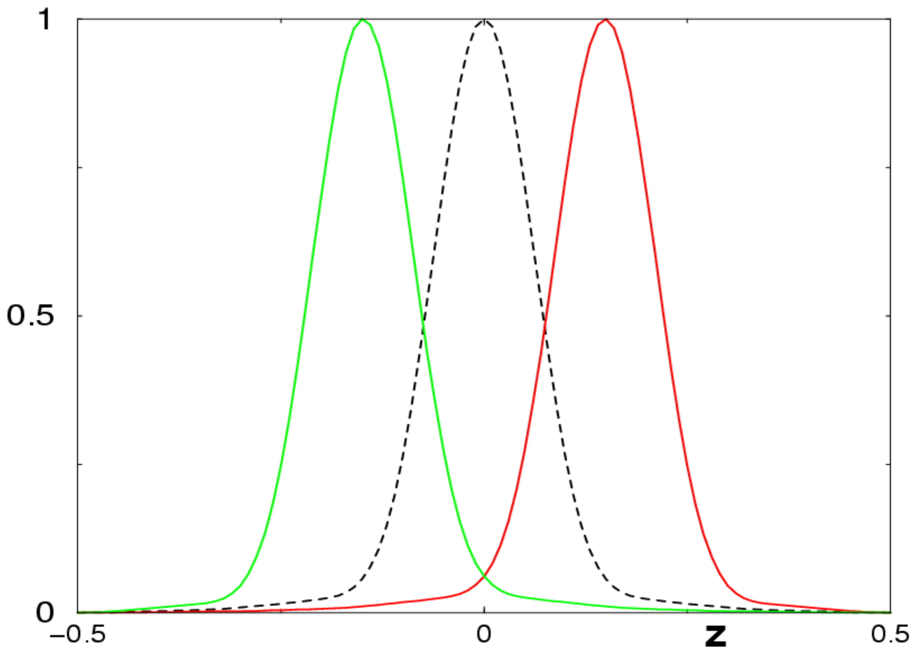

Figure 5: u_z-components of the disturbances with $|q| = 0.05$ (solid lines) together with u_z for the basic roll solution (dashed line) as functions of z in the case $\gamma = 0.2, Q = 10^9$.

below the neutral curve.

For reasons of symmetry there are always two disturbances with the same growthrate, one dominating in the region $z < 0$, the other in the region $z > 0$. The profiles of the u_z-components of the disturbance velocity fields are shown in figure 5 together with the corresponding profile of the basic roll solution. As must be expected the maxima of the disturbance amplitudes lie at the value z_0 where the disturbance roll axis parallels the magnetic field, i.e. $|z_0|$ =arctan$(q/a_c)/\gamma \approx 0.15$ in the present case in agreement with the solid curves of the figure. The disturbances of figure 5 correspond to $\epsilon = 0.076$ where their growth rate reaches a maximum according to figure 4. At higher values of ϵ the disturbances of maximum growth are shifted further away from $z = 0$ in order to reduce the overlap with the steady finite amplitude solution.

6. Conclusion

The properties of convection rolls in the presence of a twisted magnetic field may eventually contribute to an improved understanding of convection in the penumbra of sunspots. It has not been the goal of this paper to develop a model of penumbras. Instead we hope to have elucidated some typical features of the interaction between magnetic fields and convection. The fact that the conditions on the sun differ considerably from the Boussinesq approximation loses some of its importance through the property that the convection rolls are confined to rather thin layers for large values of G.

The theory also does not depend significantly on the property that the magnetic field is horizontal. In fact, the two-dimensional solution depending on y and z does not change when the direction of gravity and of the basic temperature gradient are turned by the angle θ away from the z-axis towards the x-axis except that in section 2 $\hat{g}$ and $T_2 - T_1$ must be replaced by $\hat{g}\cos\theta$ and $(T_2 - T_1)\cos\theta$. In particular equations (9) through (12) as well as the corresponding nonlinear extensions remain unchanged. A third component, u_x, proportional to $\sin\theta$ is added to the velocity field, but this does not affect the basic finite amplitude roll solution as is evident from the analogous case of longitudinal rolls in an inclined convection layer (Clever and Busse, 1977). Since the solution depends only weakly on the conditions at the boundaries for large values of G it should not matter much that the boundaries will no longer be horizontal or isothermal for $\theta \neq 0$.

The assumption of a constant twist is, of course, an idealization and convection rolls are likely to appear first in regions where the twist is relatively small. Twisted magnetic fields in the penumbra of sunspots appear to be a rather common phenomenon which seems to be related to the property that sunspots usually exhibit a small rotation relative to the surrounding solar atmosphere (Brown et al., 2003).

The occurrence of convection rolls in sublayers with discretely changing orientation is an interesting and unusual phenomenon which demands further investigations. The interaction of finite amplitude rolls at different levels z_0 can be attacked only through a fully nonlinear numerical analysis. A study of steady solutions with multiple sublayers is in progress.

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