EXPERIMENTS WITH TRAVELLING WAVES IN ELECTROHYDRODYNAMIC CONVECTION

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The experimental setup is the following: A nematic liquid crystal (purified MBBA or Merck-PhaseV) is sandwiched between two transparent electrodes rubbed to produce a prefered direction on the orientation of the molecules. The thickness (13-50μm) was adjusted with polymeric mylar sheets, and the whole cell was sealed. The cell is embedded in an isothermal ($\pm$0.1K) box. A shadowgraphic image of the pattern (Rasenat et al. 1989) is observed with a CCD-camera mounted on a polarising microscope and digitized (512x512 pixels, 256 grey scales). Applying an ac-voltage to the cell leads to convection setting in at a critical driving voltage V_c, as shown in Fig.1. Here a direct bifurcation to travelling waves (TW) is observed in the whole conductive regime and even in the dielectric regime, unlike measurements shown elsewhere (Hirakawa & Kai 1977, Joets & Ribotta 1988, Rehberg et al. 1988a,b). The solid line is obtained by linear stability analysis of the Leslie-Erickson equation (Zimmermann & Thom 1988), where MBBA parameters (25°C) are used and the electrical conductivity σ has been adjusted to fit the threshold voltage where the dielectric and the conductive instability meet. This theory predicts a steady bifurcation. A hopf bifurcation from the spatially homogenous state, which would explain the TW observed in the experiment, has not been found for a range of material parameters likely to include the realistic values for MBBA. The agreement of our experiments (Rehberg et al. 1988a) with the theoretical predictions for temporal modulation of a direct hopf bifurcation with O(2)-symmetry (Riecke et al. 1988, Walgraef 1988), however, strongly supports the idea that a hopf bifurcation is responsible for the observed TW.

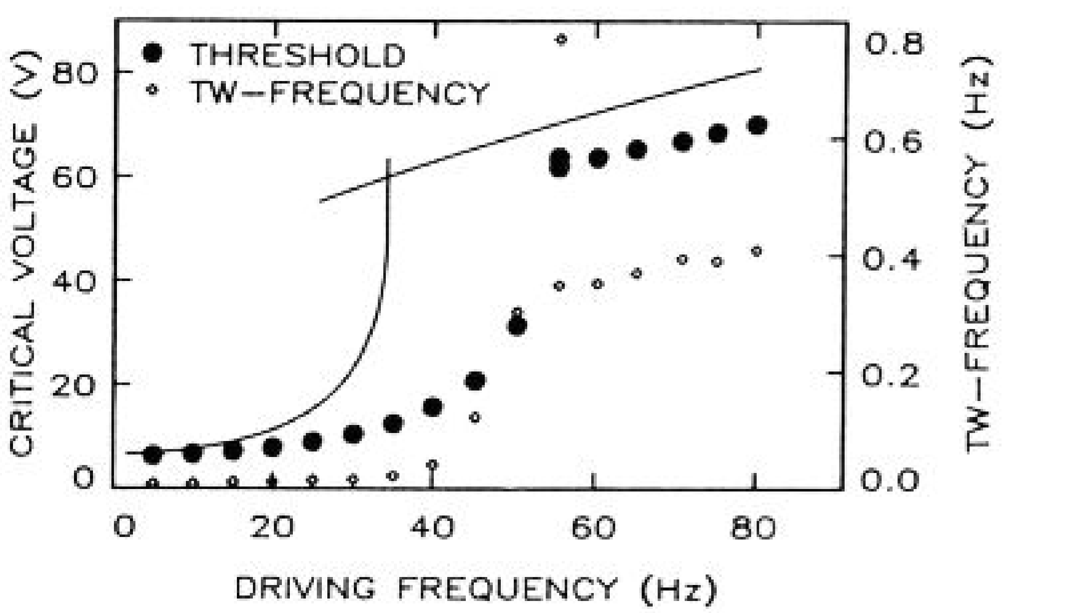

Fig.1. V_c (solid circles), and critical frequency (open diamonds), MBBA, 54μm. Theory: solid line.

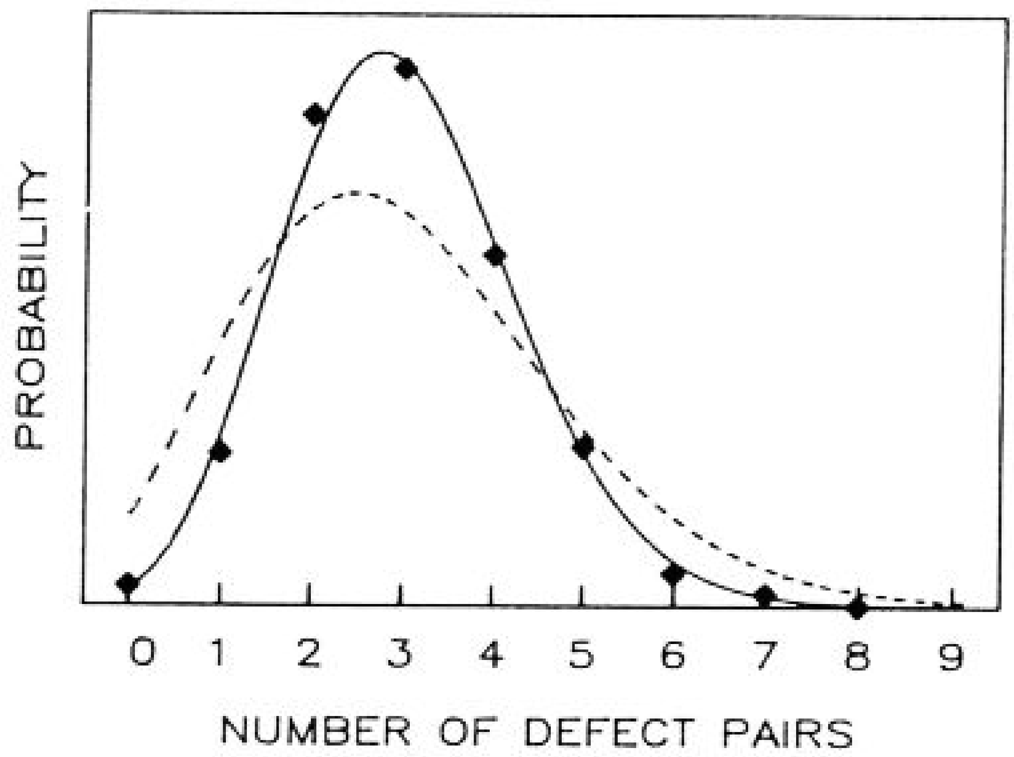

Fig.2.Distribution of defects, *steady* pattern, MBBA, 25μm. Solid line: predicted curve; dashed: Poisson.

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In the experiment (Rehberg et al. 1988b), the transition to turbulence in TW convection is characterized by the appearance of topological defects. Their statistical distribution is in very good agreement with a prediction based on the idea that the probability to create a defect pair is only dependent on $\epsilon=V/V_c-1$, while the annihilation rate is proportional to the number of defects squared (Gil et al. 1988). These arguments should apply to TW as well as to steady patterns. Fig.2 shows the curve obtained in steady convection. The squared poisson distribution (solid line) fits the data well.

We performed measurements in a 13μ cell (Phase V, cutoff frequency 100Hz) in order to measure some of the coefficents of the amplitude equations

$$\partial_t A_1=(\mu+i\nu)A_1+\chi A_2-(1+i\beta)A_1|A_1|^2+(\gamma+i\delta)A_1|A_2|^2$$
$$\partial_t A_2=(\mu-i\nu)A_2+\chi A_1-(1-i\beta)A_2|A_2|^2+(\gamma-i\delta)A_2|A_1|^2 \qquad (1)$$

which govern the dynamics of oscillatory states under the influence of a temporal forcing $V(t)=1+\epsilon+b\cos(\omega_e t)$, with $\omega_e\approx 2\omega_h$, ($\omega_h$: critical frequency) where A,B,C defined by $\mu=\epsilon/A$, $\nu=B\mu-(\omega_e/2-\omega_h)$, $\chi=b/C$ have to be determined experimentally (Riecke et al. 1988).

Fig. 3 shows a measurement of the onset of travelling waves. Light intensity scans on a line perpendicular to the roll axis are taken and subtracted from a reference line measured without convection. The temporal average of the rms-values of these differences provides information about the amplitude of convection and is plotted as a function of the driving voltage. The critical threshold for the onset of convection is thus determined to be $8.77V_{eff}$. The finite RMS-values obtained at negative ϵ are caused by intensity fluctuations and camera noise. The frequency of the TW observed above this voltage is shown to decrease monotonically with increasing ϵ. The solid line is obtained by linear regression analysis allowing the determination of the critical frequency ω_h.

The linear stability analysis of equations (1) shows that the convection threshold for $\mu<0$ is defined by the relation $\mu^2+\upsilon^2=\chi^2$ which defines an hyperbola in the ω_e-b plane. Fig.4 shows the corresponding measurement together with a fit to a hyperbola, where A,B,C and ω_h are adjustable parameters. Thus the linear coefficients of the amplitude equations (1) can be determined, although more measurements are needed to improve the accuracy of the results.

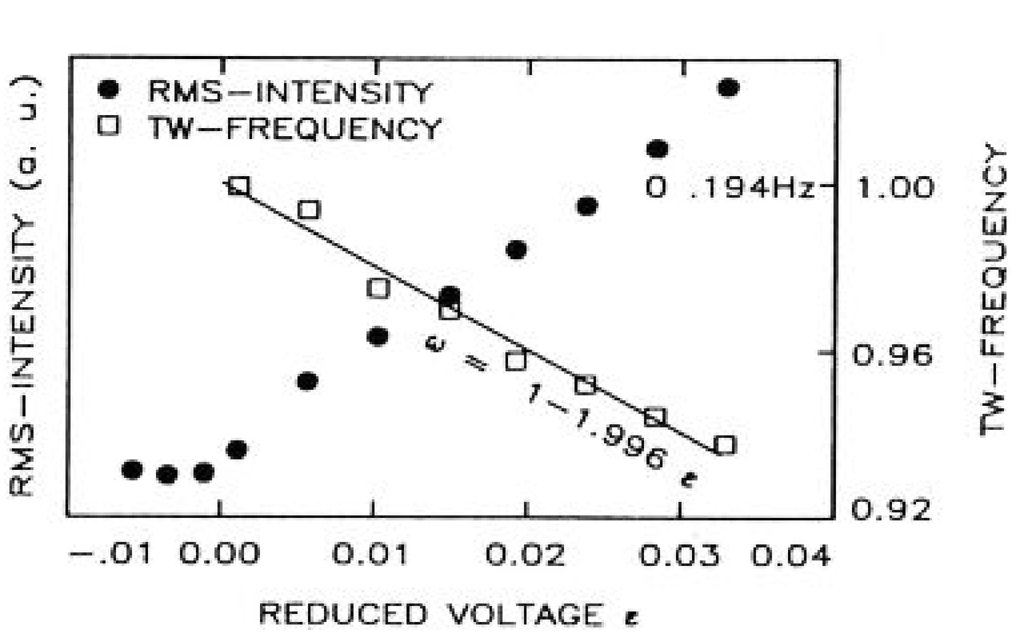

Fig. 3. The amplitude of convection increases with ϵ, and the frequency of the TW decreases. The solid line is the linear regression line. (Phase V, $13\mu m$)

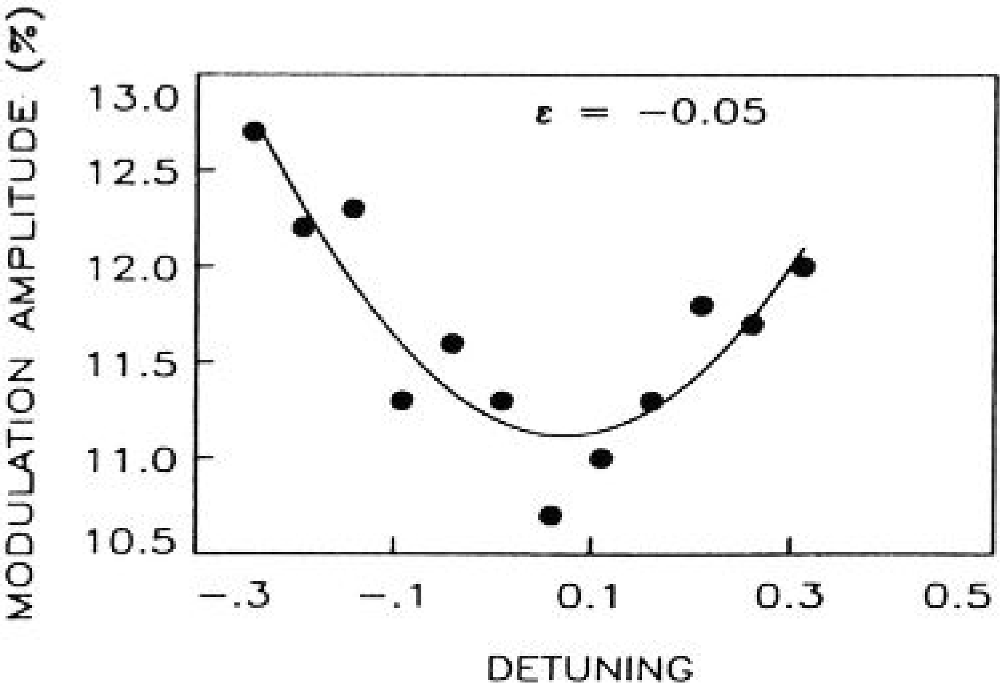

Fig. 4. The threshold of convection in the ω_e-b plane (Phase V, $13\mu m$) The adjusted hyperbola (solid line) is described by A=0.453s, B=-0.130, C=1.00s, $\omega_h=0.198s^{-1}$

One can then even calculate the nonlinear parameter β=B-A·$\Delta\omega_{TW}/\Delta\epsilon$, thus separating the linear from the nonlinear frequency shift (unlike Chiffaudel et al. 1987). By the slope of the frequency shown in Fig.3, β is determined to be -0.183. Next we study the nonlinear resonance curve obtained by measuring the amplitude of the convection as a function of the modulation frequency for a fixed negative ϵ and a fixed modulation amplitude. The result is shown in Fig.5. Note that the transitions to standing waves are supercritical ones. The solid line is the formula given by Riecke et al. (1988), where the unknown nonlinear parameters δ and γ had been adjusted to produce a reasonable similarity to the measurement.

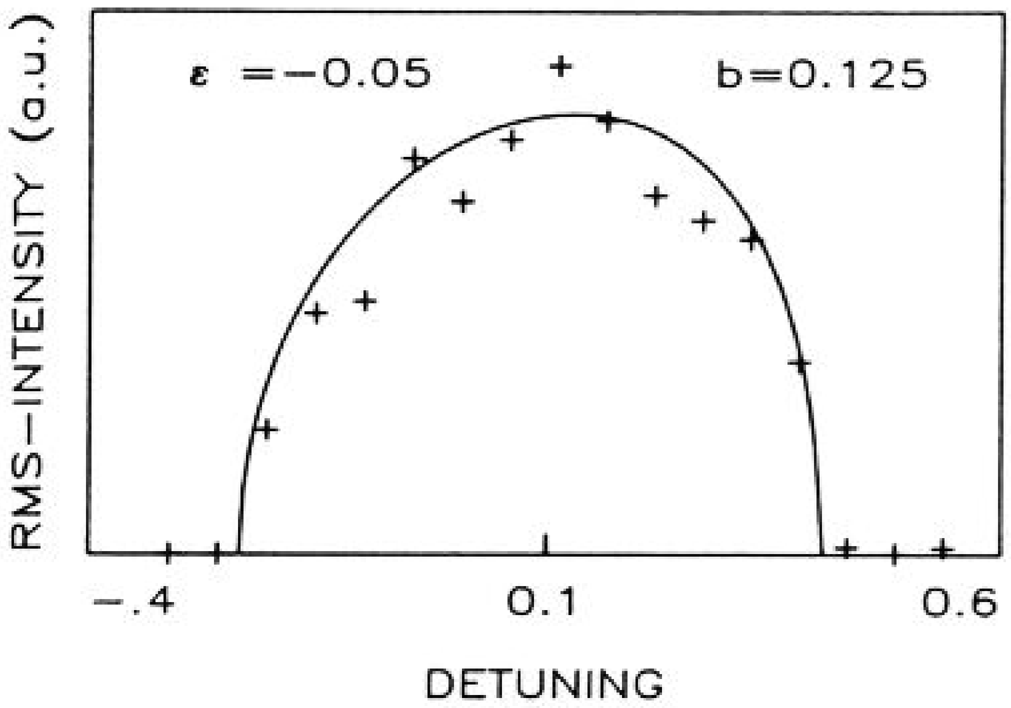

Fig. 5

The amplitude of the standing wave convection measured as a function of the modulation frequency (+). The solid line is the theoretical curve, with δ and γ adjusted. Note that convection occurs at the subcritical value ϵ=-0.05.

Acknowledgements

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