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Measuring the growth rate of electroconvection by means of thermal noise

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The growth rate of the the critical mode of electroconvection in a nematic liquid crystal is measured by analyzing the correlation function of subcritical thermal director fluctuations. The measured number agrees with the one obtained by using a deterministic method, namely the analysis of decaying convection rolls after a sudden quench of the control parameter from supercritical to subcritical values. The measured growth rates do not agree with results from a two-dimensional linear stability analysis using the full nematodynamic equations.

1. Introduction

Electroconvection in a thin layer of a nematic liquid crystal is established as the Drosophila of anisotropic pattern forming systems. One of the reasons for the popularity is its richness: Even very close to onset convection patterns can occur in the form of steady oblique rolls, traveling oblique rolls, steady normal rolls (Williams domains), traveling normal rolls and parametrically excited standing waves (dielectric rolls), depending on the frequency of the driving ac-voltage [1]. The steady instabilities can be described by an amplitude equation of the form [2]

$$\tau_0 \partial_t A = \epsilon A + \xi_\parallel^2 \partial_{xx}^2 A + \xi_\perp^2 \partial_{yy}^2 A - g|A|^2 A, \qquad (1.1)$$

where the sign of the nonlinear coefficient g determines whether the bifurcation is a supercritical or subcritical one. It has been shown recently that the steady oblique rolls [3] and the steady normal rolls [4] do not appear via a forward bifurcation, as predicted by the theory [2] and as assumed in most experimental works. The convection rather starts via a subcritical bifurcation, with a hysteresis smaller than one percent of the control parameter. Such a small hysteresis is often hard to detect due to spatial inhomogeneities. A qualitative disagreement like this between theory and experiment raises the desire for a critical comparison between the measured and the calculated values of the linear coefficients of the amplitude equation.

A subcritical bifurcation makes it hard to analyze the nature of the instability experimentally, because the pattern which forms once the jump of the order parameter has taken place might very well have not too much in common with the linear unstable mode. As an example, the formation of steady normal rolls seems to take place via a subcritcal instability with respect to traveling waves [4], and the onset of oblique rolls seems to be even more complex [3]. In order to check the nature of the instability experimentally, it seems advantageous to have a tool which excites all possible modes at the same time, so that by inspection of the decay rates of the mode of interest one obtains a measure for its distance from criticality. It has been shown recently that nematic liquid crystals offer such a tool, namely the intrinsic thermal noise, which leads to hydrodynamic fluctuations of measurable intensity in the neighborhood of the hydrodynamic instability [4,5].

In this paper we would like to propose that this fact offers a convenient measuring tool for testing the linear stability of different modes of convection. The convenience stems from the fact that the thermal noise can be thought of as being white, at least in the sense that it contains all possible modes. We demonstrate this method by measuring the decay rate of normal rolls via an analysis of subcritical fluctuations, and compare this method with the classical one of inspecting the decay of fully developed convection rolls.

2. Experimental setup and procedure

The experimental setup is very similar to the one described in ref. [5]. We used a layer of thickness $d = 24\,\mu$m of the nematic liquid crystal N-(p-methoxybenzyldiene)-p-butylaniline (MBBA). A preferred direction of the rolls above onset is enforced by rubbing of the polymer-coated glass slides which confine the sample. The temperature is controlled to $25 \pm 0.01°$C. The mean-square value of the AC-voltage applied to the transparent electrodes serves to define the control parameter $\epsilon = V^2/V_c^2 - 1$, where V_c is the critical voltage for the onset of convection. The cutoff-frequency of the sample is at $f_c = 220$ Hz.

A red light emitting diode (LED) illuminates the sample through a polarizer. The microscope objective forms an image of the layer located 5d above the upper boundary of the liquid crystal. This image is detected by a Peltier-cooled thermostatted array of 1024 photo diodes, with a spacing of 0.025 mm and a height of 2.5 mm. In order to speed up the readout time during the transient processes, we use only the central 512 diodes of the line camera. The exposure time for the diodes is 0.088 seconds. The light intensity is digitized with 14 bits, which corresponds to 16 348 gray levels.

The use of such a line camera offers the advantage of a faster readout time when compared with a two-dimensional charge-coupled device array. Another advantage when observing two-dimensional patterns with an amplitude described by $I(x,y) = I_0 \sin(qx)\sin(py)$ is the fact that modes with a $p \neq 0$ will not be detected. To be more precise, the camera has a transfer function for these modes according to $I^2_{\text{meas}} = I_0^2 \sin^2(pl_y)/(pl_y)^2$, where l_y is half the length of the photo diodes along the y-direction. This is not a problem for the normal rolls we are considering here, but makes such a line camera unfavorable for oblique rolls.

The idea for the experimental determination of a growth rate σ using intrinsic thermal noise is based on the underlying assumption that the amplitude A_c for the convective mode with the critical wavenumber is very small and can thus be described by a linearized equation:

$$\dot{A}_c = \sigma A_c + 2F\,, \qquad (2.1)$$

where F represents δ-correlated Gaussian white noise. This model leads to a correlation function:

$$\text{Cor}\,(\Delta t) \sim e^{\sigma \Delta t}\,. \qquad (2.2)$$

Thus, the growth rate σ, which is negative for the subcritical values of the control parameter considered here, can be determined by an analysis of the measured correlation function, as has been described in detail in ref. [5]. It seems useful to point out that this method is very similar to measure the decay time of director fluctuations via dynamic light scattering [6]. That method, however, measures a signal which is quadratic in the director deformation, while the shadowgraph method used here provides a linear response.

As an alternative, σ can be measured by means of a deterministic method, namely by fitting an exponential decay to the time dependence of the order parameter after a sudden quench to subcritical values of the control parameter. This method had been used in connection with the shadowgraph method in ref. [7] for the nematic liquid crystal Merck Phase V, and in ref. [8] for MBBA. In these works, the root mean square of the measured light intensity signal has been used. As suggested in ref. [9], however, it might be advantageous to do a Fourier analysis of the measured light intensity signal and use the fundamental mode as a more robust estimator of the order parameter.

3. Experimental results

Fig. 1 presents the result for determining σ at a driving frequency of 50 Hz. The data shown as solid squares have been obtained by accumulating the spatio-temporal correlation function in the way described in ref. [9] over a time interval of 12 minutes, and the growth rate σ of the critical mode is extracted form these functions following the procedure described in ref. [5]. The data points can be reasonably well fitted by a streigth line, whose slope defines the characteristic time τ_0. Note that no data were taken in the immediate neighborhood of the critical point where the growth rate approaches zero, because here one might have to worry about nonlinear effects, which would make our simplified analysis inapplicable. A close inspection shows that the data do show a systematic deviation in the neighborhood of the critical point, but whether this fact can already be interpreted as an indication for the presence of nonlinear effects is unclear at the present time.

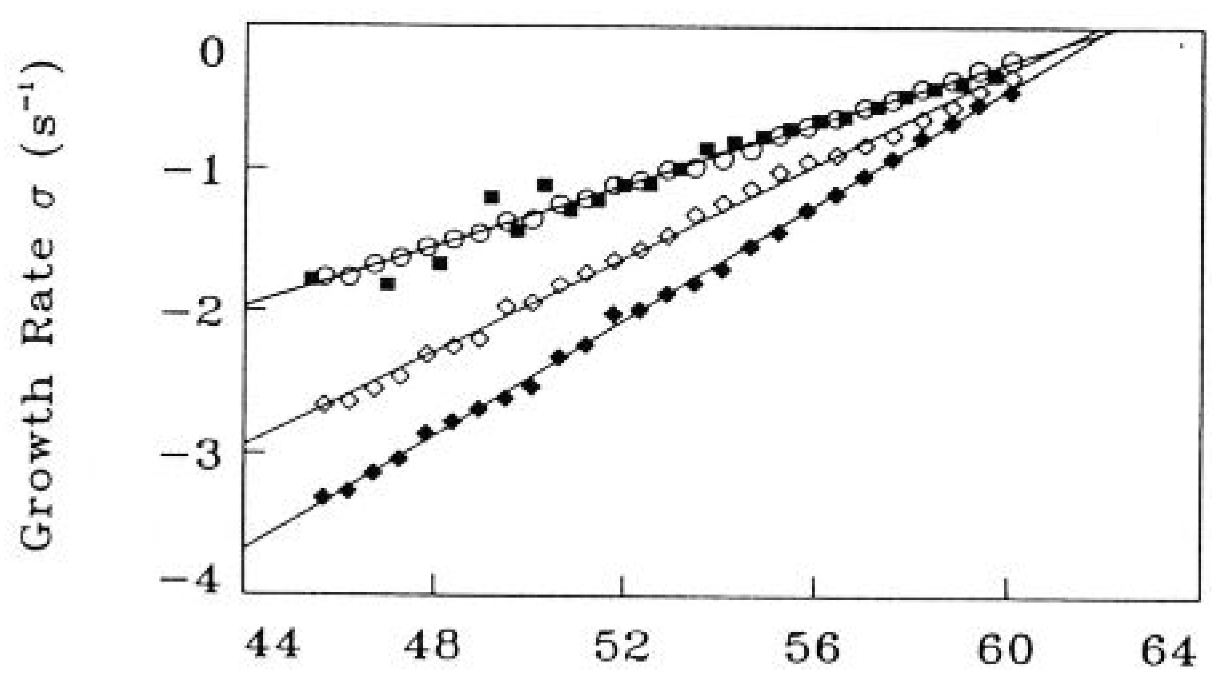

Fig. 1. The growth rate as measured by various methods at different subcritical values of the driving voltage. The solid squares are obtained by analysis of the correlation function. The open circles correspond to the decay rate of the fundamental after a quench of the order parameter. The second harmonic (closed diamonds) decays twice as fast. The effective decay rate of the rms (open diamonds) is between these two values.

The other data in fig. 1 have been obtained by jumps from a small supercritical value of $\epsilon = 0.01$ to the different subcritical values of the driving voltage indicated by the x-axis. From the different alternatives to extract numbers from the temporal decay of the contrast measured along a line perpendicular to the roll pattern, we recommend most to measure the decay rate of the fundamental (open circles). The second harmonic, being due to a quadratic effect in the shadowgraph method, must decay twice as fast, and the measurements (closed diamonds) are indeed in agreement with this prediction. The amplitude of the second harmonic is a very sensitive function of the focal heights of the microscope, which is often hard to determine exactly, thus its use cannot be recommended. The rms is a mixture of the different modes, and thus its effective decay rate (open diamonds) will be something in between the values for the fundamental and the higher harmonics, as indicated by the measurement. The rms-method will only give correct results if the amplitude of all other harmonics can be neglected, but because the relative strength of these modes is a sensitive function of the focal heights and the value of the order parameter itself, this cannot always be guaranteed. For the data presented here we have focussed the microscope to a height of 5d above the liquid crystal. When focusing closer to the middle of the cell, the second harmonic is less pronounced and the analysis of the rms gives results being closer to the upper line in fig. 1. Moving the focal plane further away from the cell tends to indicate a faster decay rate. In conclusion, using the rms for estimating growth rates cannot be recommended.

The main purpose of this paper is to demonstrate that the determinstic method (in connection with analyzing the fundamental mode) and the noise method do give the same results. At small values of the control parameter, the data obtained via inspection of the correlation function show a somewhat larger scattering, which is

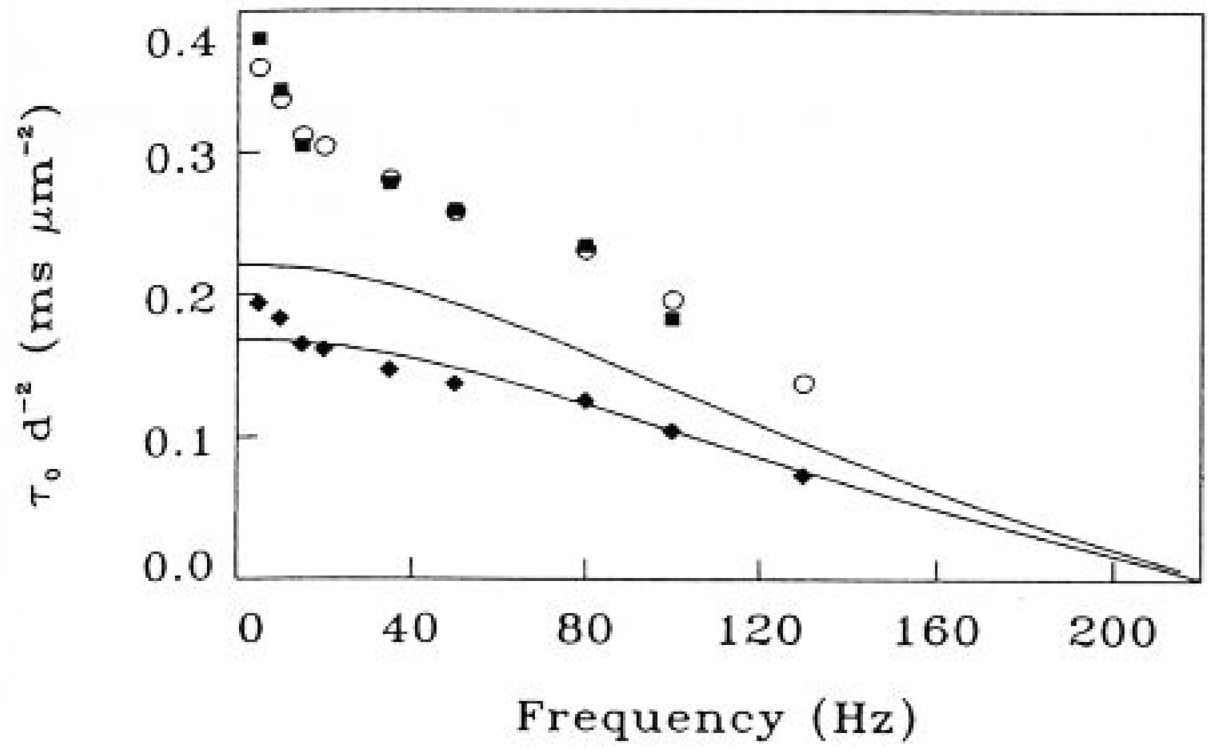

Fig. 2. The slope τ_0 of the growth rates as a function of the driving frequency. The solid curves are theoretical estimations corresponding to the different data sets MBBA I (above) and MBBA II (below). The solid squares are obtained using the noise method, and agree with the open circles obtained via a deterministic decay of rolls. The decay of the second harmonic (solid diamonds) is about twice as fast.

due to the fact that the fluctuation intensity is very small here.

The slope of the lines $\sigma(V^2)$ serves to define the characteristic time τ_0, which can be compared with the predictions from a linear stability analysis. In fig. 2 such a comparison is presented. The lines have been calculated using the theory summarized in ref. [2] (but by means of an independent computer code) and are in agreement with the results presented there. In order to give a hint about the range of τ_0 to be expected due to the experimental uncertainties of the material parameters, the curves for the data set MBBA I and MBBA II as tabulated in ref. [2] are both presented. In both cases the electrical conductivity has been adjusted in order to match our measured cutoff frequency of 220 Hz. The data points obtained with the noise method (solid squares) and via analysis of the fundamental mode using the deterministic method (open circles) are presented as well. Both data sets agree with each other within the experimental uncertainty. The increase of τ_0 at frequencies below 10 Hz is unexplained. It is in agreement with the general experimental experience, that the convection pattern behaves different at very small frequencies, and possible explanations might be DC-effects like charge injection.

Both data sets do not agree with the theory very well. A disagreement of a similar magnitude was found in refs. [4,5]. We conclude that the theory is not applicable for our cell. It is known that the theory does contain simplifications which might fail to be valid for thin cells. Measurements in thicker cells might thus clarify the situation. Alternatively, our boundary conditions might not be well modeled by the ideal strong anchoring without pretilt that is assumed in the theoretical analysis. It has been shown that spatially modulated boundary conditions might even change the qualitative character of the bifurcation [10], thus it seems very reasonable to assume a quantitative change due to such an effect as well.

In ref. [8] a good agreement of the measured growth rate with the line for the data set MBBA II of ref. [2] data was found by using the rms of the light intensity. That method tends to indicate a faster decay rate and thus a smaller value of τ_0. In order to illustrate this point we have included the growth rates obtained by analyzing the second harmonic in fig. 2 as well (solid diamonds). These data seem to agree with the theory, but this is in our case just an artifact caused by the higher harmonics of the light intensity signal as explained above. At this point we can only conclude that additional measurements in other cells are necessary in order to propose a systematic disagreement between theory and experiment. We would like to stress, however, that there seems to be very little evidence for proposing an agreement at the present time, and that the noise method seems to provide a particularly useful tool for further investigations.

4. Summary and discussion

In summary, we have shown that the measurement of τ_0 via intrinsic noise is a convenient alternative to deterministic methods. One disadvantage of the method could be the smallness of the noise – the fluctuations might not be readily detectable in thicker samples. If the fluctuations are detectable, however, the method offers a number of advantages. i) The smallness of the fluctuations guarantees the linear approximation of the shadowgraph method to be valid. ii) Even modes which never become unstable, i.e. whose growth rate remains negative, can be measured. iii) The method even works in the neighborhood of a subcritical bifurcation.

The noise method should be suitable to measure all the linear coefficients of the amplitude equation for electroconvection. In addition to the measurement of the linear decay rate as presented here, it is possible to determine the linear correlation lengths $\xi_{\|}$ and $\xi_{\perp}$ [11]. We expect such a measurement to be especially enlightening in the neighborhood of the so called Lifschitz-point [1], where a comparison with the prediction from a linear stability analysis was unsuccessful so far, presumably because of the fact that the subcritical bifurcation observed here made it impossible to work at small enough amplitudes for making contact with a linear stability analysis [3]. An experimental attempt to resolve this puzzle by analyzing subcritical fluctuations in the neighborhood of this point is in progress.

If the thermal noise is simply used as a measurement tool in the way presented in this paper, it is essential to stay within the linear description of the fluctuations, i.e. far enough away from threshold. In order to answer the more fundamental questions about the nonlinear interactions of fluctuations in nonequilibrium systems, however, it seems extremely interesting to perform these measurements closer to the critical point.

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References

[1] For a recent review see W. Zimmermann, MRS Bulletin 16 (1991) 46.

[2] E. Bodenschatz, W. Zimmermann and L. Kramer, J. Phys. France 49 (1988) 1875.

[3] M. de la Torre Juárez and I. Rehberg, Phys. Rev. A 42 (1990) 2096.

[4] I. Rehberg, S. Rasenat, M. de la Torre Juárez, W. Schöpf, F. Hörner, G. Ahlers and H.R. Brand, Phys. Rev. Lett. 67 (1991) 596.

[5] I. Rehberg, F. Hörner, L. Chiran, H. Richter and B.L. Winkler, Phys. Rev. A 44 (1991) R7885.

[6] P.G. de Gennes, The Physics of Liquid Crystals (Clarendon, Oxford, 1973).

[7] I. Rehberg, B.L. Winkler, M. de la Torre Juárez, S. Rasenat and W. Schöpf, in: Festkörperprobleme/ Advances in Solid State Physics, Vol. 29, ed. by U. Rössler (Vieweg, Braunschweig, 1989) p. 35.

[8] S. Rasenat, V. Steinberg and I. Rehberg, Phys. Rev. A 42 (1990) 5998.

[9] I. Rehberg, F. Hörner and G. Hartung, J. Stat. Phys. 64 (1991) 1017.

[10] W. Zimmermann, talk presented at this Workshop.

[11] F. Hörner and I. Rehberg, in: Proc. Workshop Physics of Pattern Formation (Kitakyushu, September 1991).