# Secondary Instabilities developed in Upwellings of high Rayleigh number Convection : Implications for Hotspots

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## 1 Abstract.

Thermal plumes are ubiquitous at high Rayleigh number convection. The idea of secondary instabilities developed from plumes has come from recent high Rayleigh number simulations by Vincent and Yuen (2000). There is also evidence from seismic tomography (Bijwaard and Spakman, 1999, Goes et al., 1999) to suggest that mantle plumes are able to branch to some extent. Skilbeck and Whitehead (1978) and Whitehead (1982) have performed laboratory experiments showing the formation of secondary instabilities from a tilted plume in a set-up in which the conduits were made by injecting oil into a more viscous heavier liquid. The purpose of this work is to show how these secondary instabilities can develop as a consequence of the interaction of the shear flow developed by the large-scale circulation and the rising plume. This will be conducted within the framework of a constant viscosity fluid because of the need to understand the fundamental physics and the parameter space in Rayleigh number before pursuing other types of rheology. First, we will employ a two-dimensional axisymmetric spherical-shell model of Moser et al. (1997) within the framework of a Boussinesq fluid. When scaled to the Earth's mantle, the computational domain of this model will have an aspect-ratio of around six. A large enough aspect-ratio is needed to generate a potent enough large-scale circulation, which is needed to bend the plumes sufficiently for inducing the secondary instabilities found in the Whitehead experiments. Second, for comparing with 3-D situations, we have used a three-dimensional Boussinesq model taken from Dubuffet et al. (2000) with a large-enough aspect-ratio of $5 \times 5 \times 1$.

In the case of constant viscosity, the nature of these secondary instabilities branching off from primary plumes requires high-resolution, because what we are trying to detect represent secondary spatial bifurcations from an already developed primary hydrodynamic instability endowed with already a boundary layer character. Therefore, we would expect length-scales to be of the same magnitude or even smaller than the boundary-layer flows at the Rayleigh number being investigated. These instabilities develop at Rayleigh numbers for 2-D case at around $3 \times 10^7$ and in 3-D in excess of $10^8$. Hence, very high spatial resolution are needed for 2-D at least 150 second-order, equally spaced finite-difference points along the vertical direction and for the more difficult 3-D situation at least 200 points along the vertical.

Fig. 1 shows a cartesian rendering of the 2-D simulations for six different Rayleigh numbers ranging from $3 \times 10^6$ all the way to $Ra = 10^{10}$. A fourth-order finite-difference scheme with equally spaced points have been used with grid points of 750 points in the radial and 4000 points along the tangential directions used for $Ra = 10^{10}$. We can observe that at $Ra = 3 \times 10^7$ there are signs for plume branching, whereas for the lowest Ra of $3 \times 10^6$, the upwellings more or less upright and do not suffer any bending from the large-scale shear flow. Thus this secondary plume bifurcation takes place at a Ra a little bit higher than $10^7$ in 2-D. As Ra moves above $10^8$, the tendency to plume branching increases and is accompanied by multiple foldings of the upwellings (see $Ra = 10^9$). These represent secondary instabilities emanating from primary upwellings, reminiscent of the instabilities found by Whitehead in his laboratory experiments.

We emphasize that in contrast to Whitehead's experiments which were conducted in a kinematically constrained situation and with two different types of fluids, the instabilities obtained here come from a thermal convection simulation with a homogeneous fluid. At higher Ra, above $5 \times 10^9$, the folding instabilities disappear as the upwellings become extremely thin and finally a quasi-layered regime emerges with small plumes hovering close to the thermal boundary layers for $Ra = 10^{10}$. This situation with separate convective systems at the top and bottom of the layer is similar to the results obtained for thermal convection at finite Prandtl number (Vincent and Yuen, 2000).

We have also investigated the three-dimensional situations in which plumes can be bent significantly and can develop secondary instabilities. For lower Rayleigh numbers we have gone up to $5 \times 10^7$ in the aspect-ratio $5 \times 5 \times 1$ box and did not find any noticeable signs of plume bending. Then at $Ra = 10^8$ there appears a bifurcation in the flow pattern and plumes are found to be bent severely by the large scale circulation produced at this high Ra and have gone to $Ra = 5 \times 10^8$ for further verification. We have used up to $1025 \times 1025 \times 257$ points in this simulation. Figure 2 shows a 3-D isosurface rendering of the $T = 0.55$ surface. We see clearly the distinct presence of a long snake-like structure which is caused by severe bending of a plume. It has clearly a complicated curved 3-D structure. A two-dimensional cross-section of the thermal fields (here the red denotes hot temperature between 0.6 and 1.0) is shown in Fig. 3. The plumes outlined in the 2-D cross section look strikingly similar to the 2-D upwellings observed in Fig. 1 for $Ra = 3 \times 10^7$. If this trend can be extrapolated upwards in Ra, then we may expect some sort of layered convection to take place in 3-D configuration for Ra between $3 \times 10^{10}$ and $10^{11}$. However, such high Ra calculations are slightly beyond the reach of massively parallel computers today, as they will require at least $5000 \times 5000 \times 1000$ grid points.

We have demonstrated here within the framework of a constant viscosity model that the secondary Whitehead instabilities can develop in a self-consistent manner in both 2-D and 3-D large aspect-ratio convection with bifurcation Rayleigh numbers of $O(10^7)$ and $O(10^8)$ respectively. These results would suggest the possibility of these secondary instabilities to arise under lower Rayleigh number conditions, since it has been shown by Malevsky and Yuen (1993) that non-linear (non-Newtonian) rheology can lower the threshold Ra for secondary instabilities to develop by at least an order in magnitude or a factor of two lower in the surface Nusselt number. Since plate dynamics involves a highly nonlinear plastic rheology, we surmise that such secondary plumelike instabilities can be developed in the upper mantle as a consequence of the interaction between the shear flow generated by plate tectonics and plumes generated from the 660 km boundary (Turcotte and Allegre, 1985, Cserepes and Yuen, 2000). These secondary instabilities can explain many hotspots without a definite age-progression trend as in the western Pacific (Mc Nutt et al., 1997).

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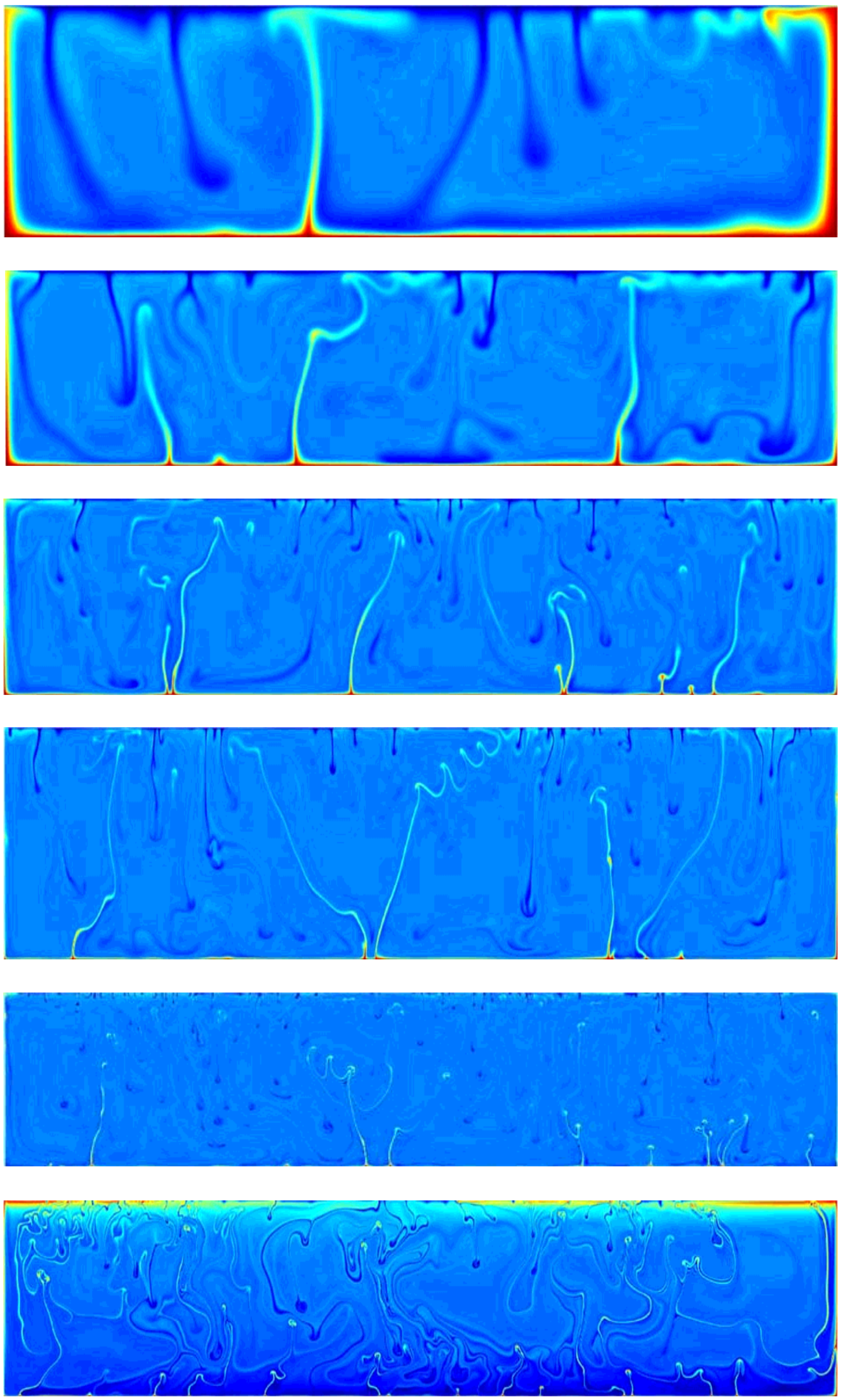


**Fig. 1.** Temperature fields of 2-D numerical simulations with purely basal heating.

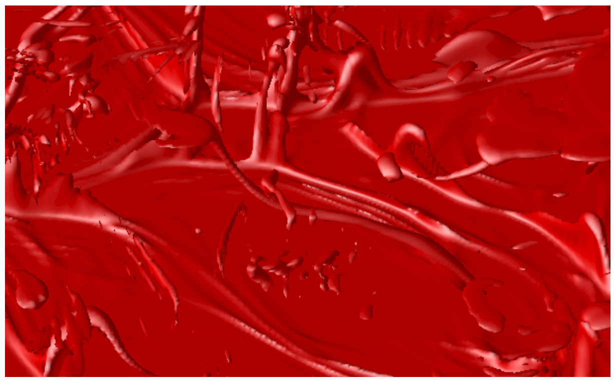

**Fig. 2.** Temperature isosurface of T =0.55 in 3-D convection with $Ra = 10^8$ in an aspect-ratio $5 \times 5 \times 1$ box, with unity being the depth.

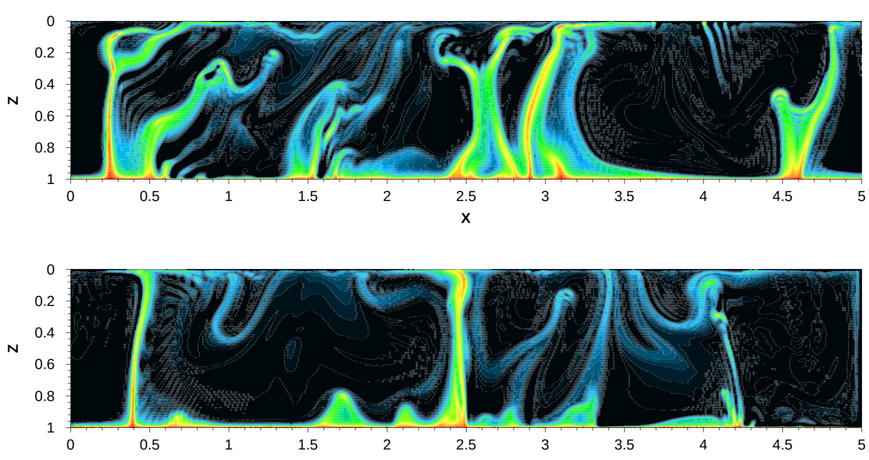


**Fig. 3.** Two-dimensional cross section of 3-D temperature field of $Ra = 10^8$ in large aspect-ratio box. Red color shows all T between 0.6 and 1.0.