Geophysical Application of Filtering and Visualizing Using Multidimensional Wavelets

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Wavelets are linear mathematical transformations (e.g. Resnikoff and Wells, 1998) which can analyze both temporal signals and spatial images at different scales. The wavelet transform is sometimes called a mathematical microscope. Large wavelets give an approximate image of the signal, while smaller and smaller wavelets zoom in on small details. Until recently most of the applications of wavelets in geophysics have been focused on the use of one-dimensional wavelets to analyse time-series of the Chandler wobble or one-dimensional spatial tracks, such as topography and gravity anomalies. Recently fast multi-dimensional wavelet transforms(Bergeron et al., 1999, 2000 a and b, Yuen et al., 2000) , based on higher-order-derivatives of the Gaussian function , have been developed and they have allowed us to construct rapidly two- and three- dimensional wavelet-transforms of geophysically relevant fields , such as geoid anomalies, temperature-fields in high Rayleigh number convection , and mixing of passive heterogeneities. We have also computed with wavelet transform the correlation of high-resolution seismic tomography between two depth levels and have constructed a two-dimensional map showing the correlation of seismic tomography for the transition zone under Europe. With wavelet transform one can view the character of a field at a particular length-scale. This continuous transform is different from orthogocal wavelet functions, which locks on to a particular hierarchy of length-scales. like a lattice. We have constructed scalograms of the two-dimensional wavelet-transformed quantities of the geoid anomalies, temperature fields in thermal convection and the scalar field depicting the hetereogeneities in mixing driven by thermal convection. With these scalograms one can discern the detail changes for the different length scales ranging from long wavelength to short wavelength , ranging from about one-tenth to one-hundredth of the entire domain . We have also constructed two proxy field variables, which are associated with the maximum of the wavelet transformed quantity, E-max, and the associated local wavenumber of the wavelet, k-max. Both E-max and k-max are functions of the spatial variables and their peak functional values indicate places with sharp variations in the field values, be it the geoid, seismic velocity anomalies or the thermal fields in convection. Both mixing dynamics and thermal convection in high Rayleigh numbers display different behavior at different length-scales, as revealed by the wavelet-transformation. Long-wavelength anomalies are mixed much faster than intermediate wavelength anomalies. The thickness of the thermal boundary layer depends also on the length-scale of the wavelet interrogation. In the geoid the ridges and subduction zones start to be discernible at around 400 km wavelength scale wavelet . Their appearance has been verified with smaller-scale wavelets down to a wavelength of around 120 km. Many spherical harmonics are required to mimick the feature capturing capability of a single wavelet at a particular scale in the geoid problem for order of the spherical harmonic going out to 256. We have compared geoid results from using a single wavelet and from a nonlinear band-pass filter in the spectral space. We have also developed analytical spherical wavelets , which involves a Bessel function and a Gaussian function, with the arc length being the argument. These spherical wavelets have been applied to decomposition of the geoid anomalies.

The two-dimensional correlation map of the seismic tomography shows that between a depth of 500 and 600 km under Europe there is an ellipse- shaped object with an area of 2000x 4000 km having a strong correlation for length scales of around 400 km. From the wavelet correlation spectra we can extract an horizontal length scale of around 100 km, which may be related to the interaction of the subdcuted material with the ambient mantle. The correlation results suggest that the thickness of the recumbent fast (cold) material is between 100 and 150 km. We hope to show here the possibilities of using multidimensional wavelets in looking at the multiscale features of several interesting geophysical problems.

References

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