Mathematical Modeling for Seismology Problems

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We obtain the exact solution for free surface displacements generated from a point source in a layered medium. The far-field results are given in cylindrical coordinates for moment tensor point sources:

$$\begin{pmatrix} u_r^{(0)} \\ u_z^{(0)} \end{pmatrix} = \sum_{i=1}^{3} \int_0^{\infty} k^2 \mathbf{j}_i L^{-1} \left[m_i \mathbf{G}_i \right] dk$$

$$u_\varphi^{(0)} = \sum_{i=5}^{6} \int_0^{\infty} k^2 j_i L^{-1} \left[m_i G_i \right] dk$$

The function of a source $\bar{u}(t)$ is modeling by Brune's model and we consider the r-th component of the displacement $u_r^{(0)}(r, \varphi, t)$ that contains $M_{zz}(t)$ (the zz-component of the seismic moment tensor M):

$$\begin{aligned} &\tilde{u}_r^{(0)}(r,\, \varphi,\, t) = \\ &\quad \frac{\sin 2\delta \sin\lambda}{2\pi\mu} \int_0^{\infty} \frac{k^2 J_1 dk}{2\pi j} \\ &\quad \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{2\alpha\beta e^{-kh\alpha} - g e^{-kh\beta}}{4\alpha\beta - g^2} M_0(k\eta) e^{k\eta t} d\eta, \\ &\quad M_{zz} = M_0 \sin 2\delta \sin\lambda \end{aligned}$$

The last equation represents the complete displacement field on the free surface for the moment tensor point source $M_{zz}(t)$, including body and surface waves. Inner integral is the sum of residues in poles of the integrands (surface waves) and profile integrals (body waves). Surface waves in the sense of normal mode are related to poles of the integrands which follow from the dispersion equations. Thus, it is possible to calculate body or surface waves. But we are interested to calculate only for poles $\eta = 0$ and $\eta = -\frac{1}{k\tau}$ of the function $M_0(k\eta)$. We obtain for the r-th component of the displacement that contains $M_{zz}(t)$:

$$\begin{aligned} &\tilde{u}_r^{(0)}(r,\, \varphi,\, t) = \\ &\quad \frac{\sin 2\delta \sin\lambda A U_0}{2\pi} \int_0^{\infty} k^2 J_1(kr) \Big[Res[V(k,\eta)] \Big|_{\eta=0} \\ &\quad + Res[V(k,\eta)] \Big|_{\eta = -\frac{1}{k\tau}} \Big] dk + u^b + u^s = \\ &\quad u^b + u^s + A + B e^{-t/\tau}, \end{aligned}$$

where u^b, u^s - displacements of body and surface waves on the free surface of the r-th component (body- and surface- wave seismograms of the displacement $\tilde{u}_r^{(0)}\,(r,\,\varphi,\,t)$).

It is shown that the function $u_r^{(0)}(r, \varphi, t)$ is directly depended upon the function of a source $u(t)$ and is obtained some function of time t $(A + Be^{-t/\tau})$ prior to P-wave arrival on the theoretical seismograms. This effect is shown on the seismogram that was recorded at the seismic station Chernivtsi (Ukraine) by 27.09.2004 for the earthquake in the Carpathian (the Vrancha region, at 09:16:22.47, h=160.3km, N45.749, E26.542, K=13.4, K-energetic class: lgM_0=0.6 K+15.5 for the Transcarpathian region).

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