Finite Prandtl Convection

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Purpose of Study

- Many fluids with large, but finite *Pr* numbers ($Pr \sim 10^4$) occur in planetary systems (mushy ice, Mg-magmas).
- These fluids are usually approximated as infinite *Pr*, same as the mantle ($Pr \sim 10^{25}$).
- $Pr \sim 10^4$ fluids still have significant inertial behaviors missed by the infinite *Pr* number approximation.

Equations of Finite Prandtl Convection

Nondimensionalized by the free-fall velocity, $U \equiv \sqrt{\alpha(\Delta T) g L_z}$

$$\nabla \bullet \mathbf{v} = 0$$

$$\frac{1}{Pr}\frac{D\mathbf{v}}{Dt} = -\nabla p + RaT\mathbf{e}_z + \nabla^2 \mathbf{v}$$

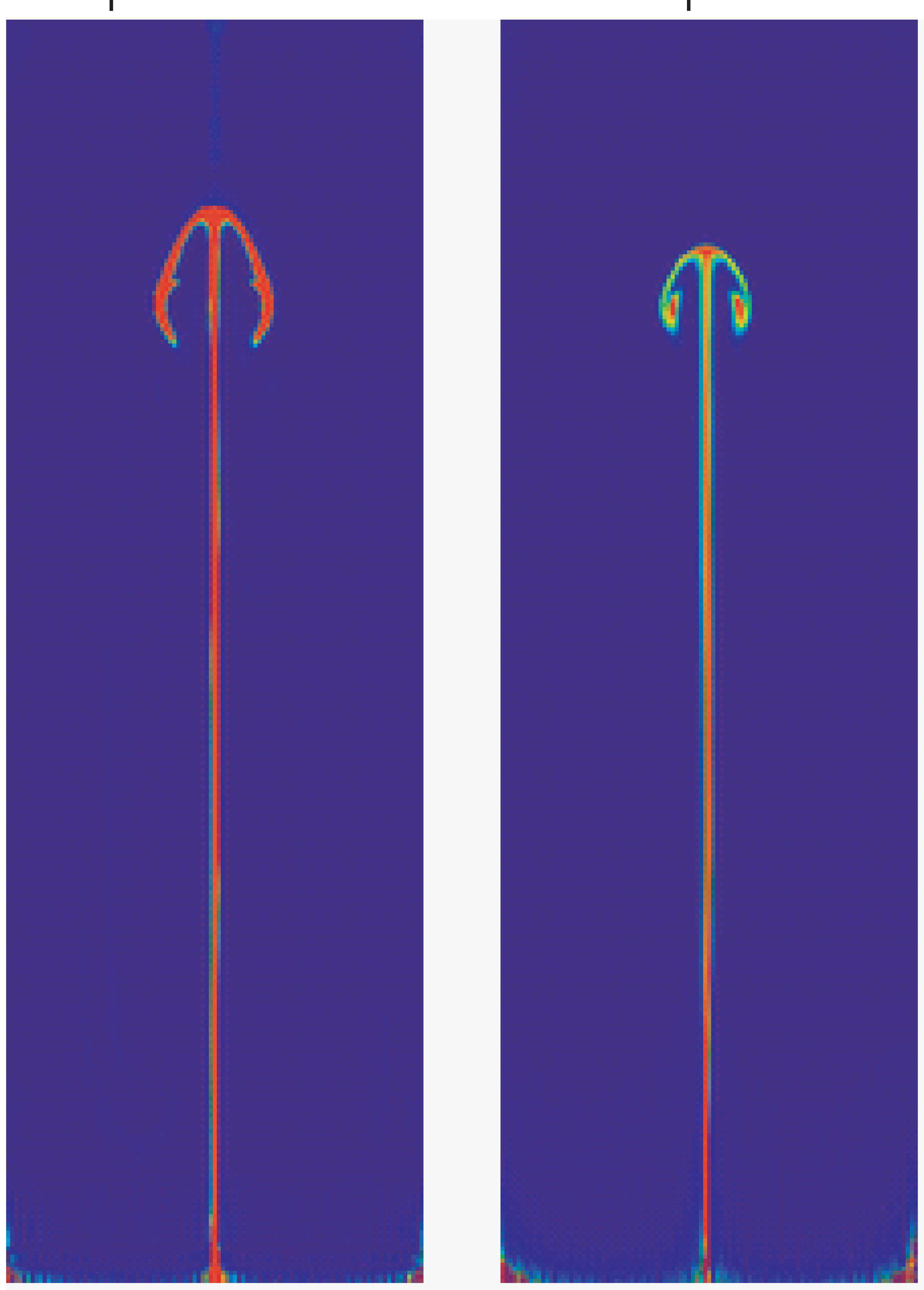
$$\frac{DT}{Dt} = \nabla^2 T$$

Rayleigh Number $Ra = \dfrac{g\alpha\Delta T d^3}{\nu\kappa}$

Prandtl Number $Pr = \dfrac{\nu}{\kappa}$

α thermal expansivity
ν kinematic viscosity
κ thermal diffusivity

Ra = 1 x 10^8 *Pr* = 2 x 10^4 *t* = t_1	*Ra* = 1 x 10^8 *Pr* = infinity *t* = $2t_1$



Temperature

Implications for Plume Heat Flux

- Prandtl number 104 plumes have significantly different behaviors from infinite Prandtl number plumes:
  □-- Grow Faster
  □-- Hotter
- Since convective heat flux = $v_Z T$, real heat fluxes are likely to be at least 2X larger than those calculated using the infinite *Pr* approximation.
- These differences tend to increase with *Ra*.
- This may have significant effects on heat fluxes in icy satellites, very hot bodies like Io, and magma ocean phases of the early terrestrial planets and moons.