Melting and Compaction in Deformable Two-phase Media

Ondrej Sramek[1], Yanick Ricard[2] and David Bercovici[1]

1 Yale University, USA   2 CNRS/Lyon1/ENSL, France

Abstract:

Melt generation and extraction are often modeled using the two-phase equations developed by McKenzie (1984) or Scott & Stevenson (1984). Usually various approximations are made to simplify the problem which may lead to some unphysical results. We have developped a generalized version of the set of equations introduced by Bercovici et al. (2001) that allows for mass transfer between the two phases and consider a self-consistent set of equations. In our description the two phases are submitted to different pressure fields whose difference is related to the surface tension at the interfaces, to the changes in porosity and to the melting rate. A kinetic relation for the melting rate arises from the second law of thermodynamics. The condition of chemical equilibrium corresponds to the usual univariant equality of the chemical potentials of each phase when the matrix and melt are motionless. In the most general form, the Gibbs-Thomson effect comes out naturally from thermodynamic equilibrium considerations.

We apply these new equations to a steady state problem of pressure release of a univariant system. We treat melting and compaction simultaneously and we observe several new effects and several possible regions near the onset of melting which correspond to various force balance situations. A consequence of matrix compaction (dilation) is a pressure difference between melt and solid which favors (inhibits) melting. Melting is favored when the extraction of melt from the matrix is efficient, i.e., when the Darcy velocity is larger then the initial upwelling velocity of the solid matrix. For parameters corresponding to pressure release melting under mid ocean ridges melting is favored and could start at most ~2 km below the standard solidus. Numerical results suggest that the movement of melt and matrix should be close to the Darcy equilibrium where the buoyancy of melt is equilibrated by the mechanical interaction between the phases. The Darcy equilibrium follows an initial stage where the matrix viscous stresses balance the Darcy friction.

- continuum averaging approach
- phases are incompressible viscous fluids
- interface between phases treated as additional thermodynamic phase
- difference of pressures in the phases related to surface tension and viscous deformation
- non-equilibrium kinetic relation for the melting rate
- equilibrium melting for infinitely fast kinetics

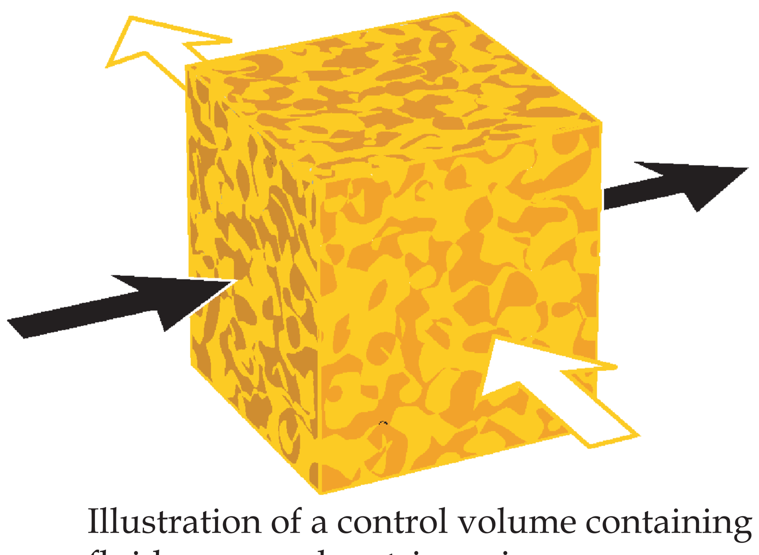


Illustration of a control volume containing fluid pores and matrix grains

1-D equilibrium melting of a univariant system

- inviscid melt
- no surface tension

- **force balance:**

$$\delta^2 \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1-\phi^2}{\phi}\frac{\mathrm{d}v_m}{\mathrm{d}z}\right) - \frac{\Delta v}{\phi} - V_B(1-\phi) = 0$$

viscous force — Darcy drag — buoyancy

- **mass conservation:**

$$\frac{\mathrm{d}}{\mathrm{d}z}[(1-\phi)v_m] = -\frac{\Delta\Gamma}{\rho_m}$$

- **energy equation:**

$$\overline{\rho v}\frac{\mathrm{d}T}{\mathrm{d}z} - T\Delta s\Delta\Gamma = c\Delta v^2 + \mu_m\frac{1-\phi^2}{\phi}\left(\frac{\mathrm{d}v_m}{\mathrm{d}z}\right)^2$$

- **melting rate:**

$$\Delta\Gamma = -\chi\left[(T-T_0)\Delta s + P_f\frac{\Delta\rho}{\rho_f\rho_m}\right]$$

$$\delta = \sqrt{\frac{4}{3}\frac{\mu_m}{c}} \quad \text{... compaction length}$$

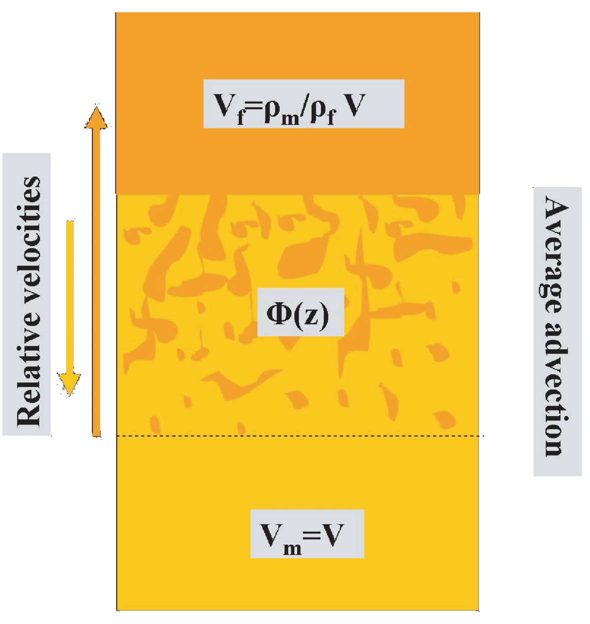
$$V_B = \frac{\Delta\rho g}{c} \quad \text{... buoyancy velocity}$$

$$\Delta v = v_m - v_f$$

$$\Delta\rho = \rho_m - \rho_f$$

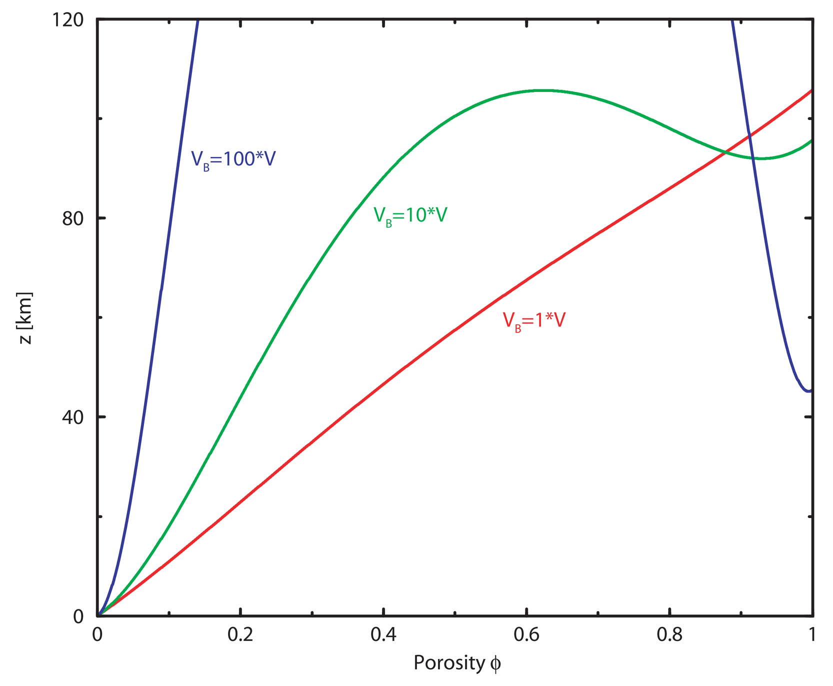
$$\overline{\rho v} = \rho_f\phi v_f + \rho_m(1-\phi)v_m$$

ϕ	porosity
$v_m,\ v_f$	matrix and fluid velocities
$\rho_m,\ \rho_f$	matrix and fluid densities
μ_m	viscosity of the matrix
c	Darcy interaction coefficient
g	gravitational acceleration
V	initial upwelling velocity
$\Delta\Gamma$	melting rate
T	temperature
Δs	entropy difference (matrix-melt)
P_f	fluid pressure
χ	kinetic rate

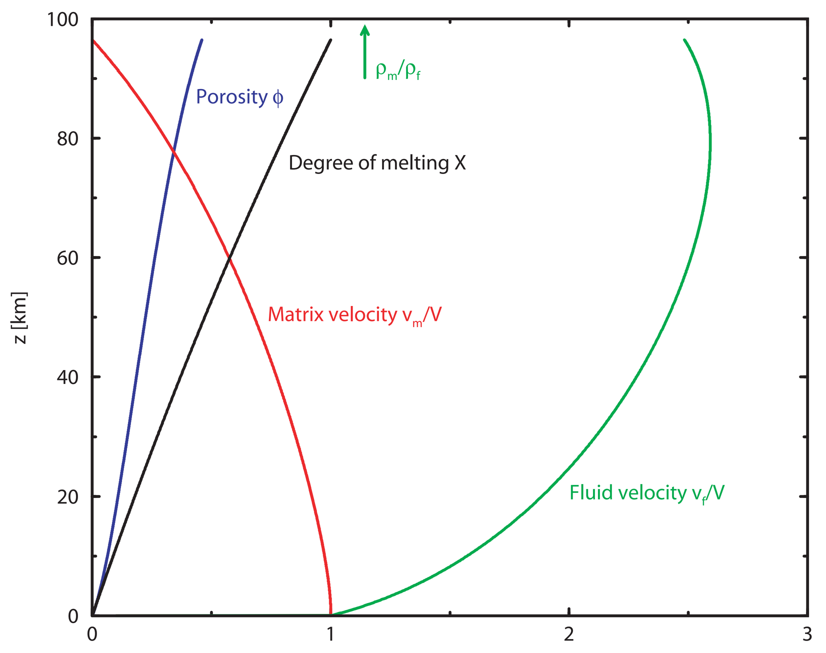


- equilibrium melting corresponds to infinite kinetic coefficient χ, then: $(T-T_0)\Delta s + P_f\frac{\Delta\rho}{\rho_f\rho_m} = 0$

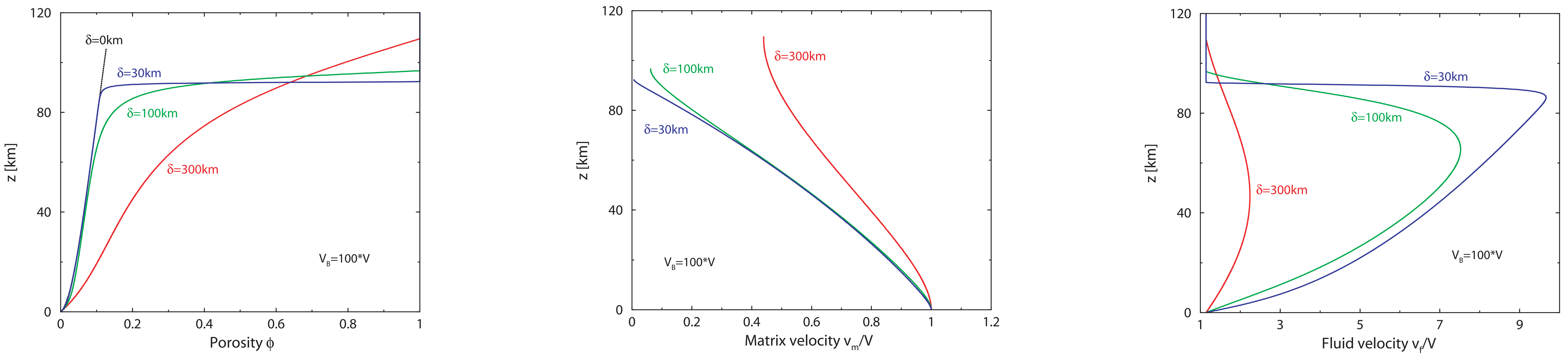
Case δ=0: Darcy equilibrium



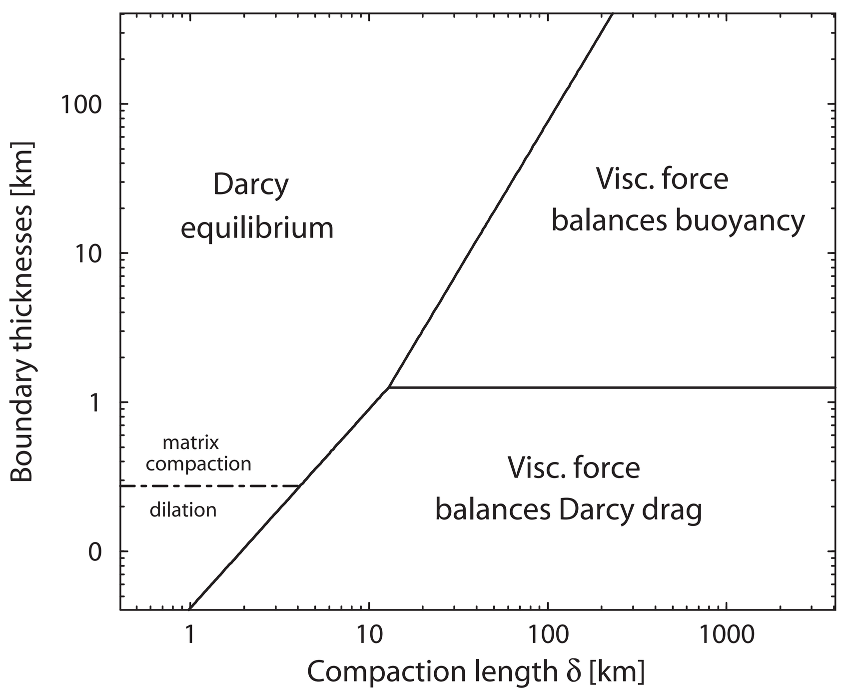
- in Darcy equilibrium buoyancy force is balanced by the Darcy drag
- mid ocean ridge:
  - compaction length small compared to the size of the melting zone, therefore Darcy equilibrium should be a good approximation
  - buoyancy velocity of the order of 100
- pure Darcy equilibrium cannot be a solution when:
  - viscous forces become important at small porosity
  - at the end of the melting zone Darcy equilibrium doesn't satisfy boundary conditions $\phi = 1$ and $v_f/V = \rho_m/\rho_f$ when $X = 1$



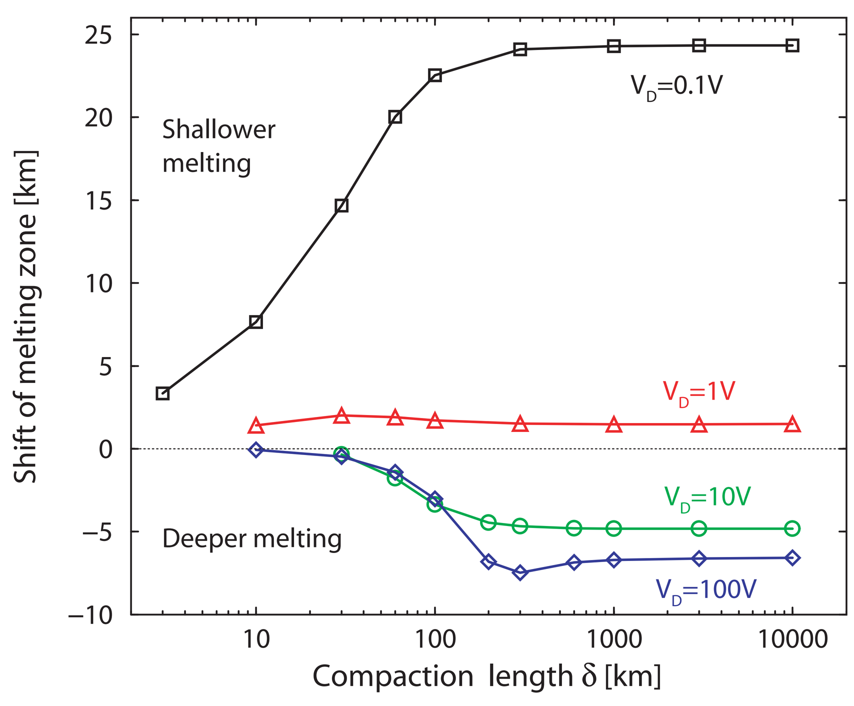
Complete solution



- at the onset of melting several regions are present that correspond to various force balances



- viscous deformation of the matrix at the onset of melting results in a pressure difference between melt and solid that favors (when matrix compacts) or inhibits (when matrix dilates) melting
- melting condition is satisfied at different depth compared to what is inferred from the average pressure



References:

Bercovici D., Y. Ricard and G. Schubert. A two-phase model for compaction and damage, Part 1: General theory. *J. Geophys. Res.*, **106**, 8887--8906, 2001.

McKenzie D. The generation and compaction of partially molten rock. *J. Petrol.*, **25**, 713--765, 1984.

Scott D.R. and D. J. Stevenson. Magma solitons. *Geophys. Res. Lett.*, **11**, 1161--1164, 1984.

Conclusions:

- Consistent set of equations where melting rate relation arises from thermodynamic considerations
- In the bulk of the melting zone Darcy equilibrium prevails, porosity remains small
- Several possible regions near the onset of melting with different force balance situations
- The dynamic pressure affects the onset of melting by several km