Westfälische
Wilhelms-Universität
Münster

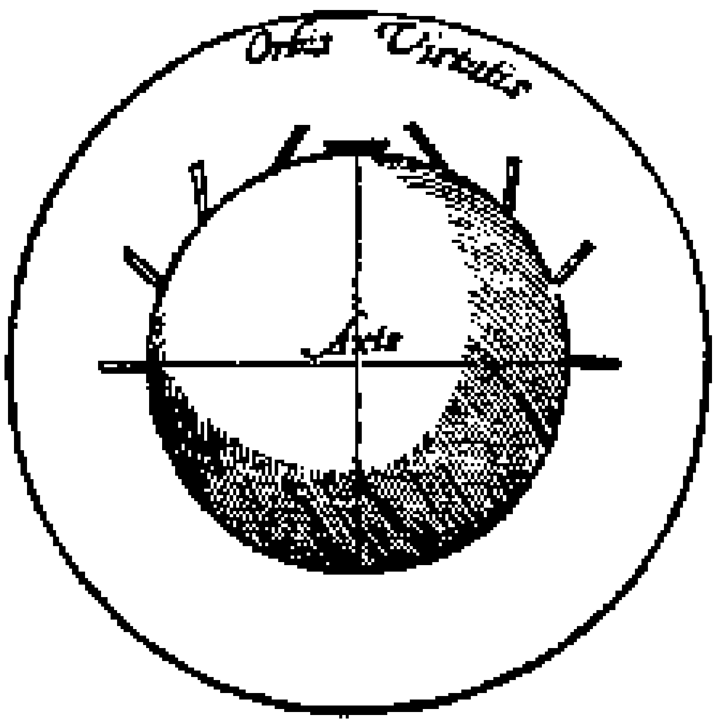
Precession driven dynamo

J. Ernst-Hullermann, H. Harder, U. Hansen

Institute of Geophysics, University of Münster, Germany

Westfälische Wilhelms-Universität Münster

The magnetic Earth

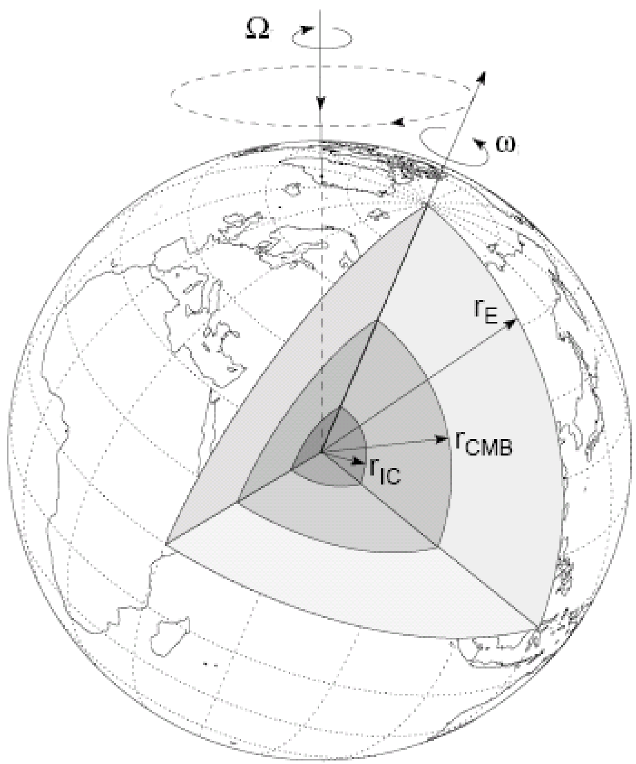


William Gilberts Terella,
“De Magnete”, 1600

- Permanent magnet in the Earth’s interior not possible
- Flows in the outer core generate the magnetic field
- The flows can be driven by precession

WESTFÄLISCHE
WILHELMS-UNIVERSITÄT
MÜNSTER

Inertial frame



adapted from: A. Tilgner (2007)

ω angular velocity

Ω precession velocity

α angel between precession and rotation axis

Boundary conditions:

$$\mathbf{u}_i(\mathbf{r}) = (\boldsymbol{\Omega} + \boldsymbol{\omega}(\boldsymbol{\Omega}, t)) \times \mathbf{r}$$ on rigid boundaries

Equation of motion:

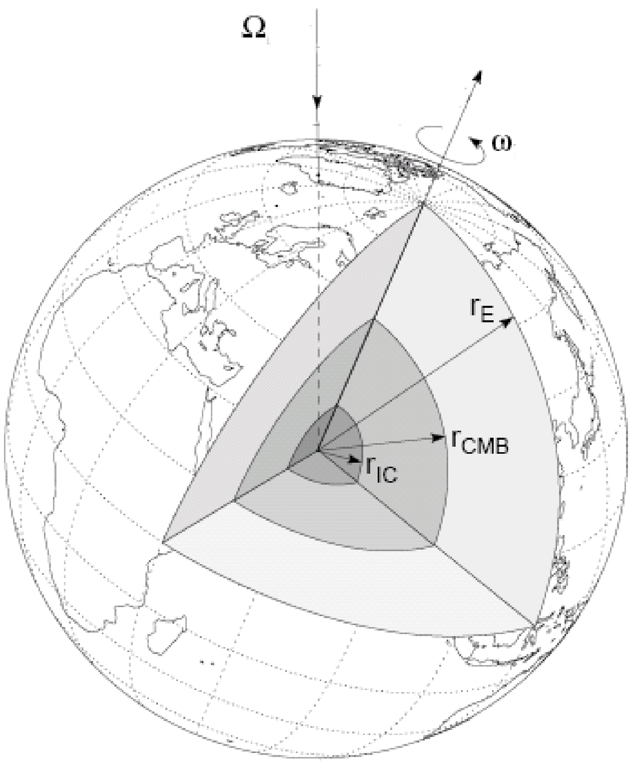
$$\frac{\partial}{\partial t}\mathbf{u}_i + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = -\nabla P + E\nabla^2 \mathbf{u}_i$$

$$\nabla \cdot \mathbf{u}_i = 0$$

Dimensionless parameter: Ekman-number

$$E = \frac{\nu}{d^2 \omega}$$

Precession frame



modified from: A. Tilgner (2007)

- Precession and rotation axis stationary
- Boundary conditions

$$\mathbf{u}_p = \boldsymbol{\omega} \times \mathbf{r} = \mathbf{e}_z \times \mathbf{r}$$ on rigid boundaries

- Coriolis force

$$\frac{\partial}{\partial t}\mathbf{u}_p + \mathbf{u}_p \cdot \nabla \mathbf{u}_p + 2\boldsymbol{\Omega} \times \mathbf{u}_p = -\nabla P' + E\nabla^2 \mathbf{u}_p$$

Mantle frame

- Poincare force

$$\frac{\partial}{\partial t}\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + 2(\boldsymbol{\Omega} + \mathbf{e}_z)\times\mathbf{u} + (\boldsymbol{\Omega}\times\mathbf{e}_z)\times\mathbf{r} = -\nabla\Pi + E\nabla^2\mathbf{u}$$

- Boundary stationary

$\mathbf{u} = 0$ on rigid boundaries

- Precession axis time dependent

$$\hat{\boldsymbol{\Omega}}(t) = \sin(\alpha)\cos(t)\mathbf{e}_x - \sin(\alpha)\sin(t)\mathbf{e}_y + \cos(\alpha)\mathbf{e}_z$$

Busse solution

- Approximate solution starting from solution for inviscid fluid
- Adds viscous boundary layers
- Flow is in 1. approximation rigid rotation $\mathbf{u} = \boldsymbol{\omega}_F \times \mathbf{r}$ with

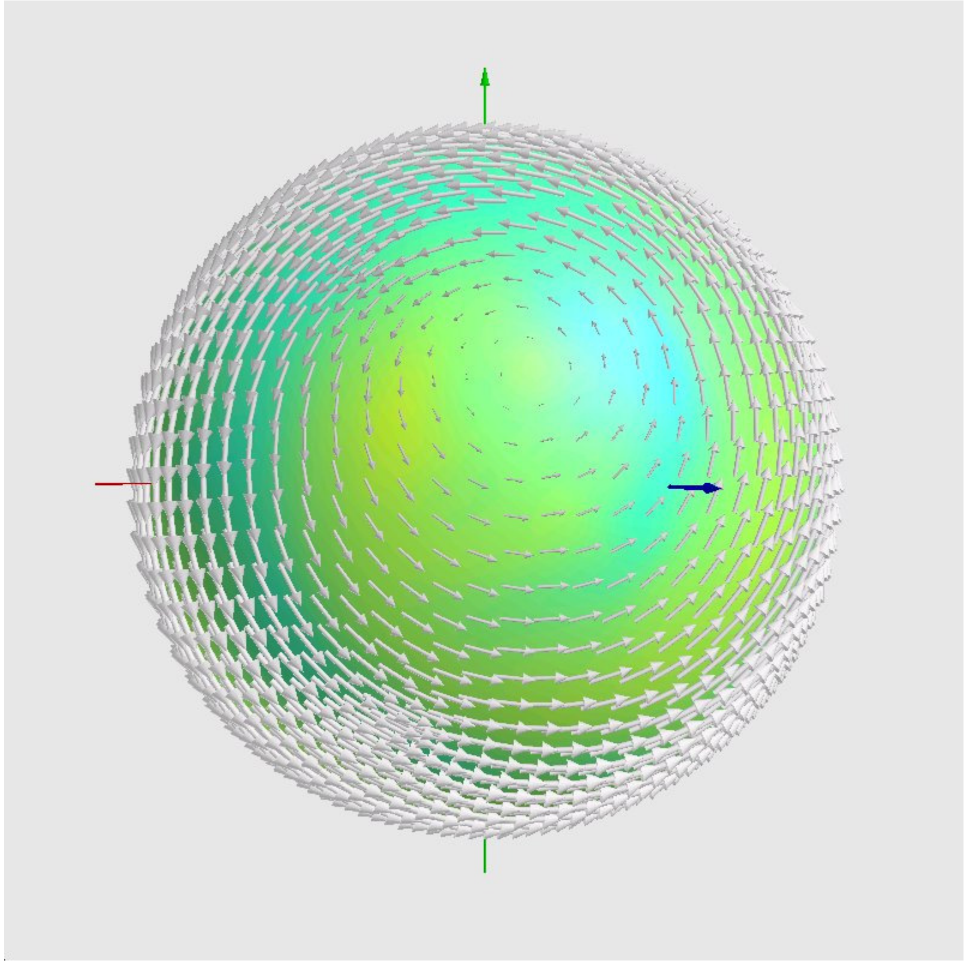
$$\frac{\boldsymbol{\omega}_F}{\omega_F^2} = \hat{\boldsymbol{z}} + \frac{A\hat{\boldsymbol{z}} \times (\boldsymbol{\Omega} \times \hat{\boldsymbol{z}}) + B(\hat{\boldsymbol{z}} \times \boldsymbol{\Omega})}{A^2 + B^2}$$

$$A = 0.259(E/\omega_F)^{1/2} + e\,\omega_F^2 + \boldsymbol{\Omega} \cdot \hat{\boldsymbol{z}}$$

$$B = 2.62(E\,\omega_F)^{1/2}$$

F. H. Busse, „Steady fluid flow in a precessing sheroidal shell“, Journal of fluid mechanics **33**, 739-751, (1968)

Results



Flow in precession frame

$$E = 10^{-4}$$
$$\Omega = -0,2$$
$$\alpha = 23,5°$$
$$\eta = 0,35$$

$$\theta = 22,4°(23,6°)$$
$$\varphi = -2,5°(-4,62°)$$
$$E_{kin} = 1,18(1,16)$$

No dynamo

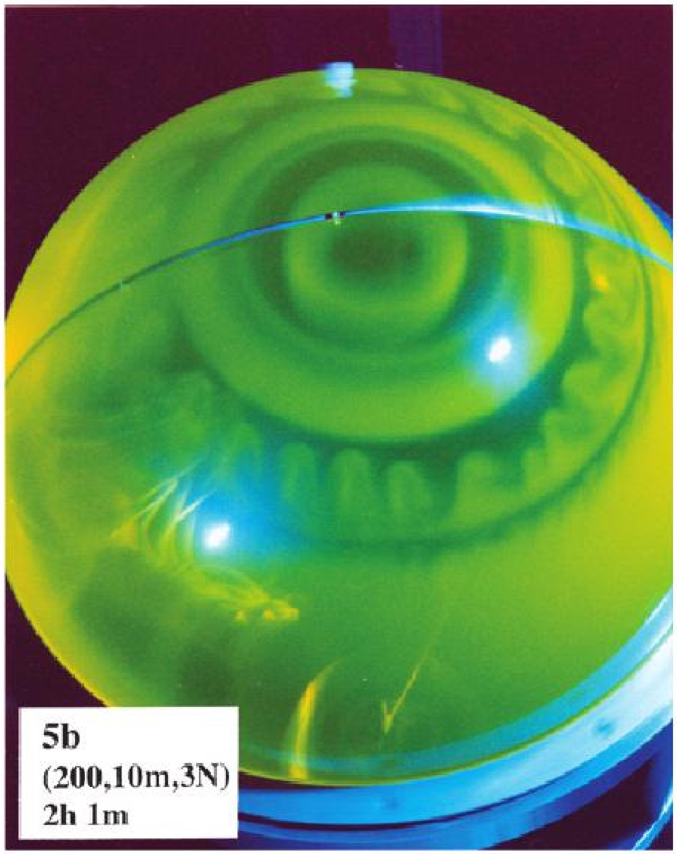
- **Anti-dynamo theorem:**
  purely toroidal flows can not sustain a dynamo-process

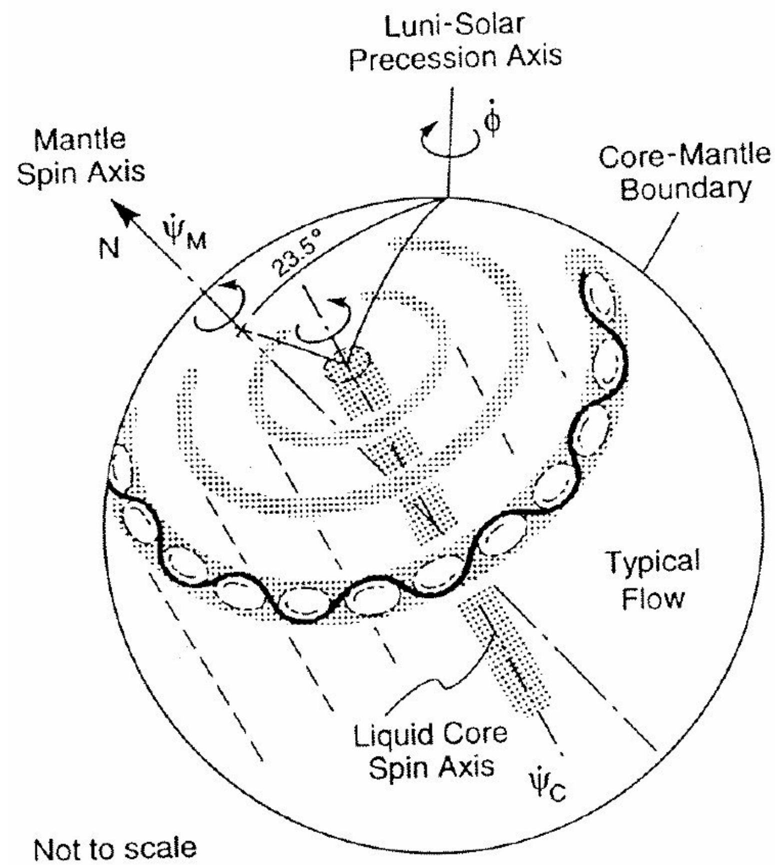
- Boundary layers do not contain enough energy for dynamo

- **But:**
  instability

Westfälische Wilhelms-Universität Münster

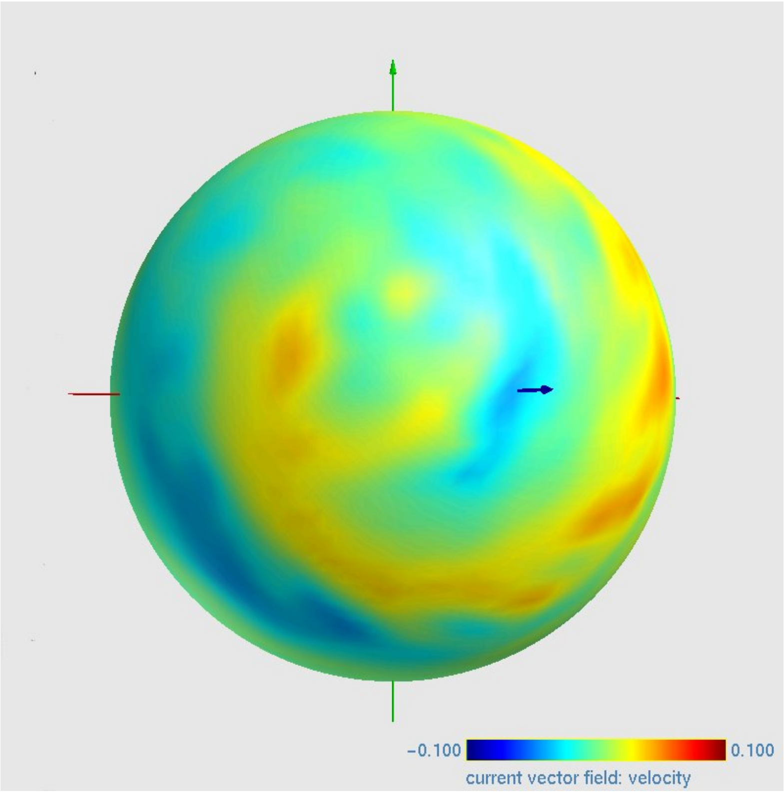
Experiments



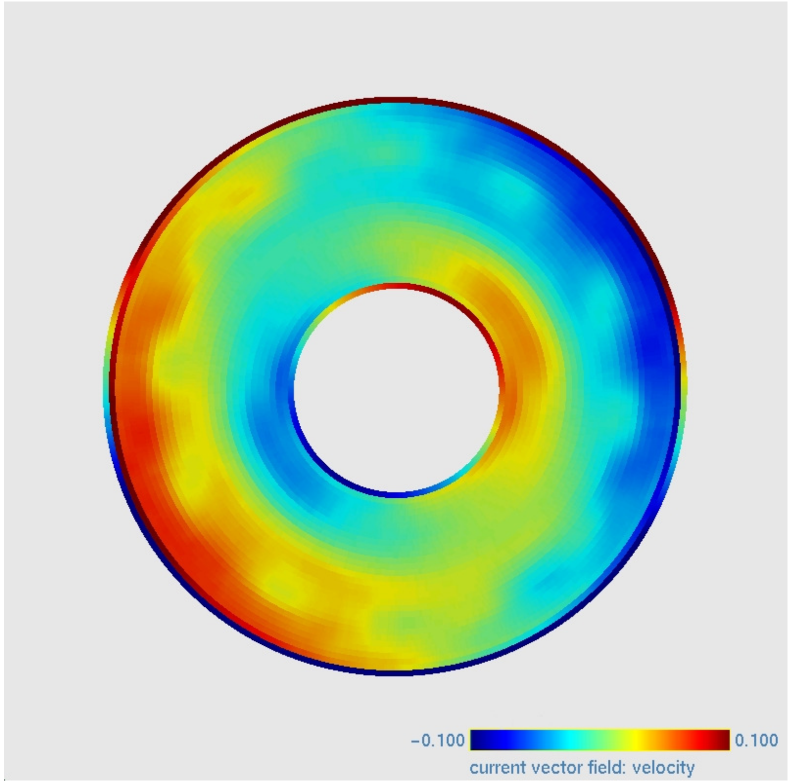


adapted from: J.P. Vanyo, „Core-mantle relative motion and coupling“, Geophys. J. Int. **158**, 470-478, (2004) Numerical simulation

u_r



u_{φ}



$E = 10^{-4}$

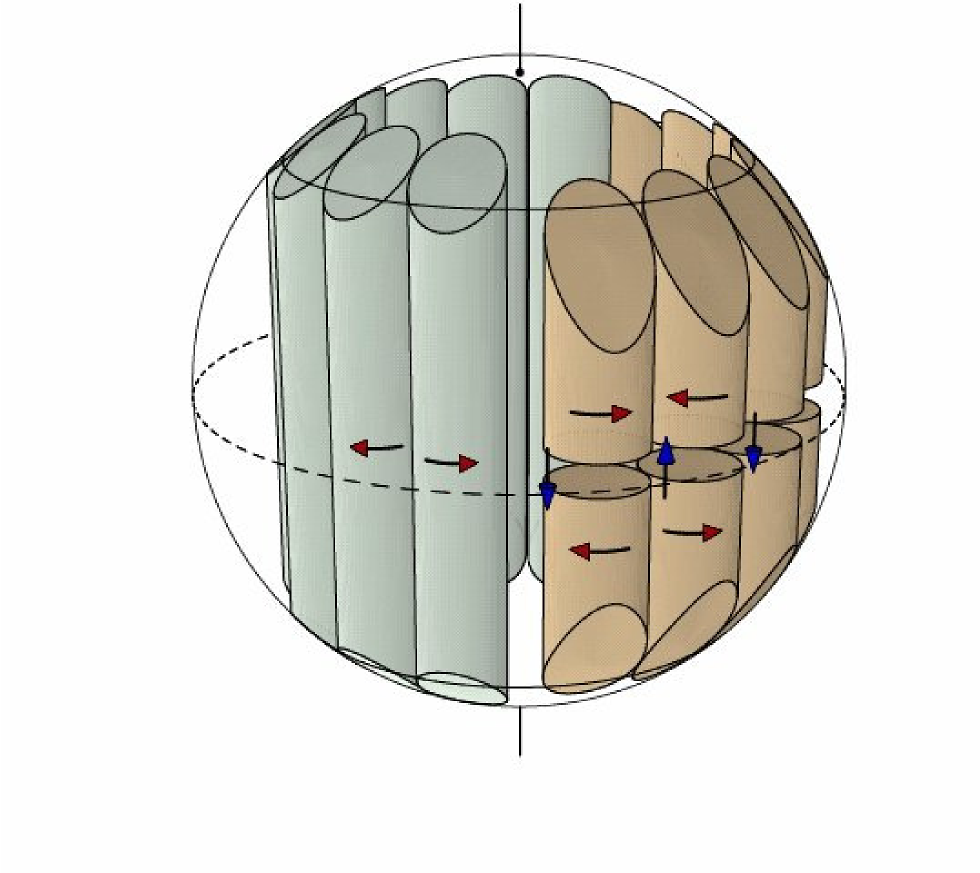
$\Omega = -0,2$

$\alpha = 23,5°$

$\eta = 0,35$

Westfälische
Wilhelms-Universität
Münster

Flow structure



adapted from: S. Lorenzani, „Fluid instabilities in precessing ellipsoidal shells“, Dissertation, (2001)

Dynamo theory

Equations of motion

Lorentz force

$$\frac{\partial}{\partial t}\mathbf{u}+\mathbf{u}\cdot\nabla\mathbf{u}+2(\mathbf{\Omega}+\mathbf{e}_z)\times\mathbf{u}=-\nabla\Pi+E\nabla^2\mathbf{u}-(\mathbf{\Omega}\times\mathbf{e}_z)\times\mathbf{r}+(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\nabla\cdot\mathbf{u}=0$$

Induction equation

$$\frac{\partial}{\partial t}\mathbf{B}+\nabla\times(\mathbf{B}\times\mathbf{u})=\frac{E}{Pm}\nabla^2\mathbf{B}$$

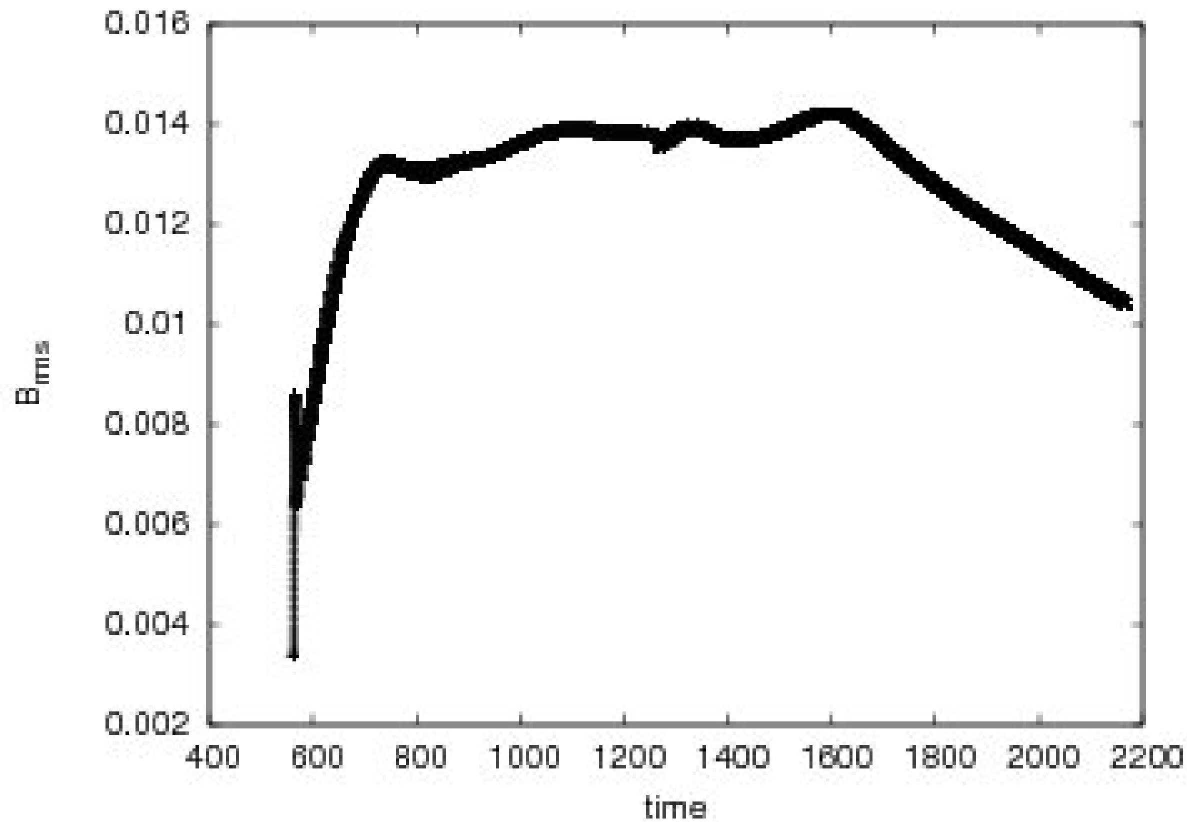
$$\nabla\cdot\mathbf{B}=0$$

Boundary condition $\mathbf{r}\times\mathbf{B}=0$

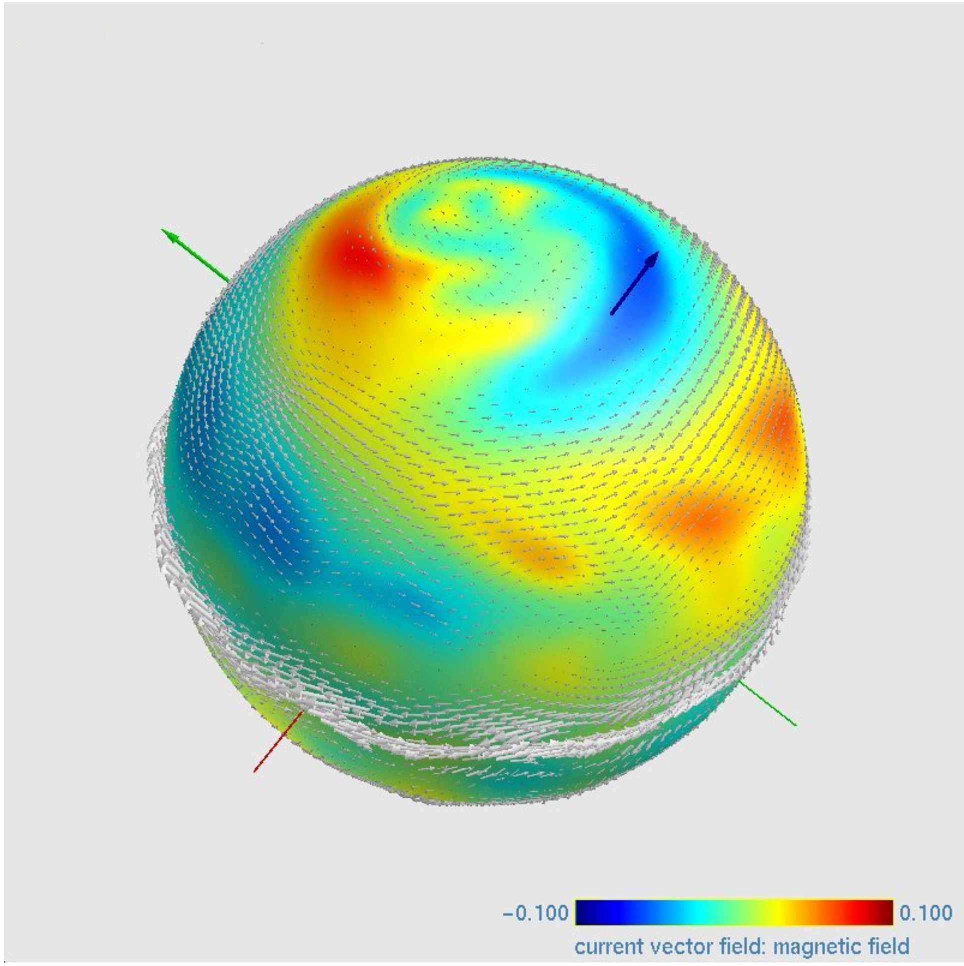
Magnetic Prandtl number $Pm=\mu_0\sigma\nu$

Results

$$E = 10^{-4}; \quad \Omega = -0,2; \quad \alpha = 23,5°; \quad \eta = 0,35; \quad Pm = 1,5$$



Magnetic field



Magnetic field produced by the flow

$$E = 10^{-4}$$

$$\Omega = -0,2$$

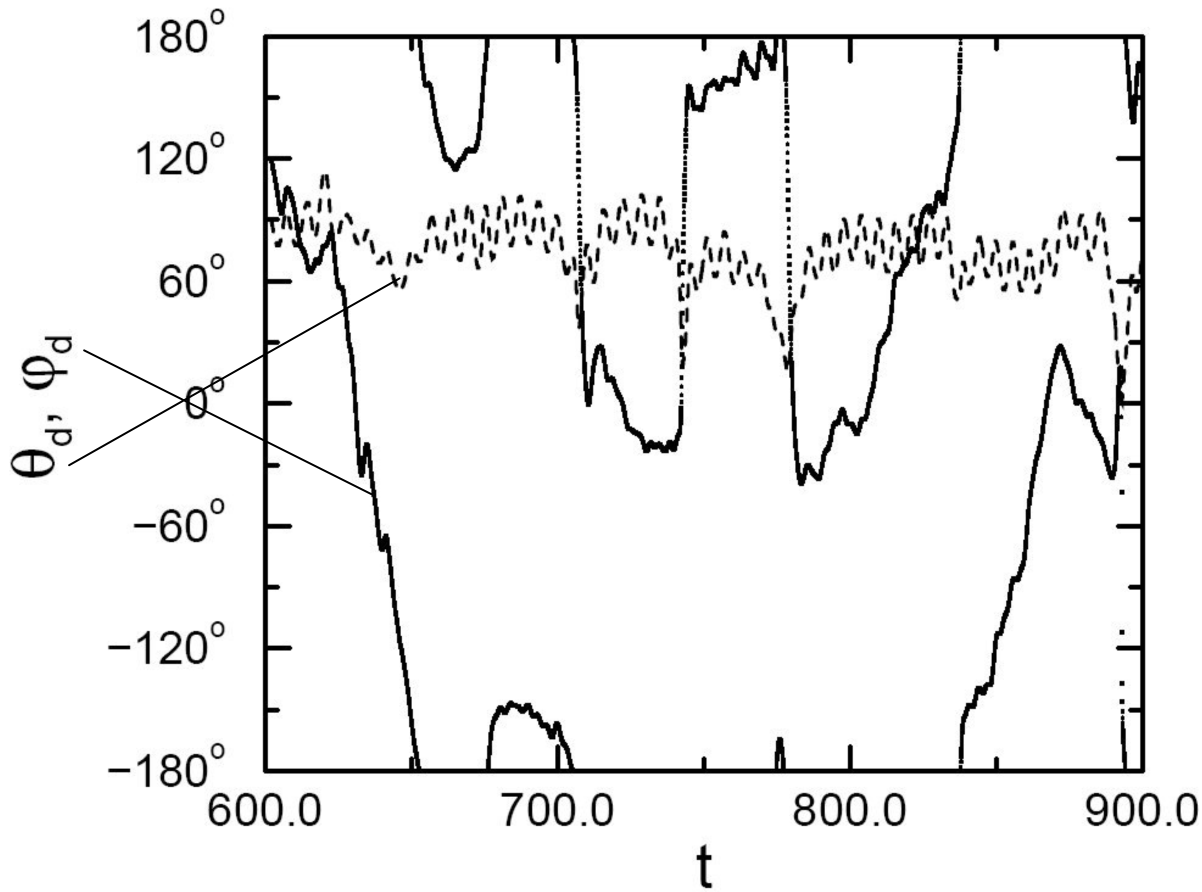
$$\alpha = 23,5°$$

$$\eta = 0,35$$

$$Pm = 1,5$$

Reversals

Stronger driven dynamo



$$E = 3 \cdot 10^{-4}$$
$$\Omega = -0,3$$
$$\alpha = 60°$$
$$\eta = 0,1$$
$$Pm = 2$$

adapted from: A. Tilgner, „Precession driven dynamos“, Phys. Fluids **17**, 034104 (2005)

Summary and outlook

- Precession drives flows in the fluid core
- Flows get unstable
- This unstable flows can drive a dynamo process

- When do you get a dynamo?
- What properties does this dynamos have?
- What changes with the parameters?
- Which effect occur with the deformation of the Earth?

Westfälische
Wilhelms-Universität
Münster

Thank you for your attention!

wissen.leben
WWU Münster

References

F. H. Busse, „Steady fluid flow in a precessing sheroidal shell“, J. Fluid Mech. 33, 739-751, (1968)

D. E. Loper, „Torque balance and energy budget for the precessionaly driven dynamo“, Phys. Earth Planet. Inter. 11, 43-60,(1975)

S. Lorenzani, „Fluid instabilities in precessing ellipsoidal shells“, Dissertation, (2001)

A. Tilgner, „Precession driven dynamos“, Phys. Fluids 17, 034104 (2005)

A. Tilgner, „Rotational dynamics of the core“, Treatise on Geophysics 8.07, (2007)

J. P. Vanyo, „Core-mantle relative motion and coupling“, Geophys. J. Int. 158, 470-478, (2004)