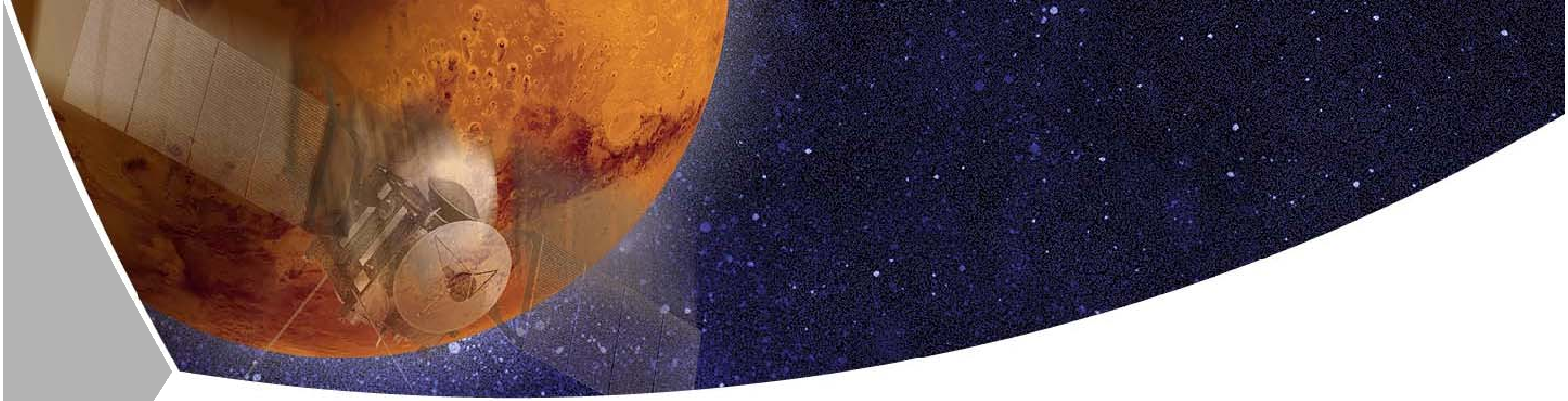

Scaling laws for internally heated mantle convection

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[1]Institute of Planetary Research, German Aerospace Center (DLR)

DLR Deutsches Zentrum für Luft- und Raumfahrt e.V. in der Helmholtz-Gemeinschaft

Westfälische Wilhelms-Universität Münster

Structure

Simulation Setup

- Standard Boussinesq
- Free slip
- Finite V/E/D 3D / 2D
- Flexible Grid
- BiCGStab solver
- Highly parallel DC
- IDL framework for evaluation / vis
- Independent C++ code, STL

$$\nabla \cdot \vec{u} = 0$$

$$\nabla \cdot \left[\eta \left(\nabla \vec{u} + (\nabla \vec{u})^T \right) \right] + Ra_Q T \vec{e}_r - \nabla p = 0$$

$$\frac{\partial T}{\partial t} + \vec{u} \nabla T - \nabla^2 T - 1 = 0$$

- Purely internally heated w/ insulated bottom
- 85 cases: 55 T-dep. & 30 T+P-dep.
- Projected spherical icosahedra grid (ar=0.55)
- Resolution: 10k lateral & 34 shells ~ 360k Nodes

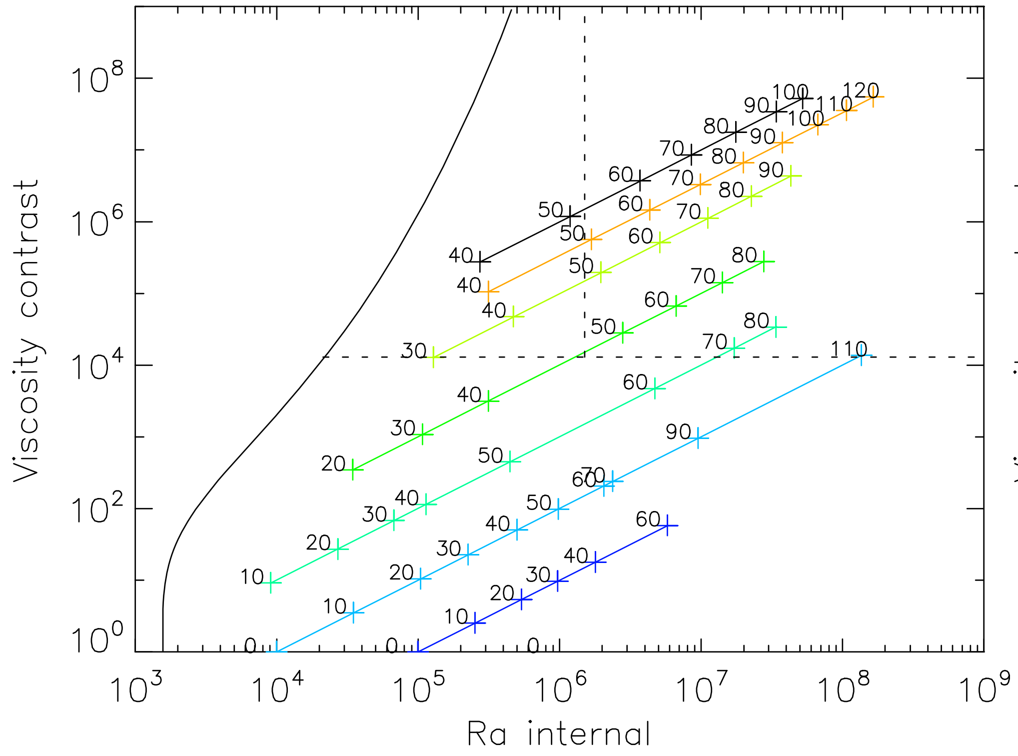
$$Ra_Q = \frac{\rho^2 g \alpha H d^5}{\kappa k \eta_{ref}}$$

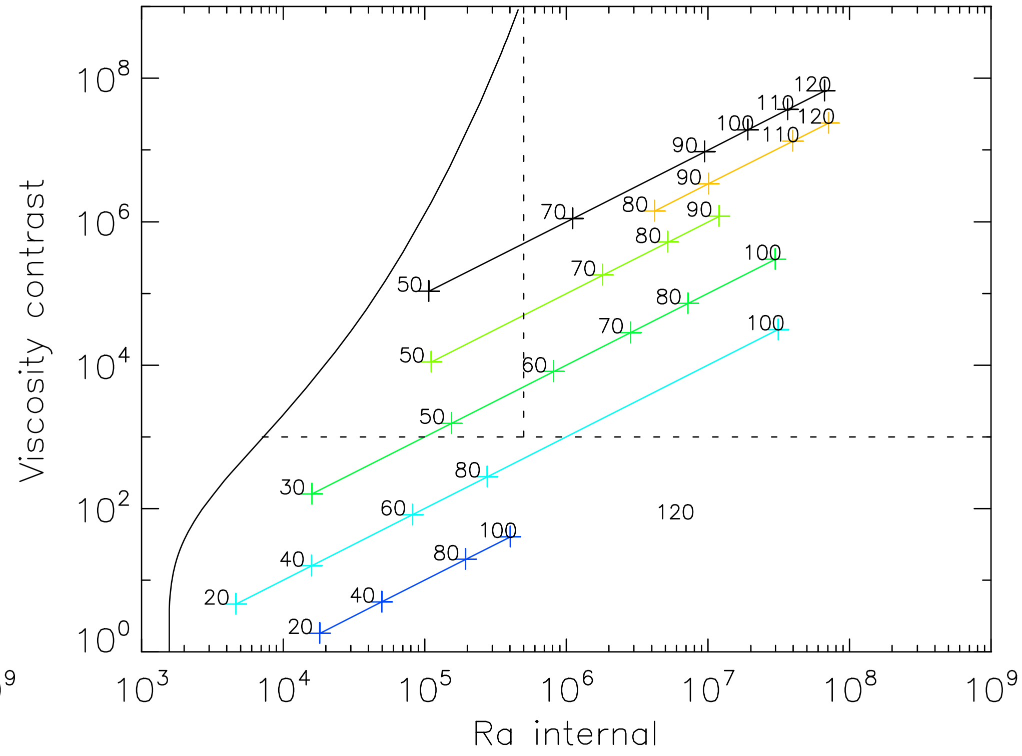
$$\eta(T) = \exp(-\gamma T + \Delta \eta_P z)$$

$$\eta_{ref} = \eta(T_S)$$

Parameter range

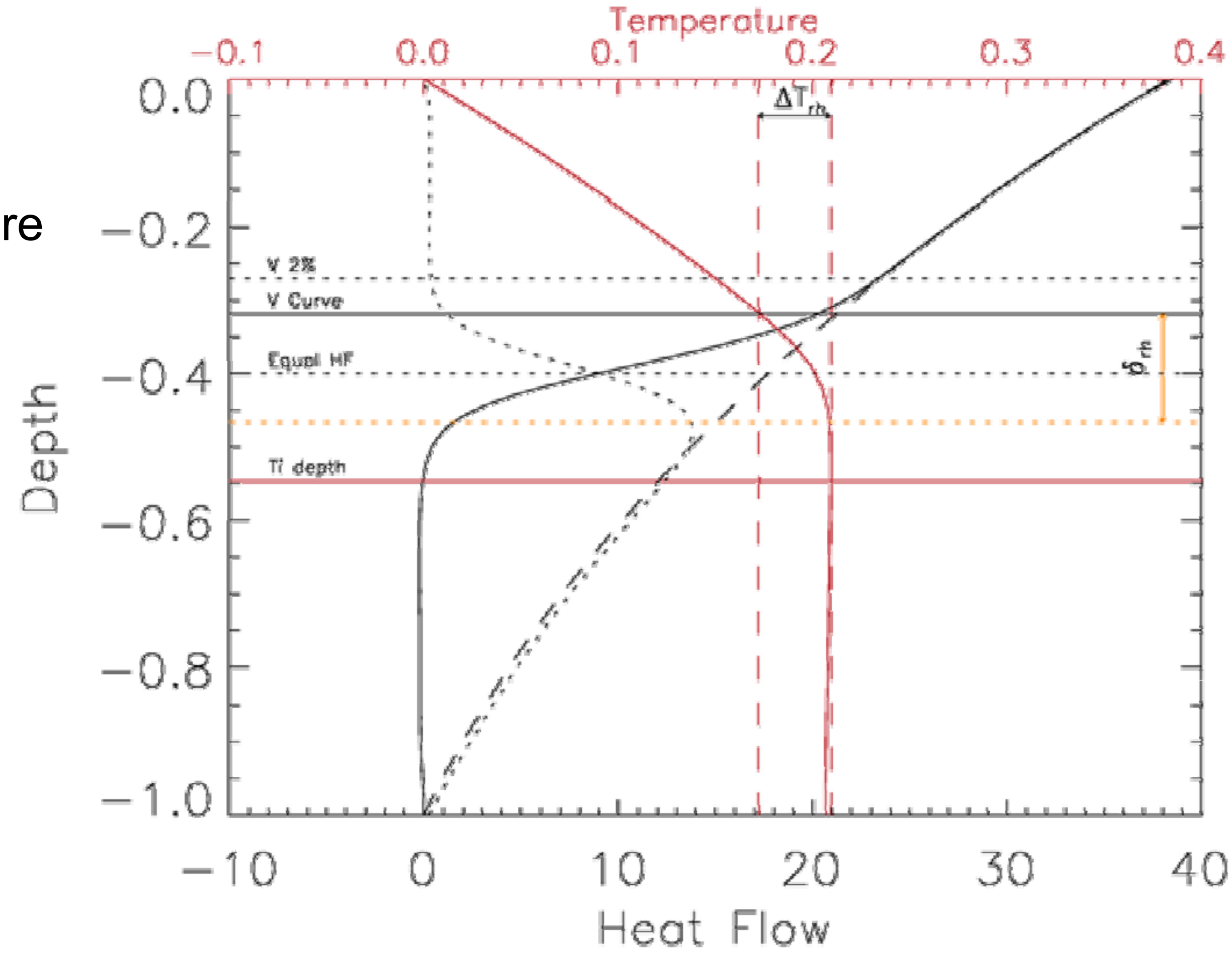
- Ra0 and γ varied for two different scenarios, with P-dep. Viscosity of 100 and without





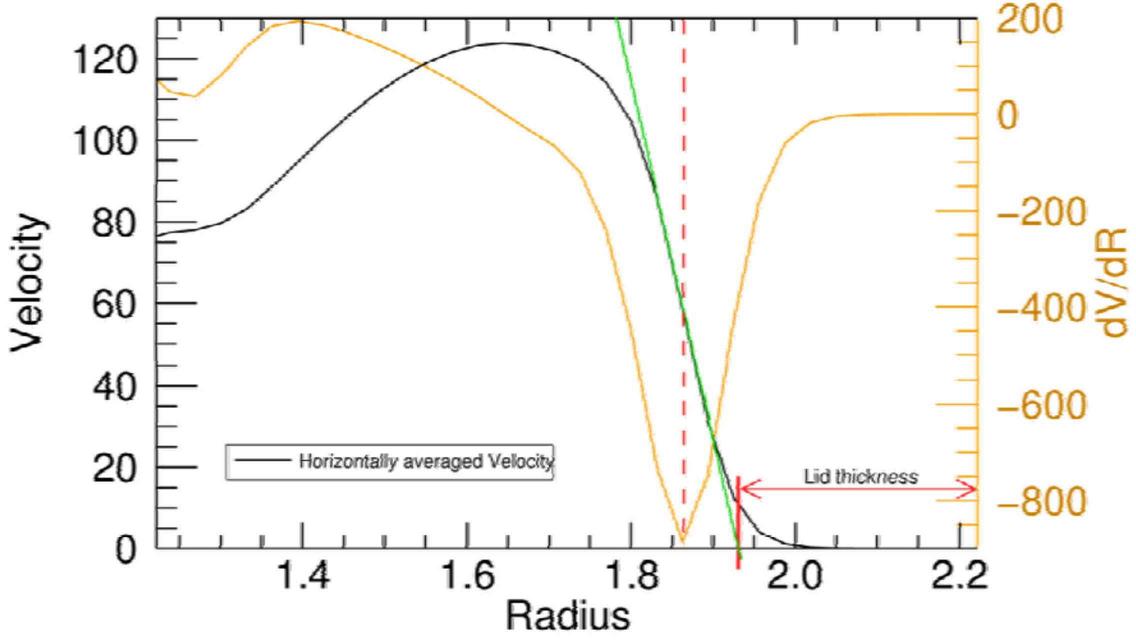
Points of interest

- Lid thickness
- Internal temperature
- Boundary layer
- Viscosity contrast
- Rayleigh numbers
- Nusselt number
- Structure
- Time dependency

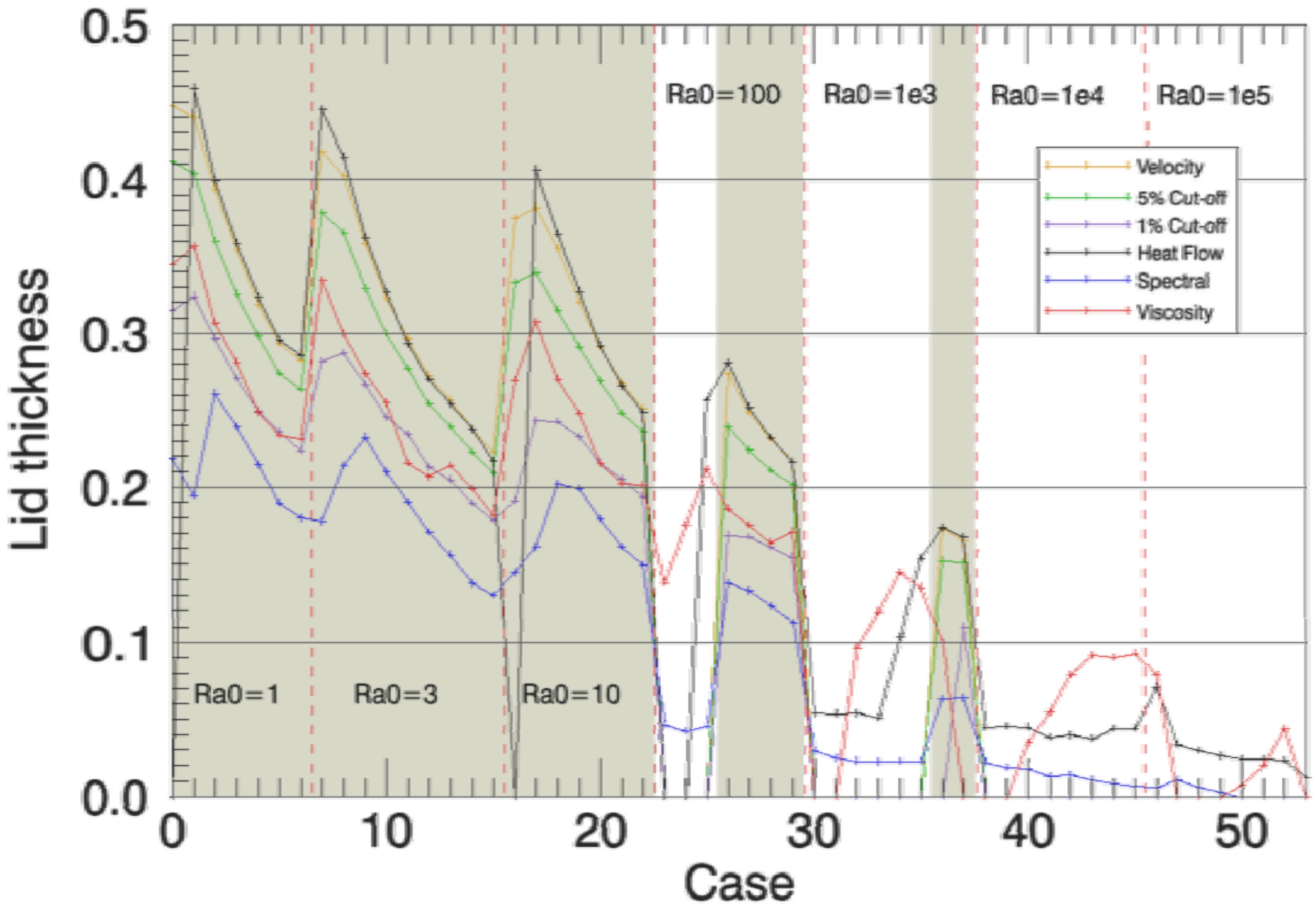


Lid thickness determination

- Various methods possible
- Delamination / erosion versus conductive heat flow
- Which one is “right”?
- Main methods:
  - Threshold in V
  - Tangent through inflexion point → (fails on some weakly convecting systems!)



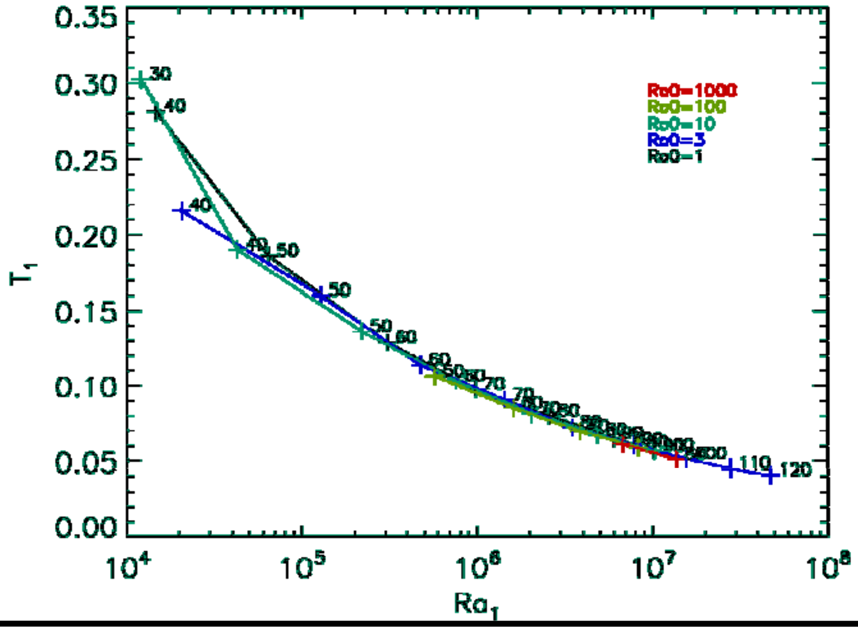
Lid thickness – A summary

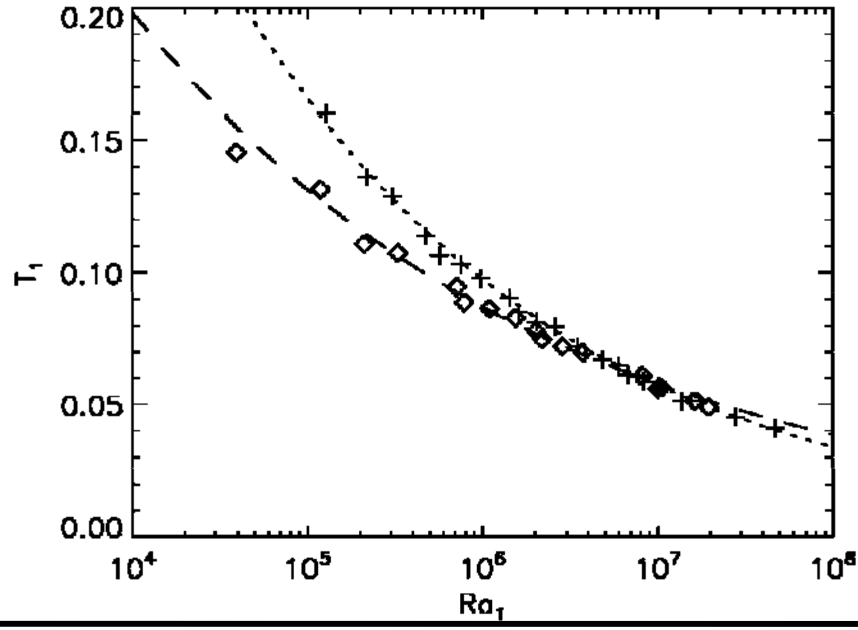


Reduction to iso-viscous parameters beneath the lid

- Measured lid thickness upper boundary
- Proves that convection beneath the lid can be treated as isoviscous

$$T_1 = \frac{\Delta T_{rh}}{d_{eff}^2} \qquad Ra_1 = Ra_i \, d_{eff}^5 \qquad T_1 = a \, Ra_1^{\beta}$$





The quality of the rheological constant with the previous lid measurement methods

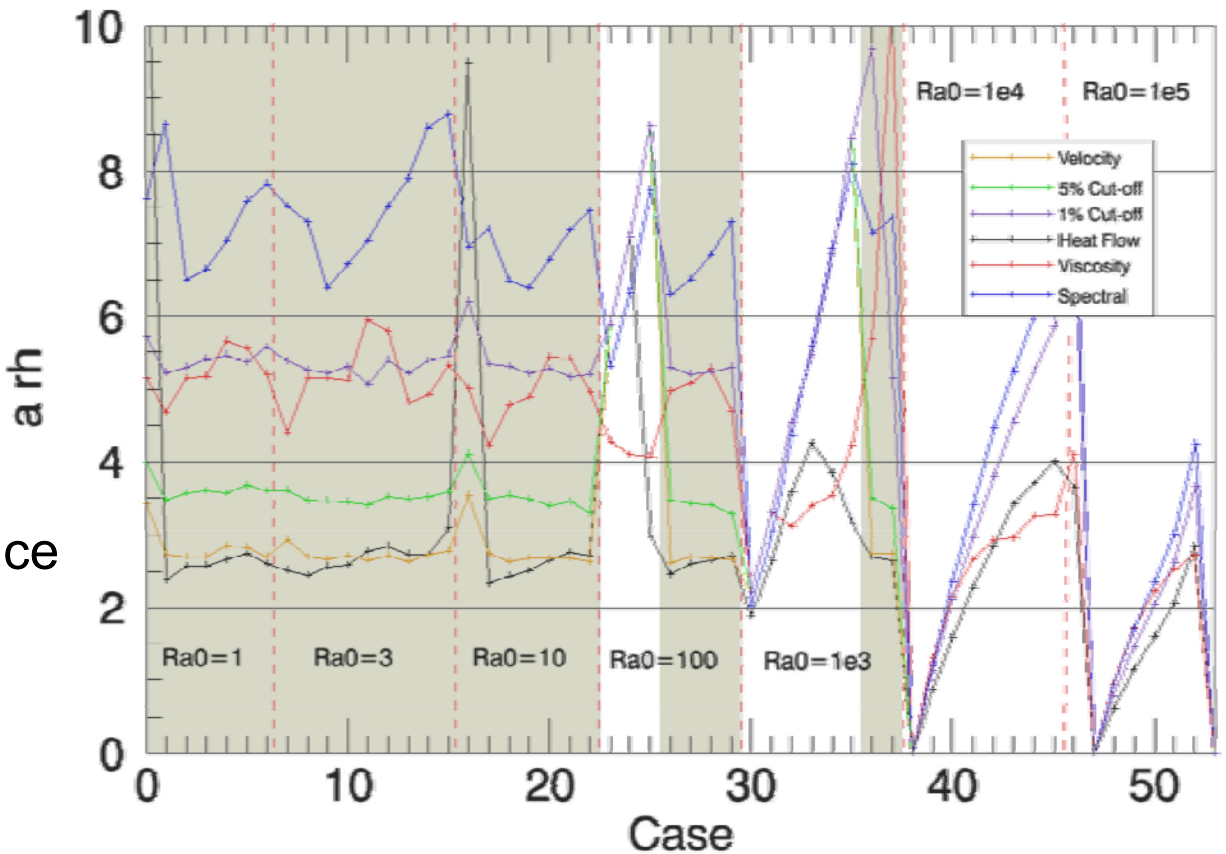
- Assume relation between temperature drop and rheological gradient

$$a_{rh} = \Delta T_{rh}\,\gamma$$

$$\Delta T_{rh} = T_i - T_L$$

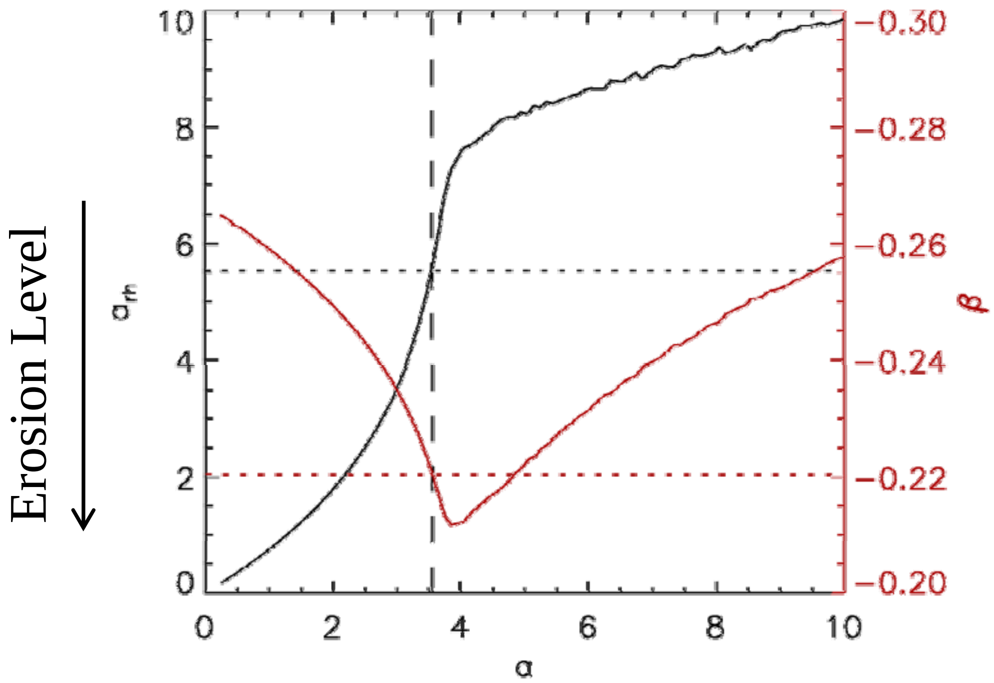
- Tangent through inflexion point of the velocity profile best choice

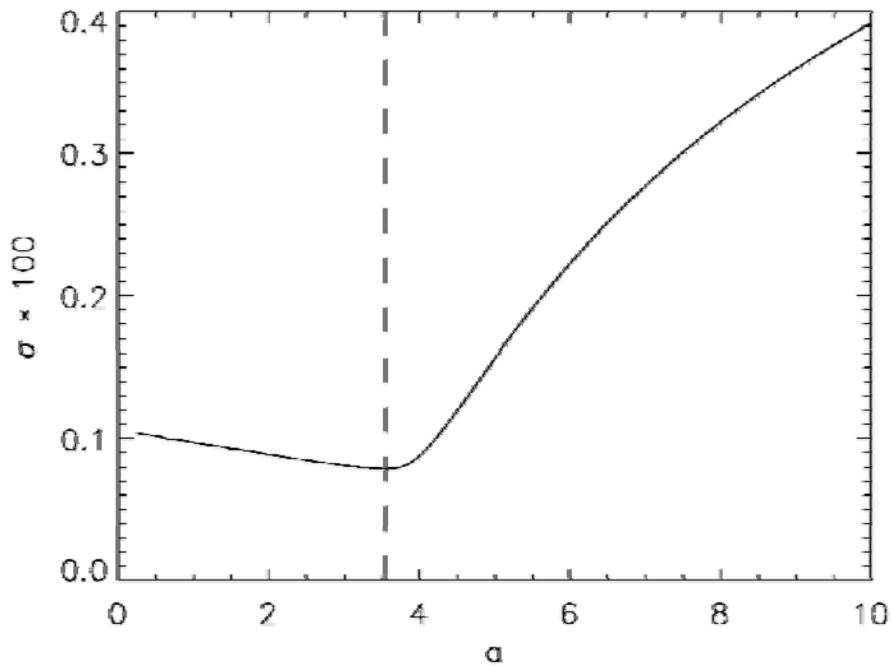
- Steady state generally higher a_rh



Internal temperature fit with the rheological constant

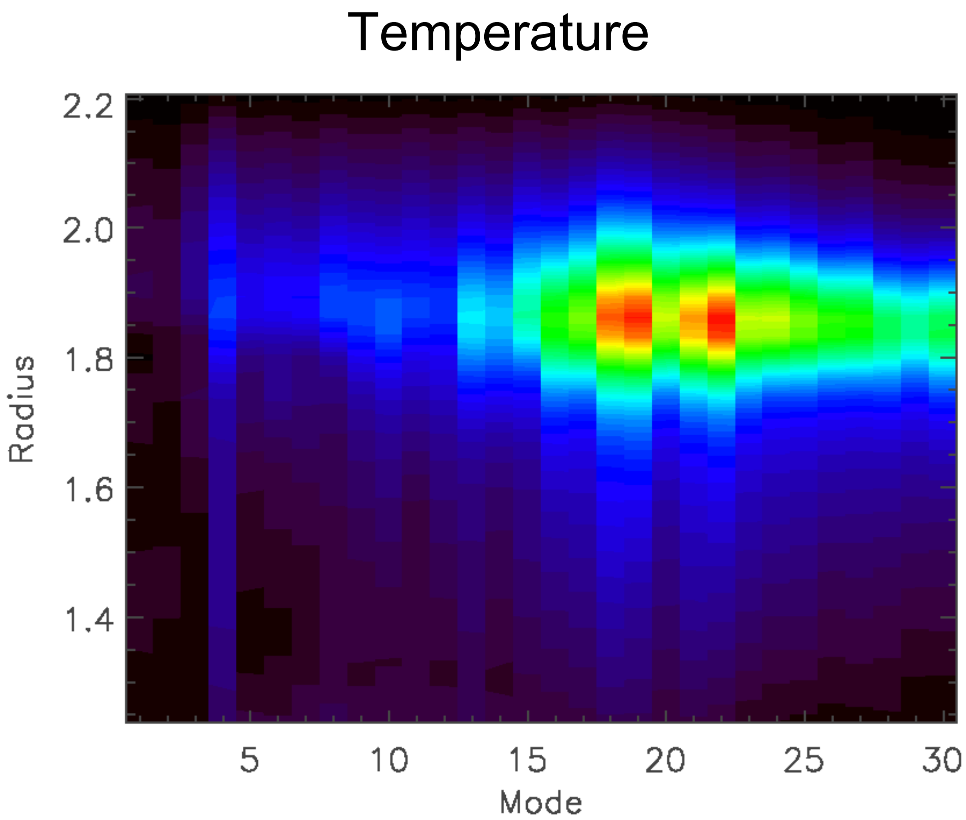
- After Davaille 92; Grasset 98; Reese 99,05; Deschamps 99
- Confirms weak dependence of all three parameters
- Choose higher a_{rh} to get a less-eroded lid

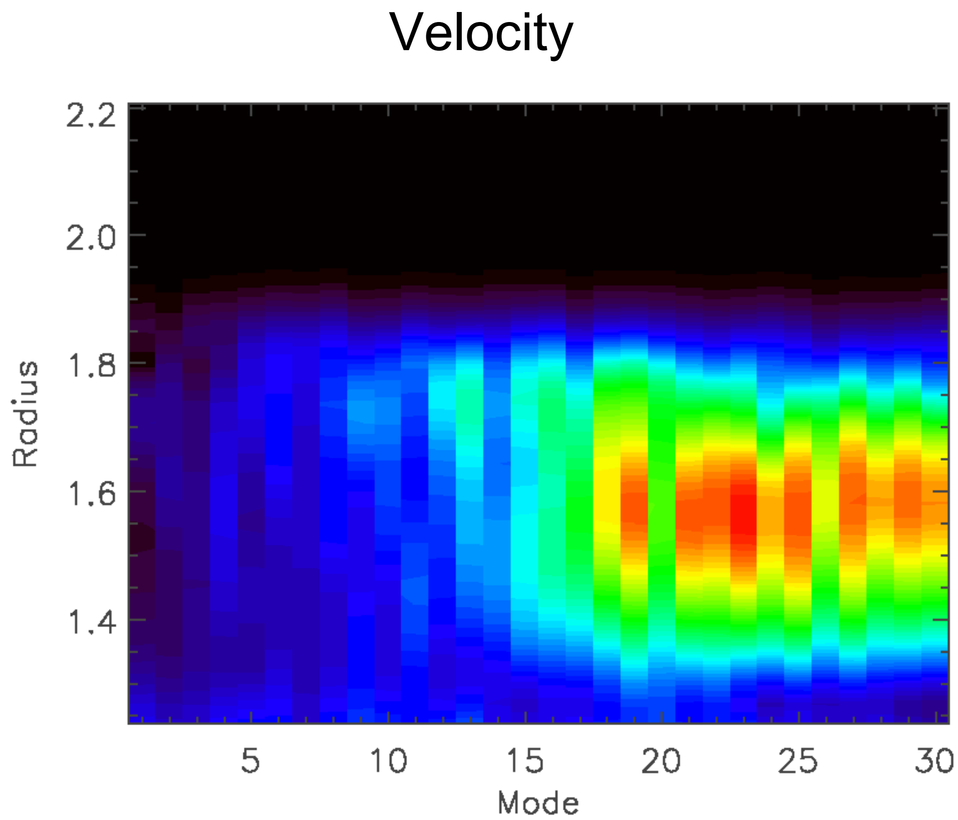




Structural analysis – Spherical Harmonics

- Example case: Ra0=1, γ=80

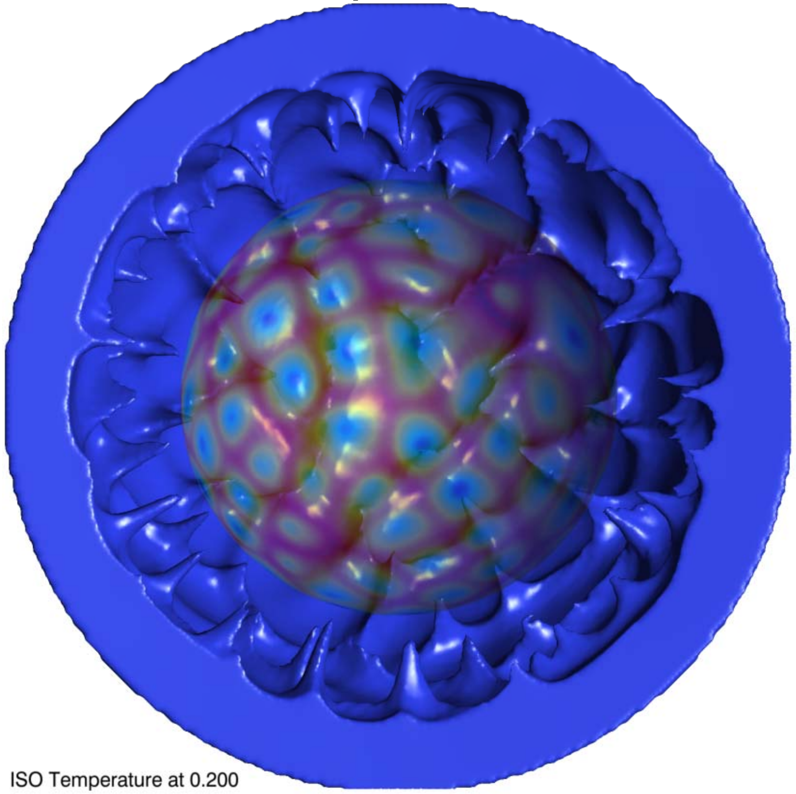




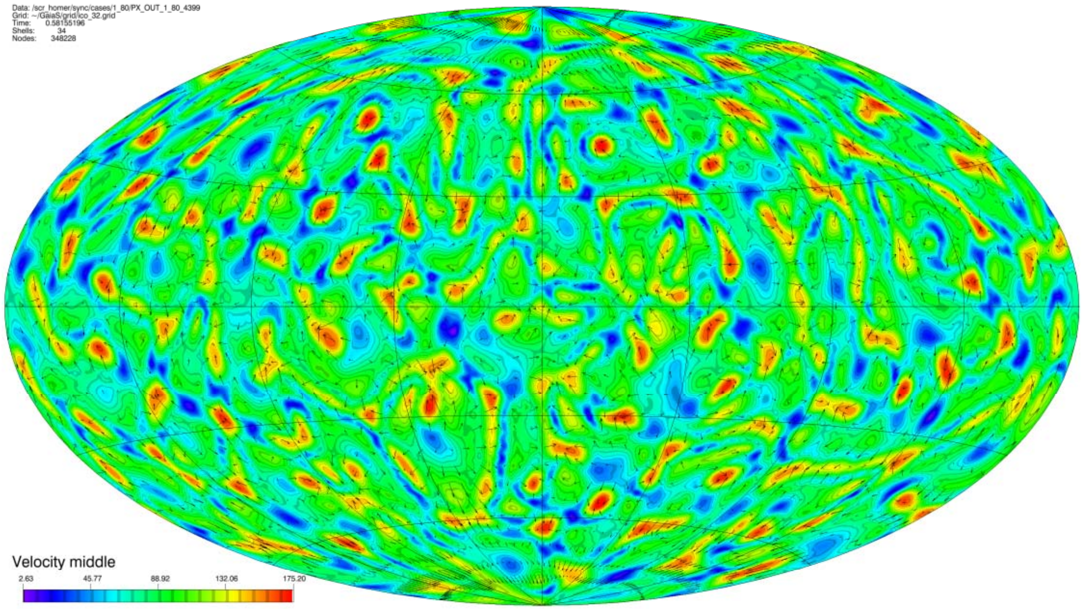
Structural analysis – Spherical Harmonics

- Example case: Ra0=1, γ=80

Temperature



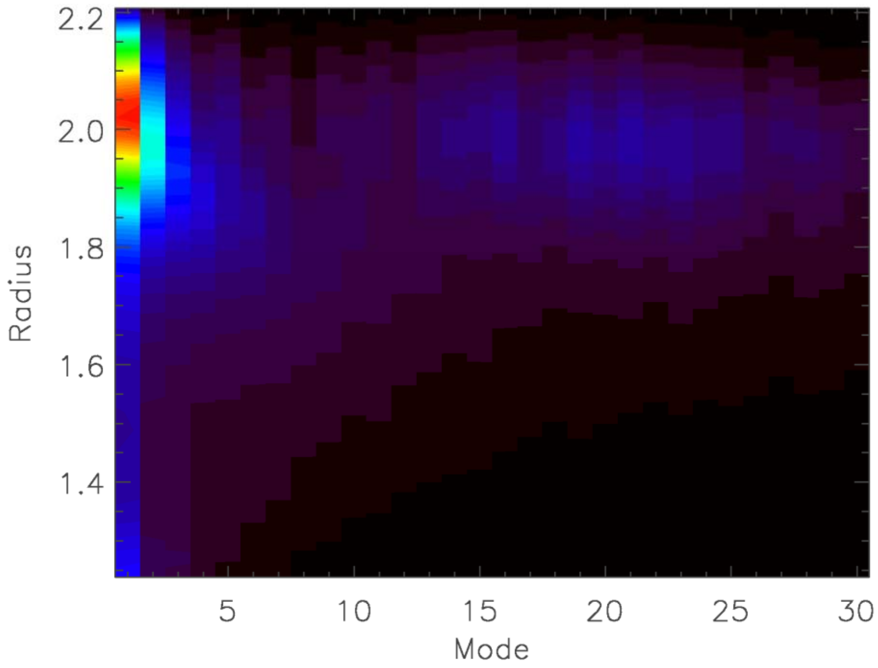
Velocity



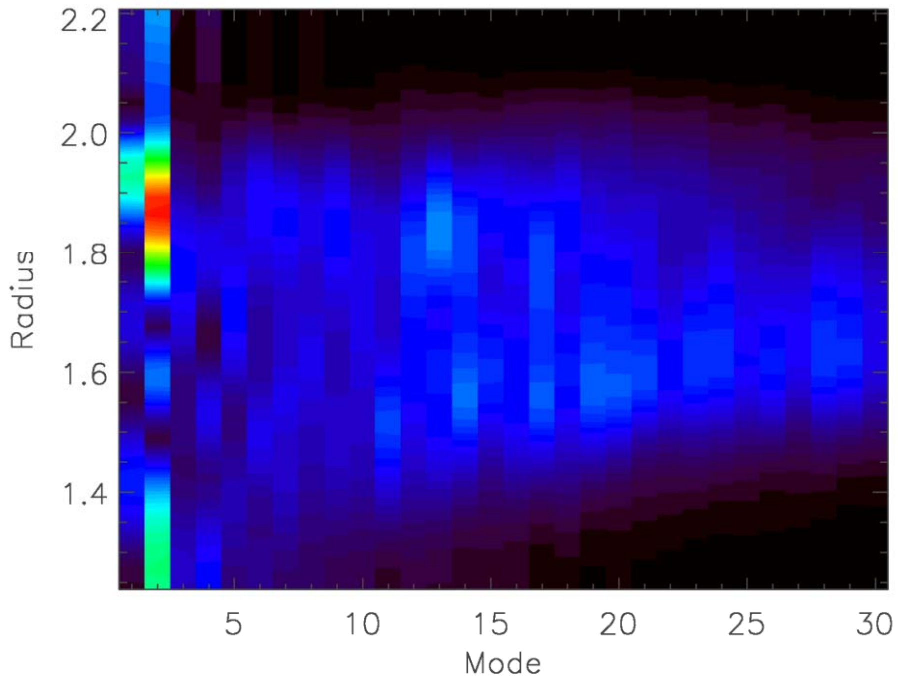
Structural analysis – Spherical Harmonics

- Example case: Ra0=1000, γ=60

Temperature



Velocity



Structural analysis – Spherical Harmonics

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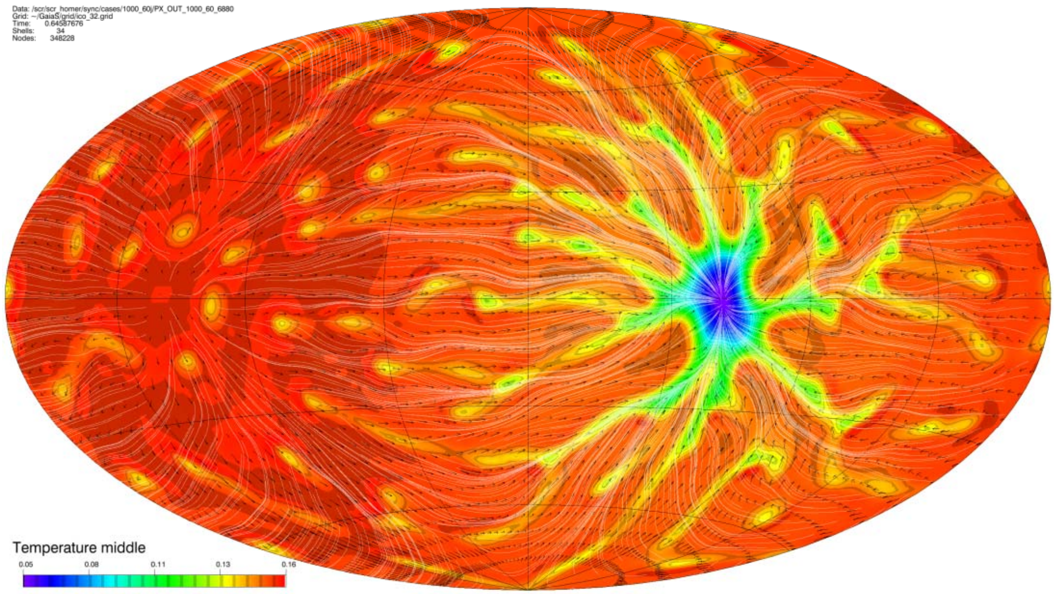
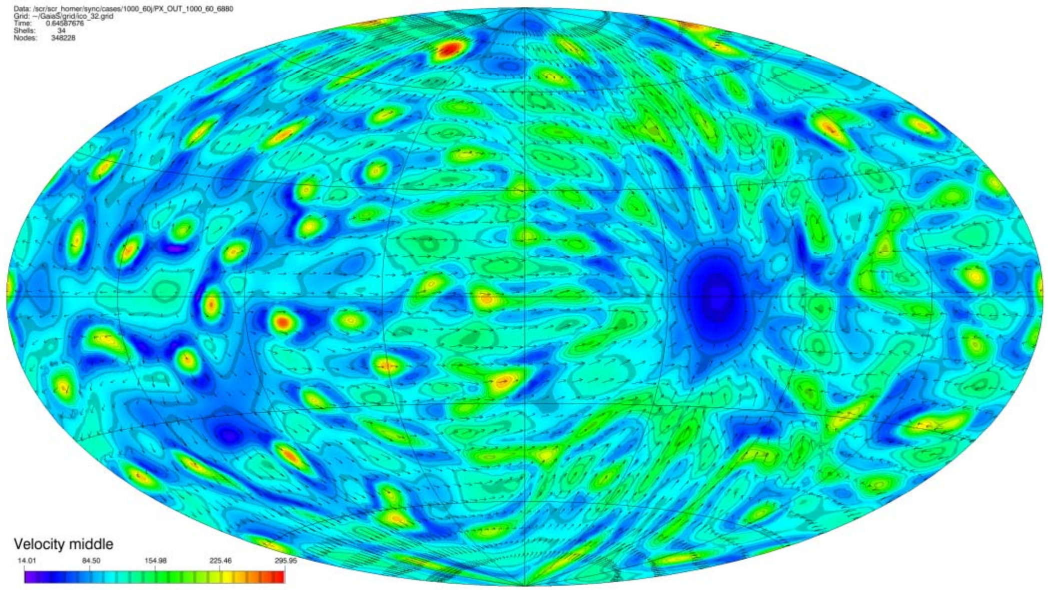
Temperature

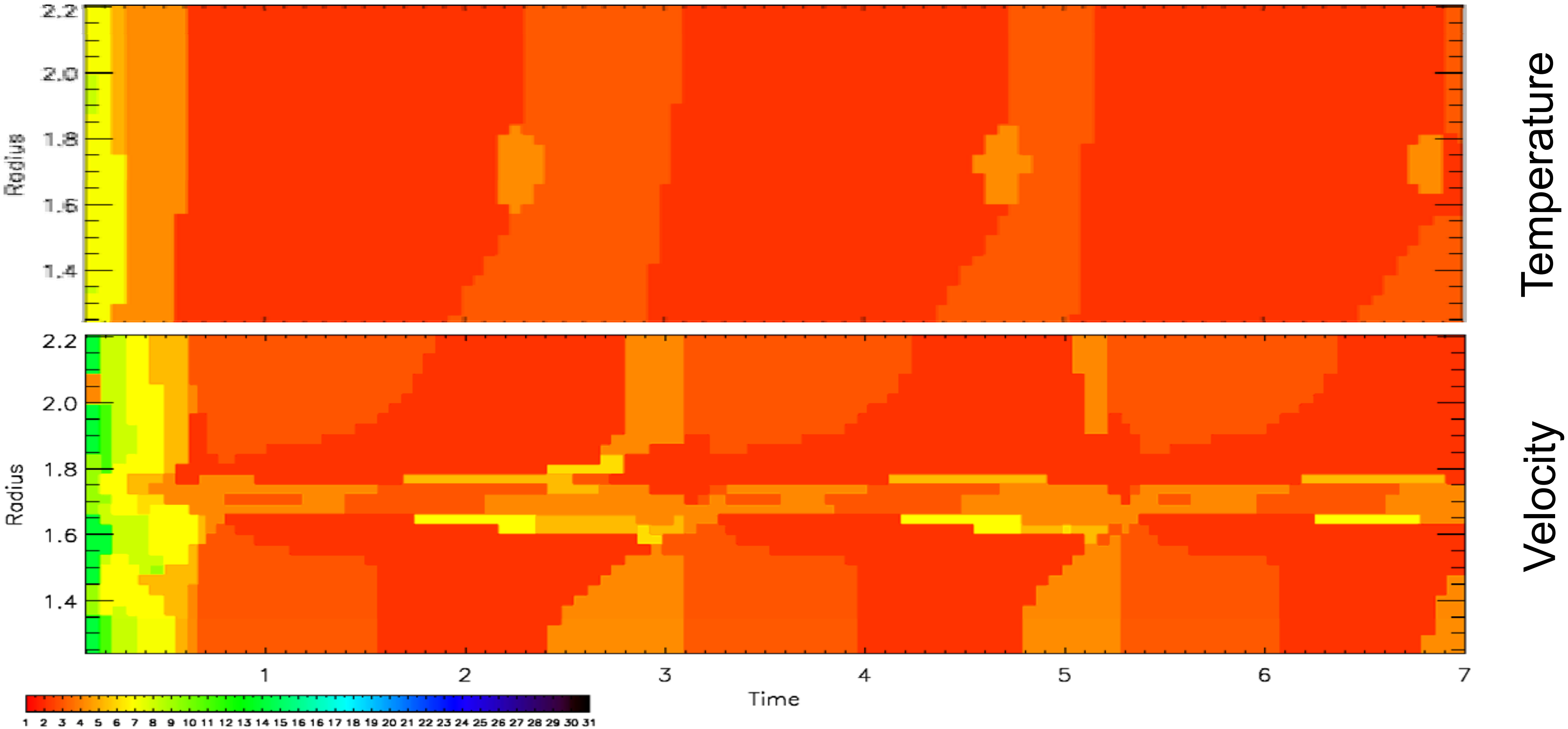


Velocity



Mode Cycling

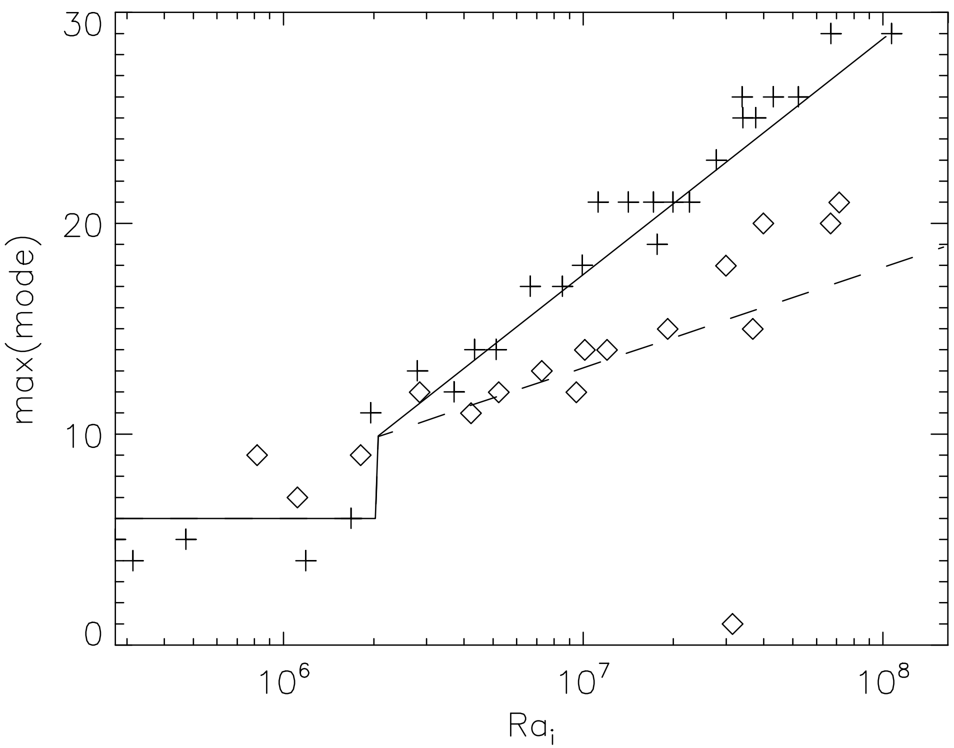
- Cycling between low dominant modes for some mobile lid cases
- Fluent transistions between quasi-steady (low-) modes

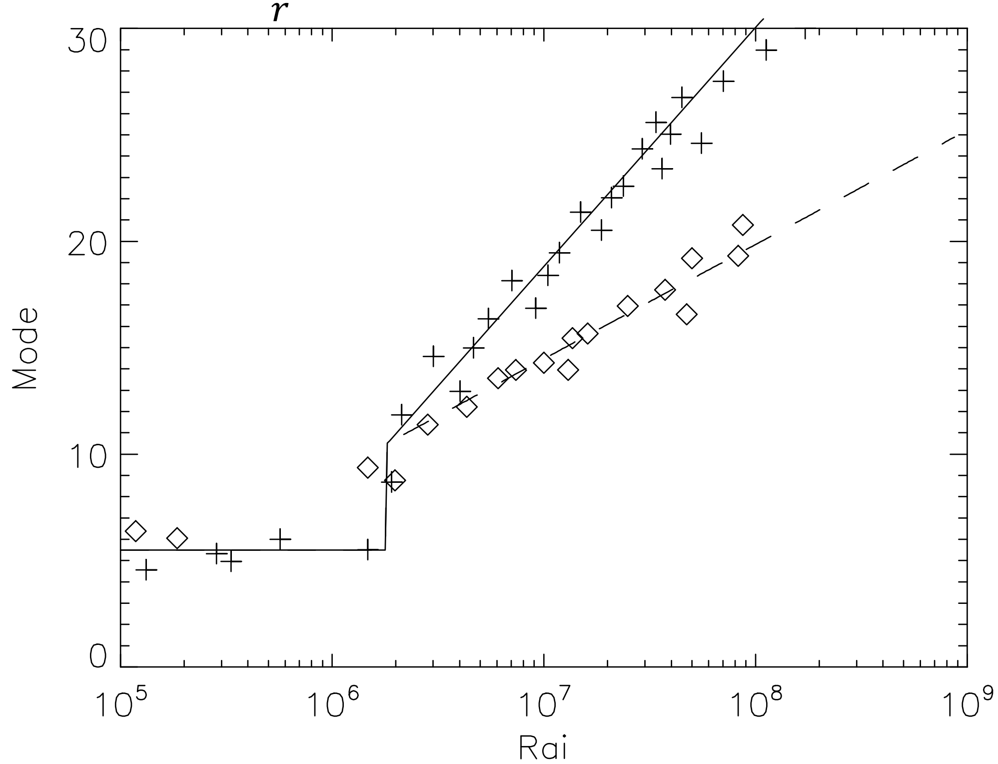


Mode – Ra fit for the stagnant lid regime

- Dominant mode not suited
- Central mode provides better representation:

$$\omega = \frac{\sum \ell_i \lambda_i}{\sum \lambda_i} \qquad \lambda_i = \sum_r \lambda_{r,i}$$



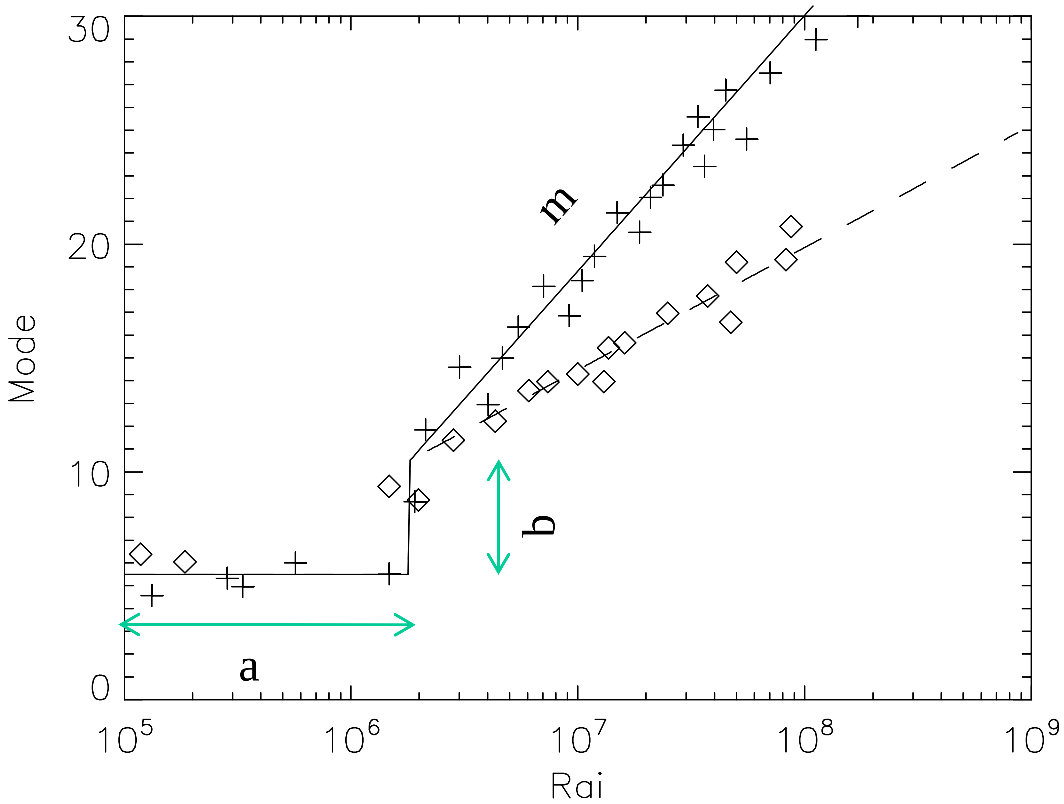


Mode – Ra fit for the stagnant lid regime

- Jump function introduced to get continuous function
- Regularized gamma funtion *P* acts as jump function

$$\omega = P(a, Ra)\, m \left(\ln(Ra) - \ln(a) + \frac{b}{m}\right) + \omega_{min}$$

- Three parameters:
  - a: Transition to time-dependent convection
  - b: Jump height
  - m: slope in the time-dependent regime
- Derived through full inversion



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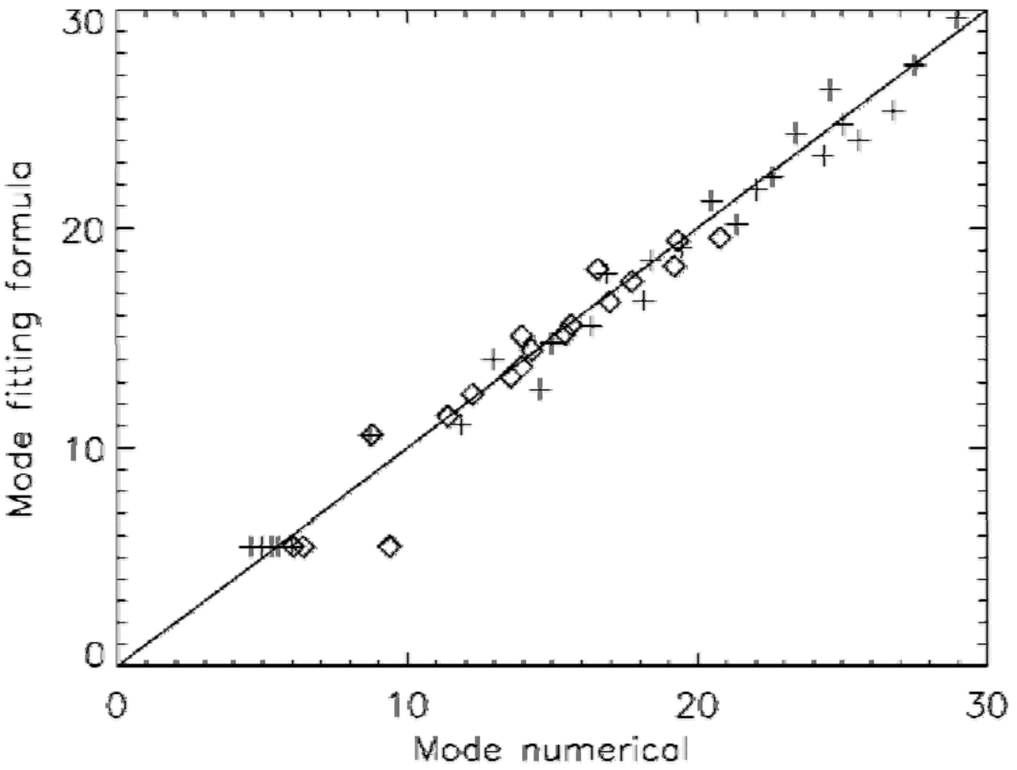
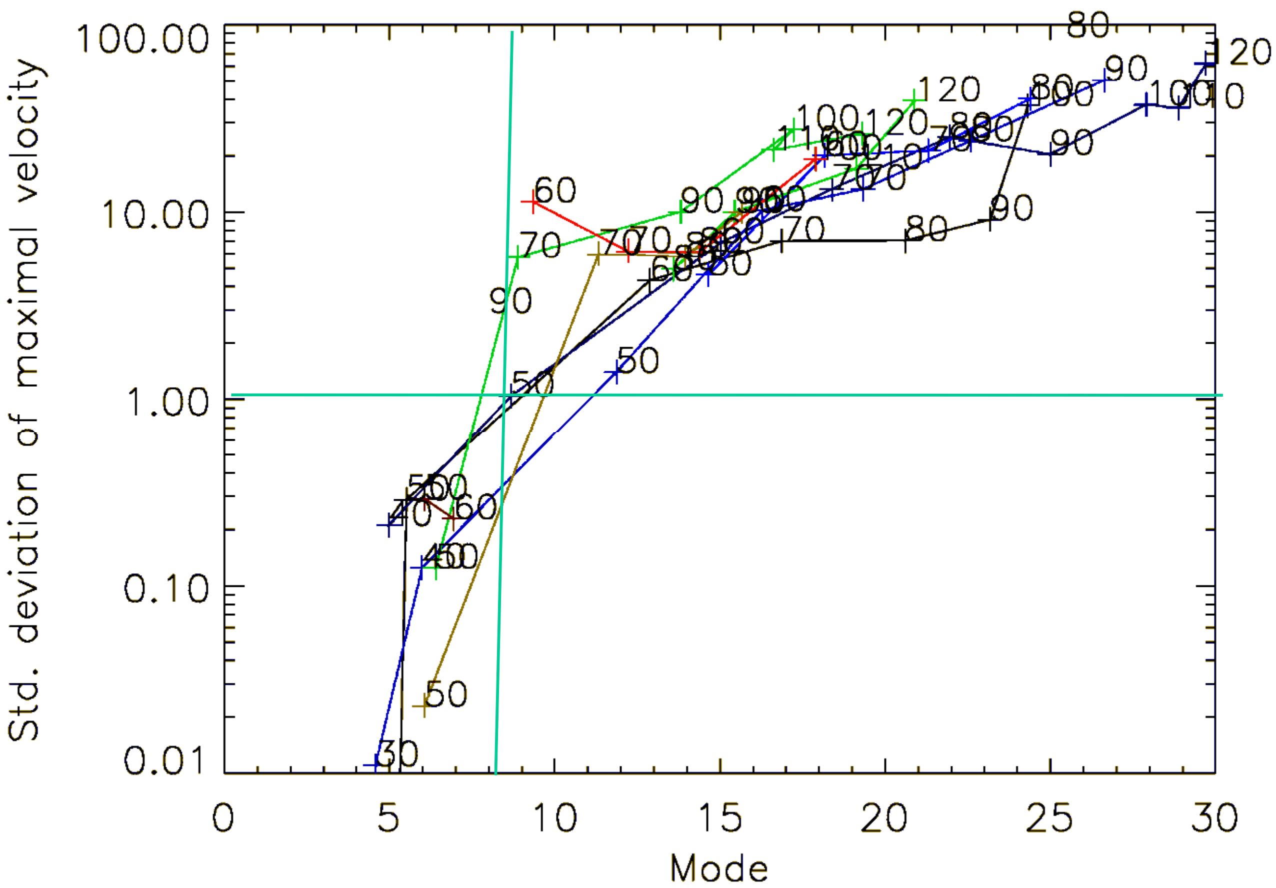


- Three parameters:
  - a: Transition to time-dependent convection
  - b: Jump height
  - m: slope in the time-dependent regime
- Derived through full inversion

Time dependent convection and its dependence on the mode

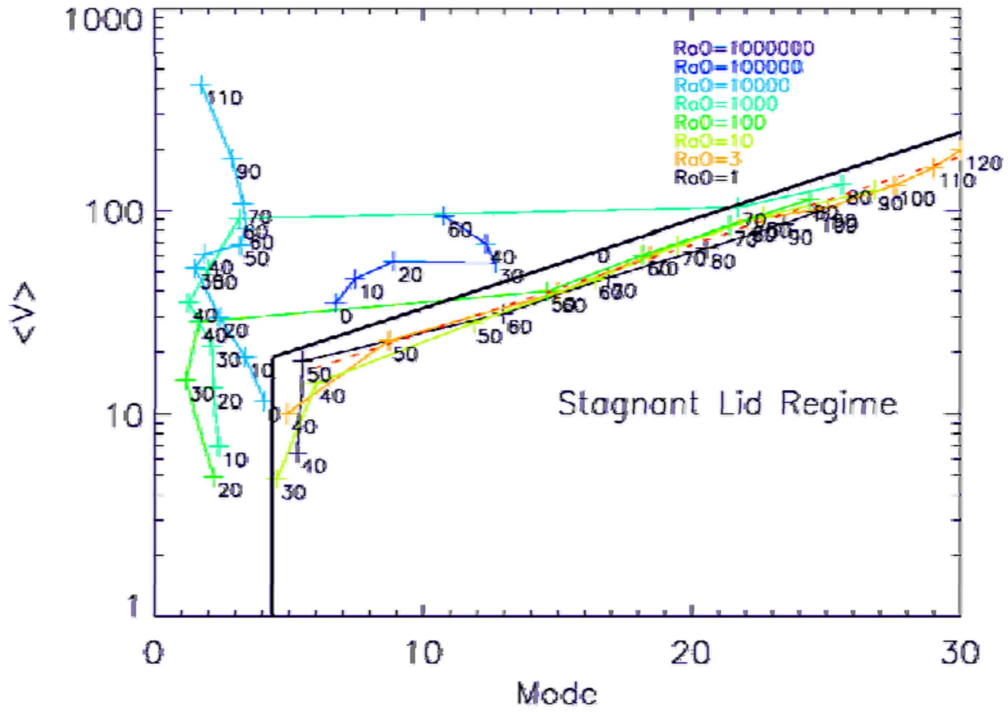
- Only transient part considered to avoid influence of IC
- One as a good boundary to avoid erroneous fluctuations
- Transition around degree 8-9 confirmed for P-dependent cases as well



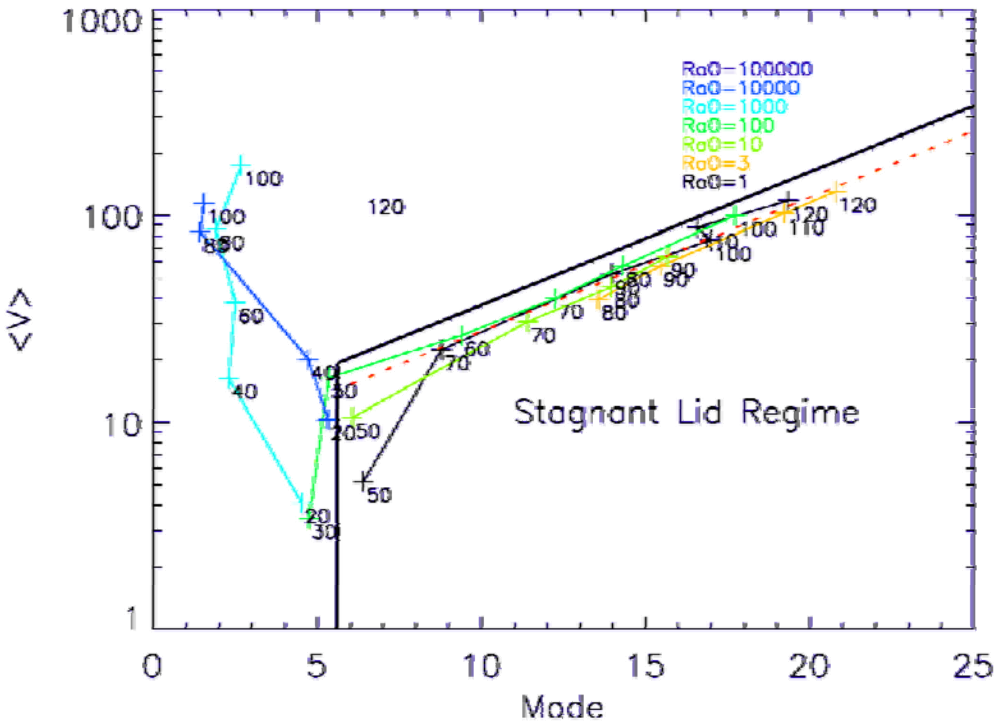
Velocity and its dependence on the mode

- Evident from correlation to Ra
- Different scaling between (quasi-)steady state and time dependent cases
- Velocity increase stronger with additional pressure dependence

Temperature dependent

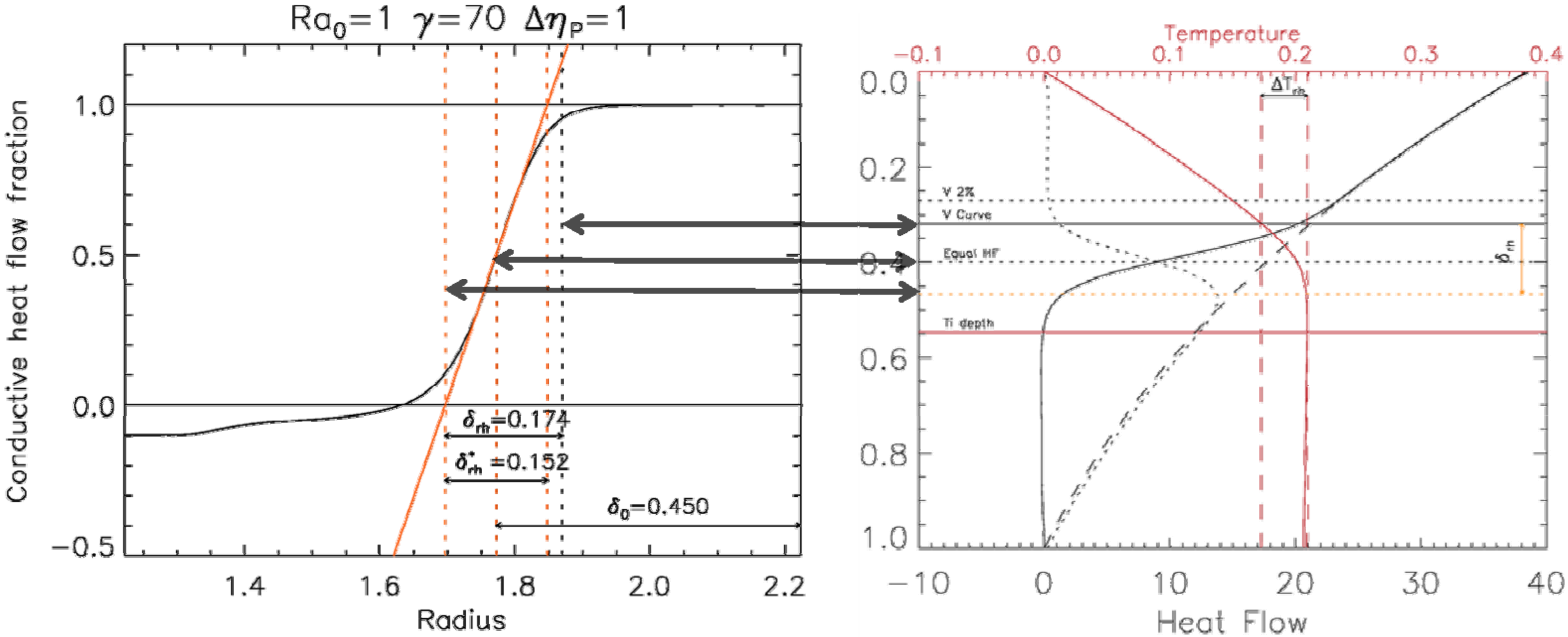


Temperature and pressure



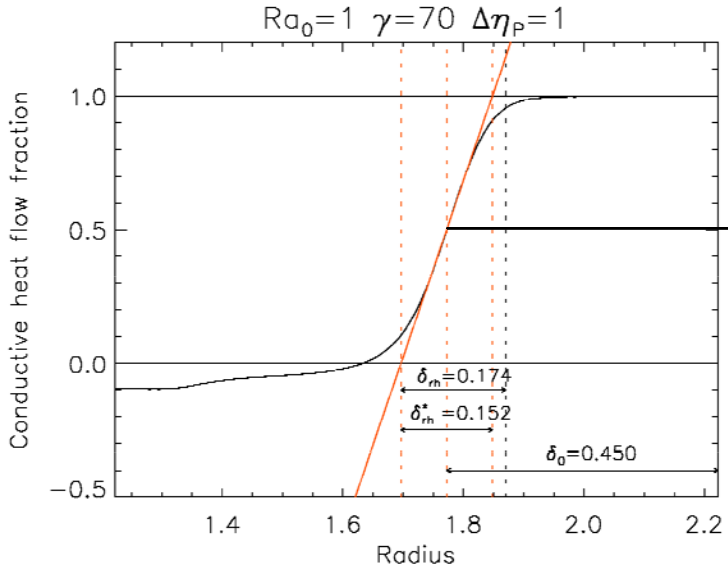
Boundary layer

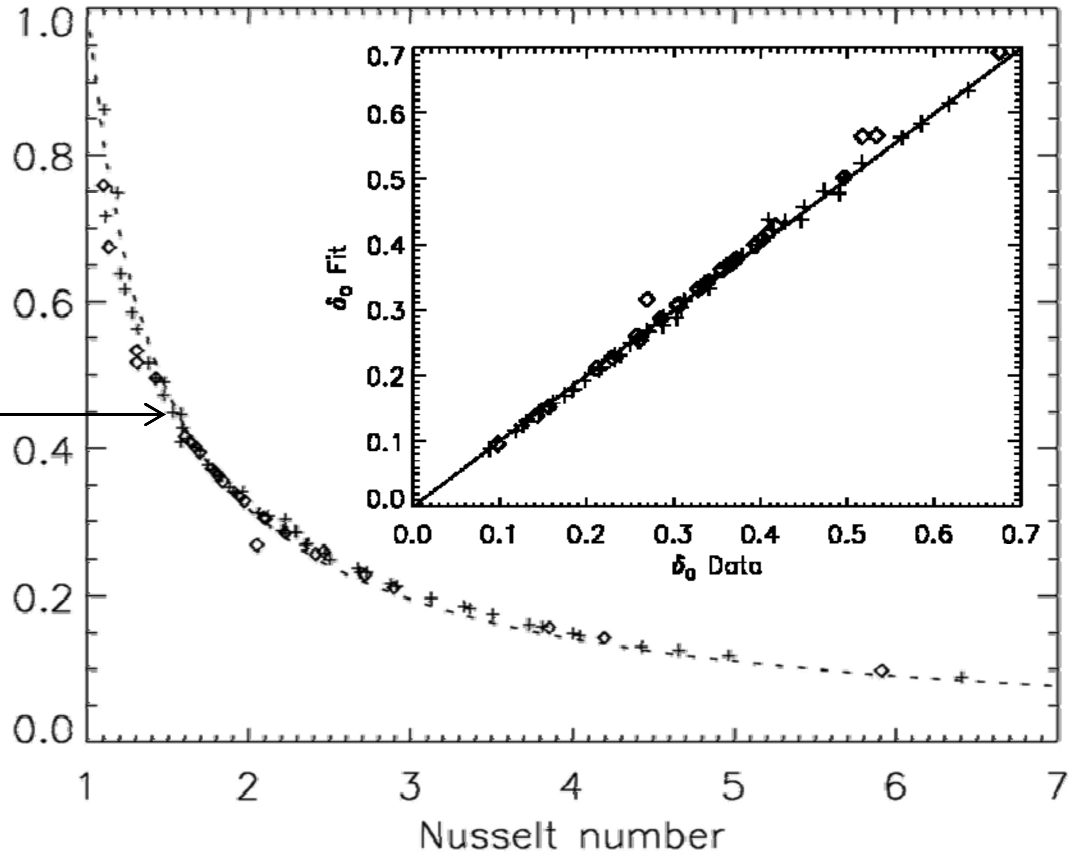
- Introducing reduced boundary layer thickness δ_{rh^*}
- Never reaches surface on mobile or isoviscous cases but centers around δ_0



Omnipresent correlation of boundary layer depth to the Nusselt number

- Correlation present for all cases, stagnant lid and mobile
- Includes pressure dependent cases marked as diamonds
- Centers around reduced boundary layer





Boundary layer thickness scaling

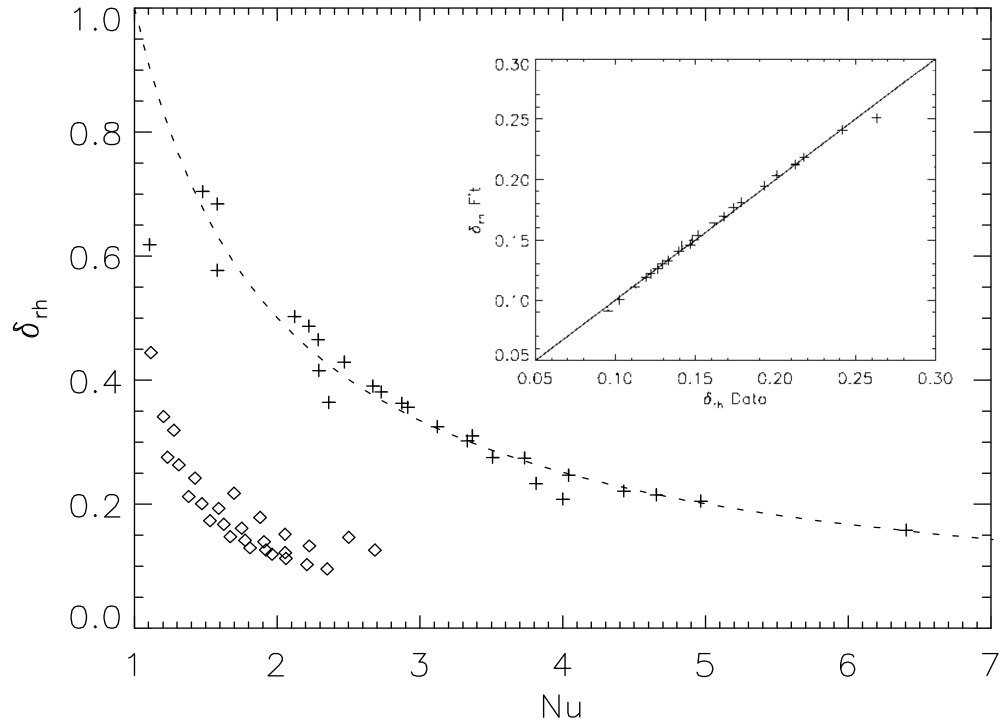
- As predicted by boundary layer theory, for all mobile & isoviscous cases:

$$\delta_{rh} \sim 2\,\delta_0 = Nu^{-1}$$

- For stagnant lid cases, the layer thickness depends on its depth and viscosity:

$$\delta_{rh} \sim \frac{\delta_0}{\gamma T_i} = \frac{\delta_0}{\log \eta_i}$$

$$\delta_{rh} = 4\,\delta_0\,(\gamma\,T_i)^{-0.84} \sim \delta_0 \eta_i^{0.059}$$



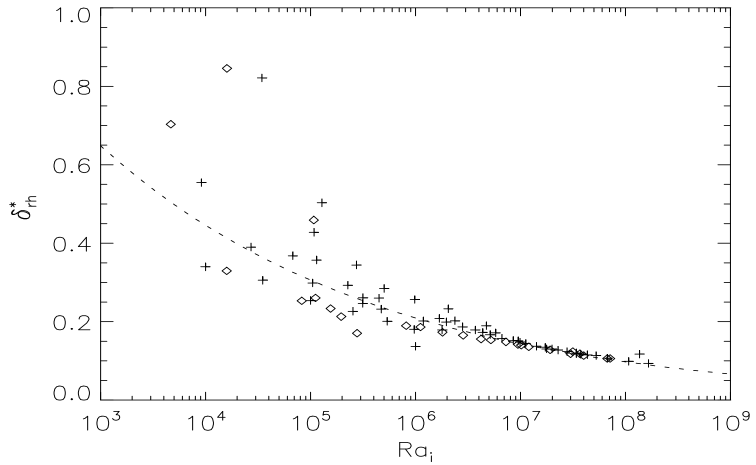
Reduced boundary layer thickness and the lid

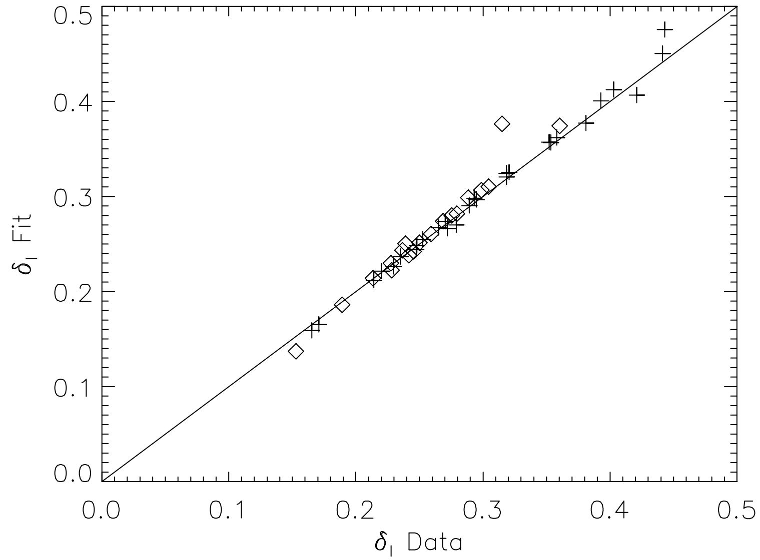
- Correlation between reduced BL and Ra only in the time dependent regime
- BUT: fits pressure dep. cases as well

$$\delta_{rh}^{*} = 2\delta_0 - \delta_b \frac{1}{2}\delta_{rh} = Ra_{H,i}^{-0.163}$$

- Resulting equation for the lid:

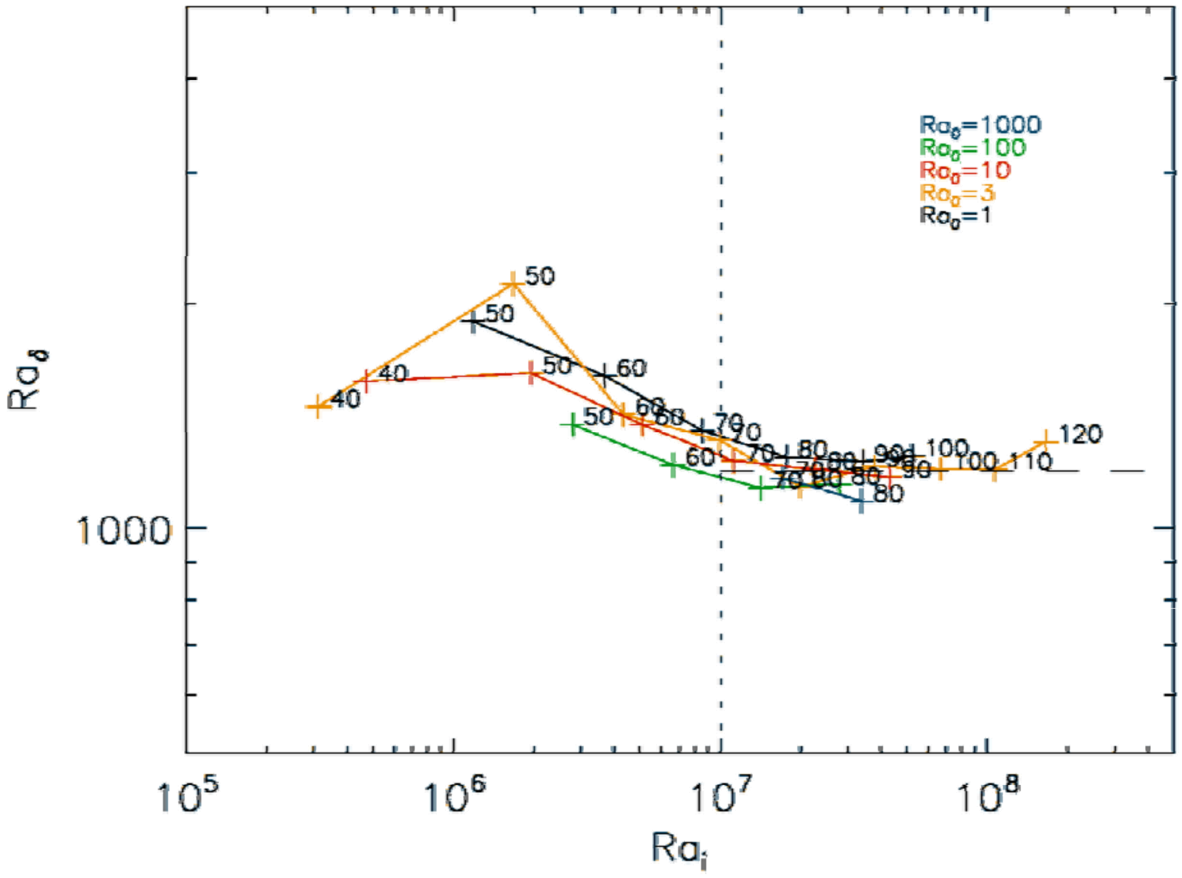
$$\delta_l = \delta_0 + \frac{1}{2}\delta_{rh}^{*} - \delta_{rh}$$





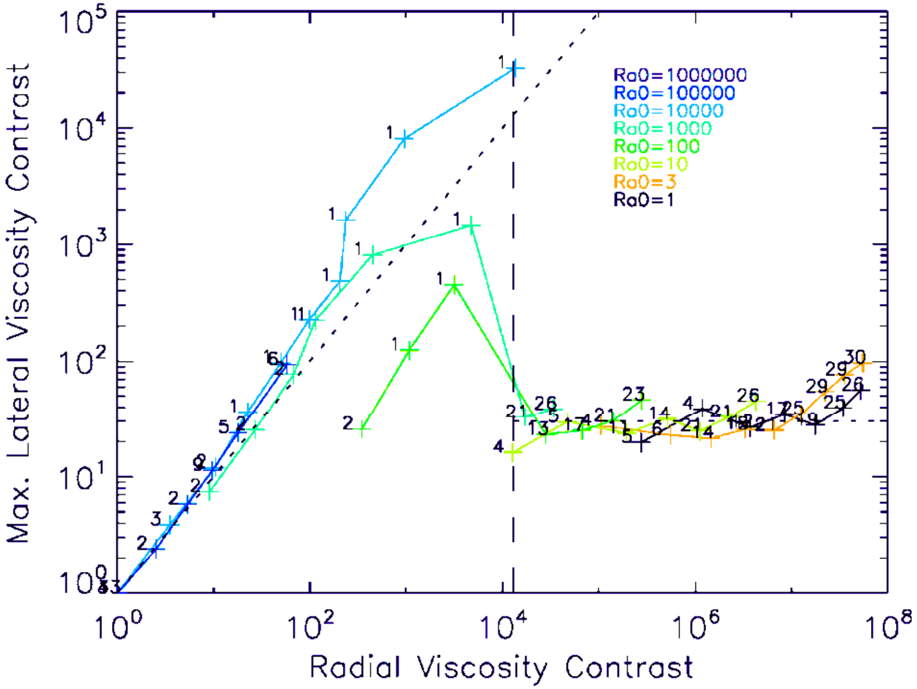
Boundary layer Rayleigh number

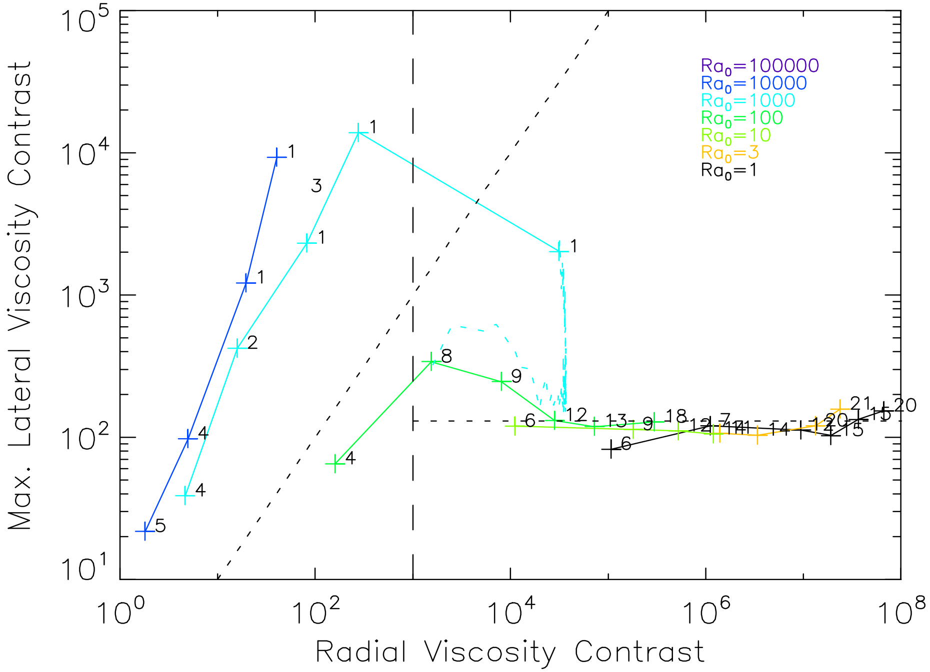
- Calculated the same way as isoviscous Ra / T
- Layer thickness used as d_{eff}
- Range similar to Deschamps & Sotin 2000
- Constant for time dependent convection with ~1193
- Unclear for P-dependence



Lateral viscosity contrast

- Approximately equal to radial contrast for mobile regime on T-dep. cases
- For T&P dep. cases, in the mobile regime, increased by the amount of $\Delta\eta_P$
- For stagnant lid cases constant: T-dep. cases ~30, T&P-dep. cases ~100





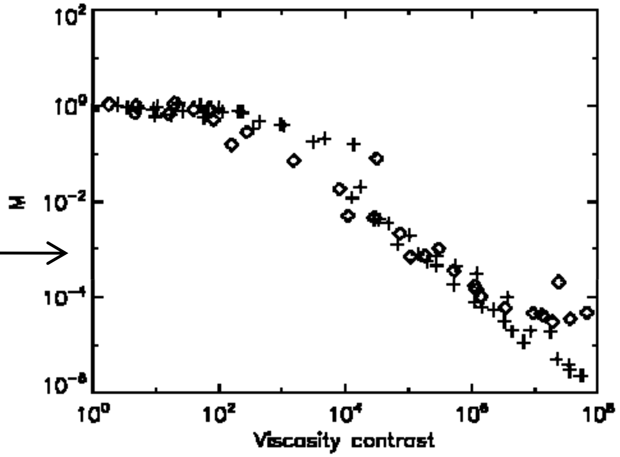
Transition to the stagnant lid regime

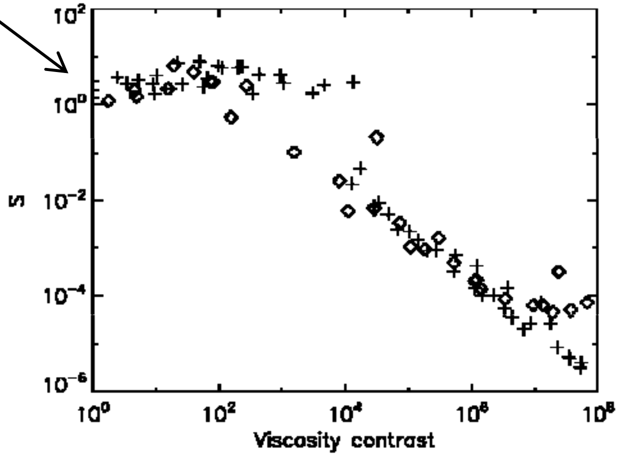
- Mobility criterions:

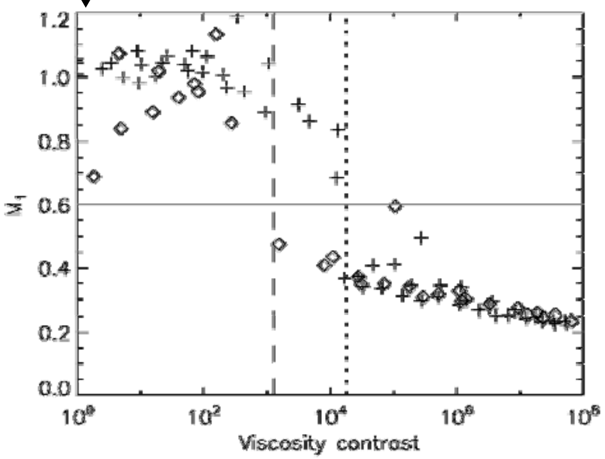
$$M_1 = \delta_{rh}\ Nu$$

$$M = v_{top} / v_{rms}$$

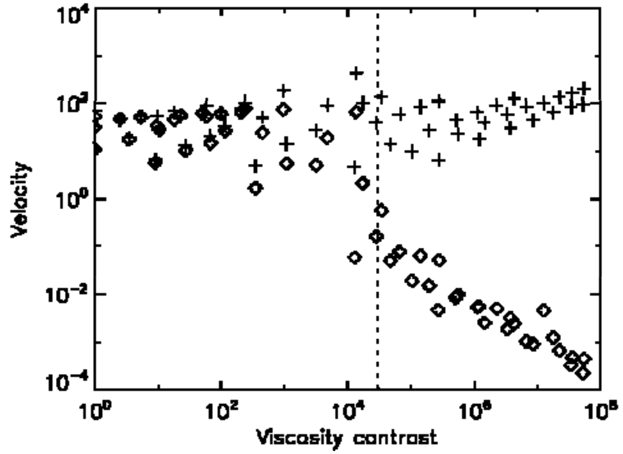
$$S = \frac{\delta_{rh}^2\ v_{top}}{\kappa d}$$



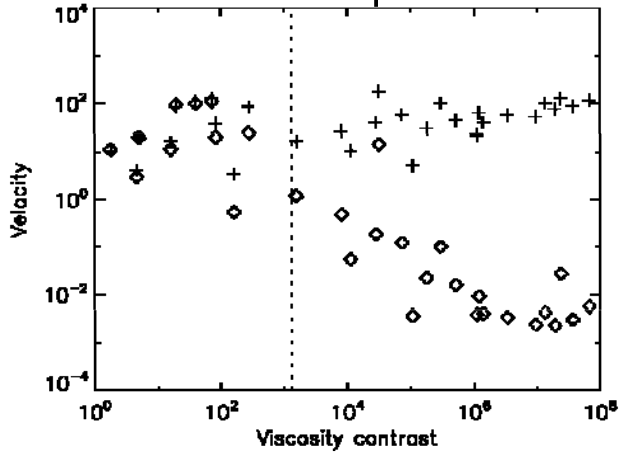




T-dep. cases



T & P-dep. cases



Direct heat flow profile reconstruction

- Regularized Gamma function *P* reconstructs heat flow profile

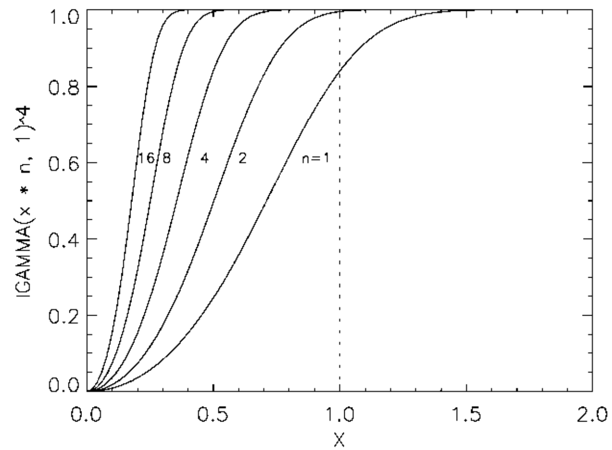
$$q(d) = q_c(d)\ \left(1 - P(\zeta\ d^{\epsilon}, 1)^4\right)$$

- Independent of regime or pressure dependent viscosity

- Only 2.5 parameters required to reconstruct complete HF and T profile

Incomplete (regularized) Gamma function

- Jump function, like error function erf

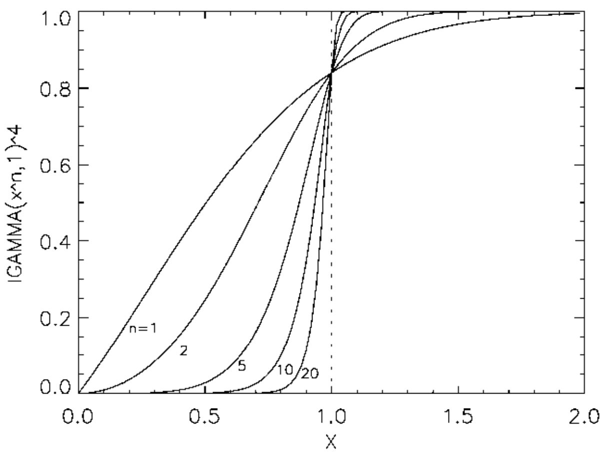


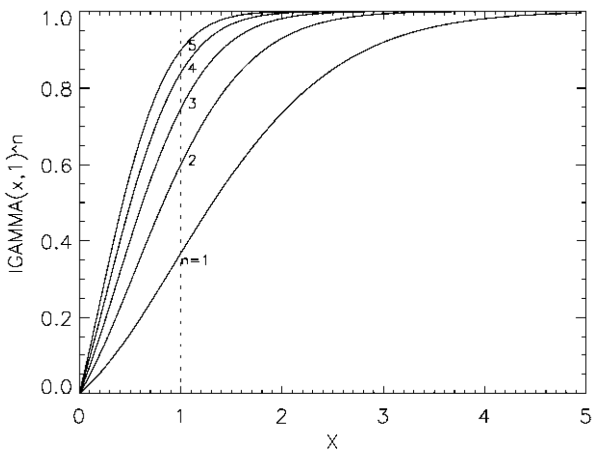
$$\Gamma(a) = \int_0^\infty t^{a-1}\, e^{-t}\, dt$$

$$\gamma(a,x) = \int_0^x t^{a-1}\, e^{-t}\, dt$$

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

$$\operatorname{erf} z = \pi^{-\frac{1}{2}}\,\gamma\left(\frac{1}{2}, z^2\right)$$



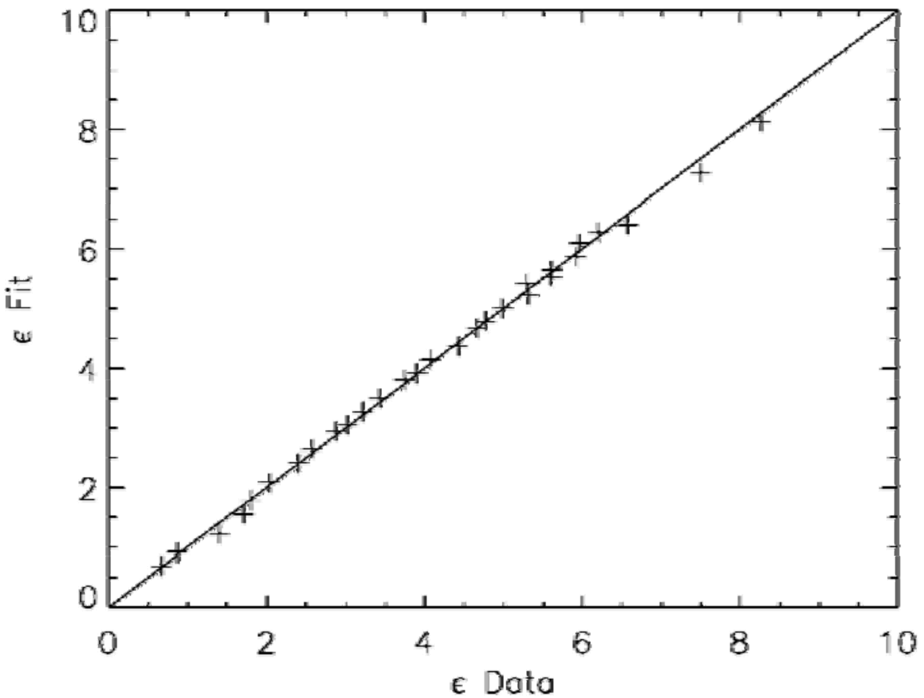


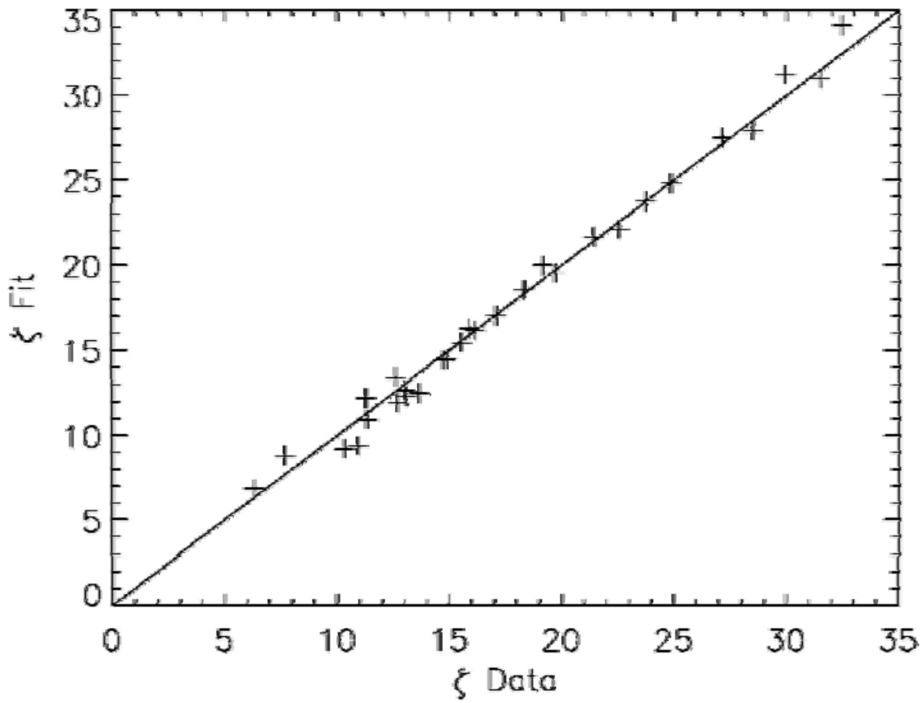
Indirect fit

- Scale ε and ζ from real input values Ra0 & γ
- Unfortunately different regimes lead again to different parameters – no „Unification“

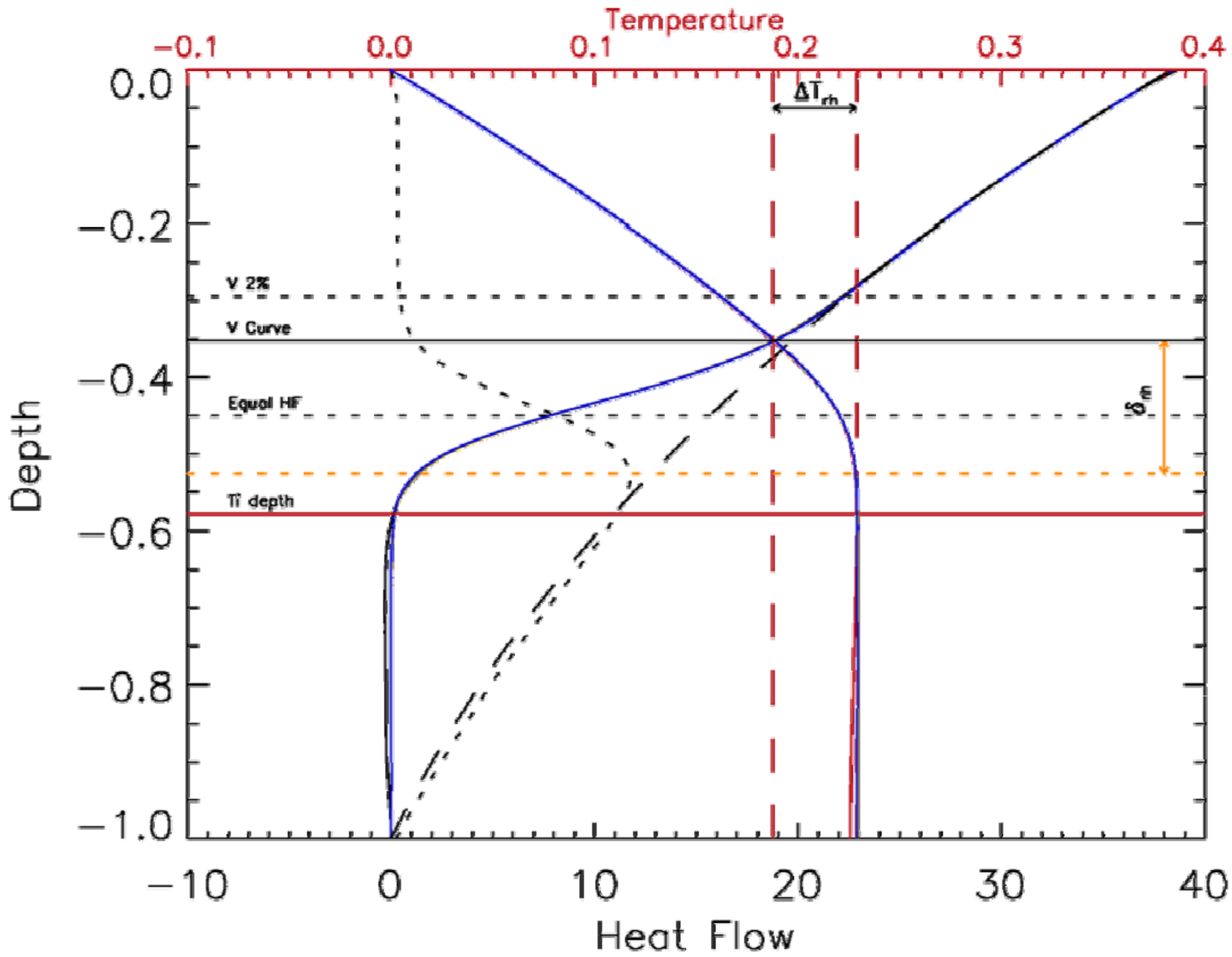
$$\epsilon = 0.864\,\gamma + 0.277\ \ln Ra_0 - 2.54$$

$$\zeta = 4.33\,\epsilon\,Ra_0^{-\frac{1}{6}} + 4.8$$

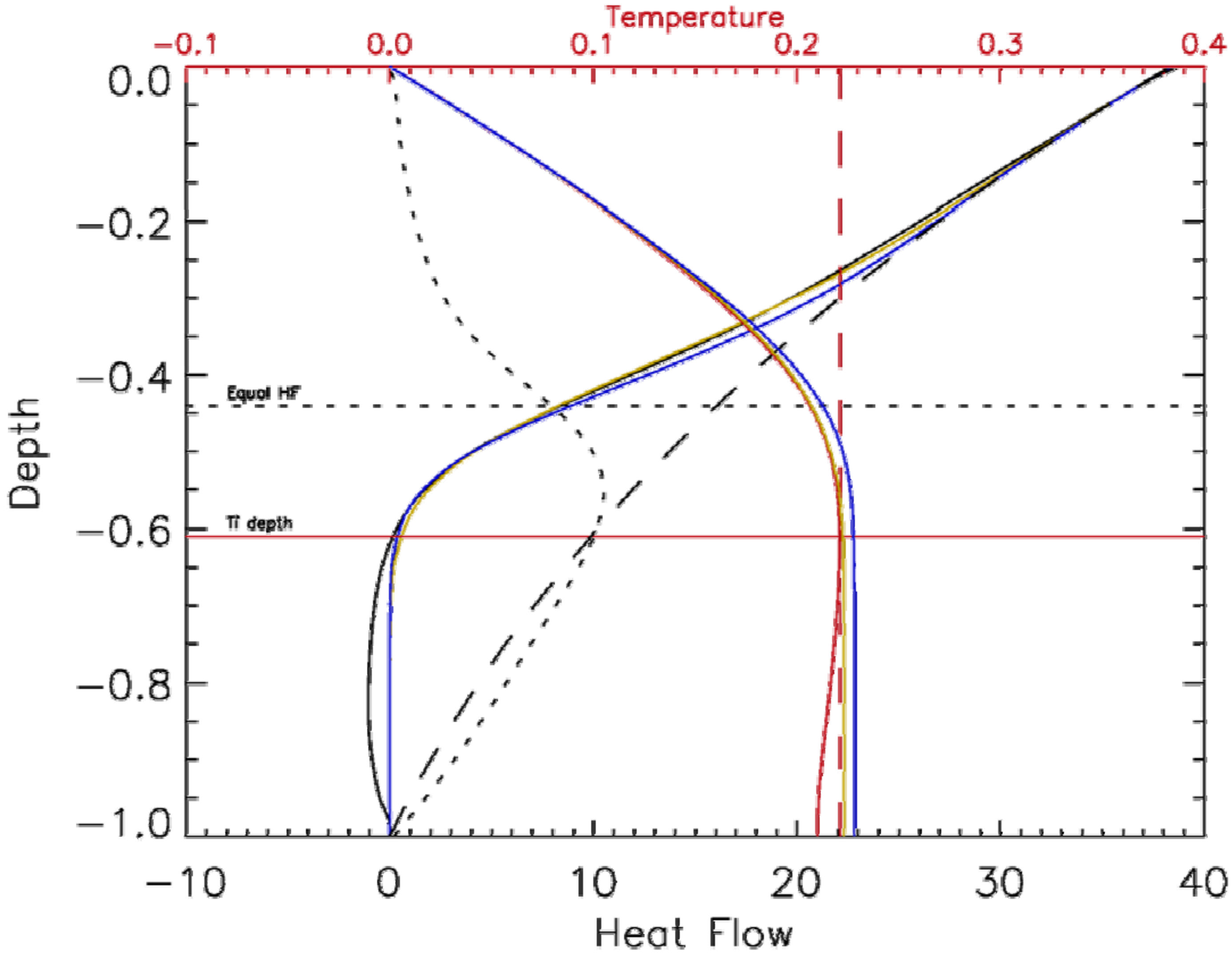




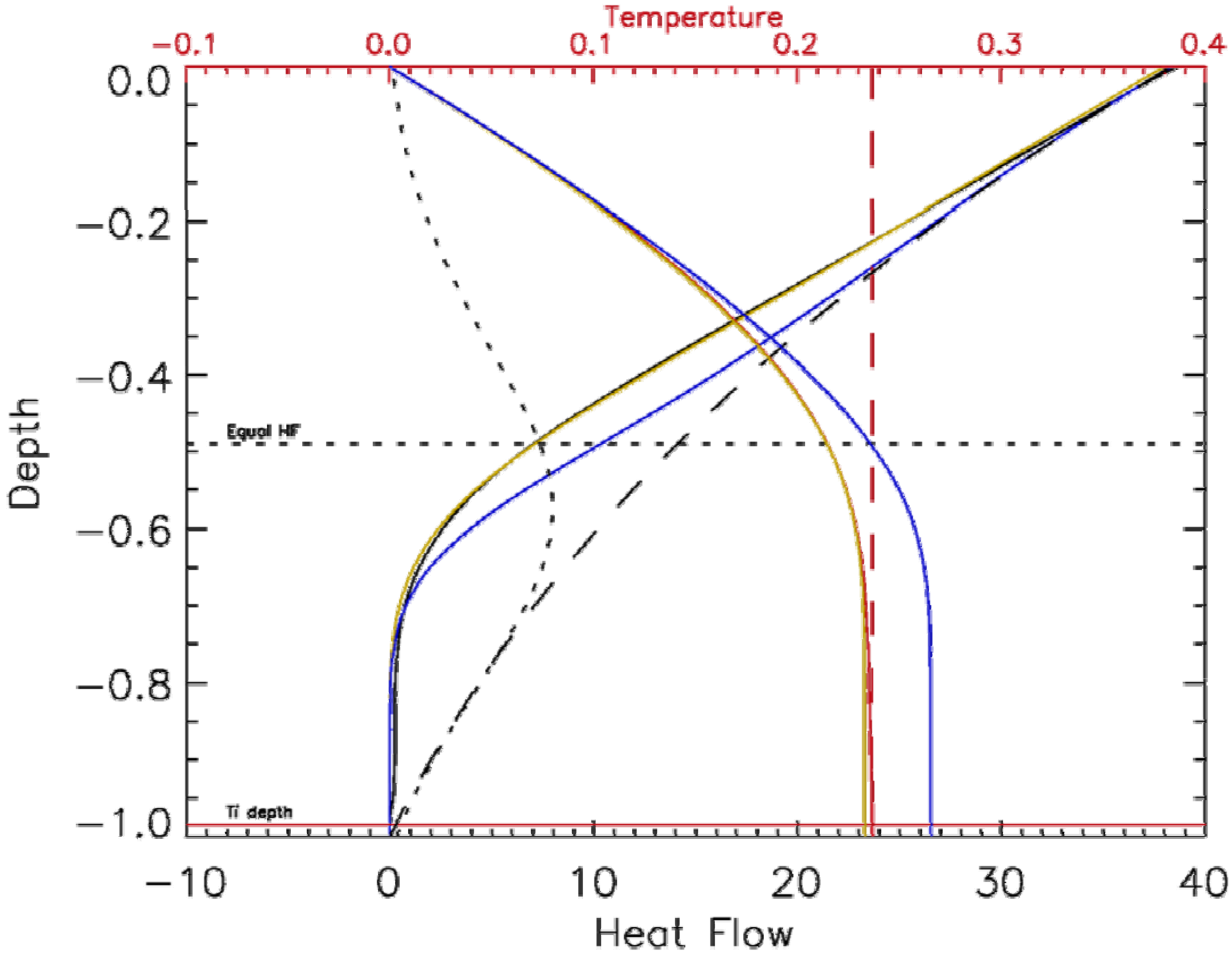
HFR examples



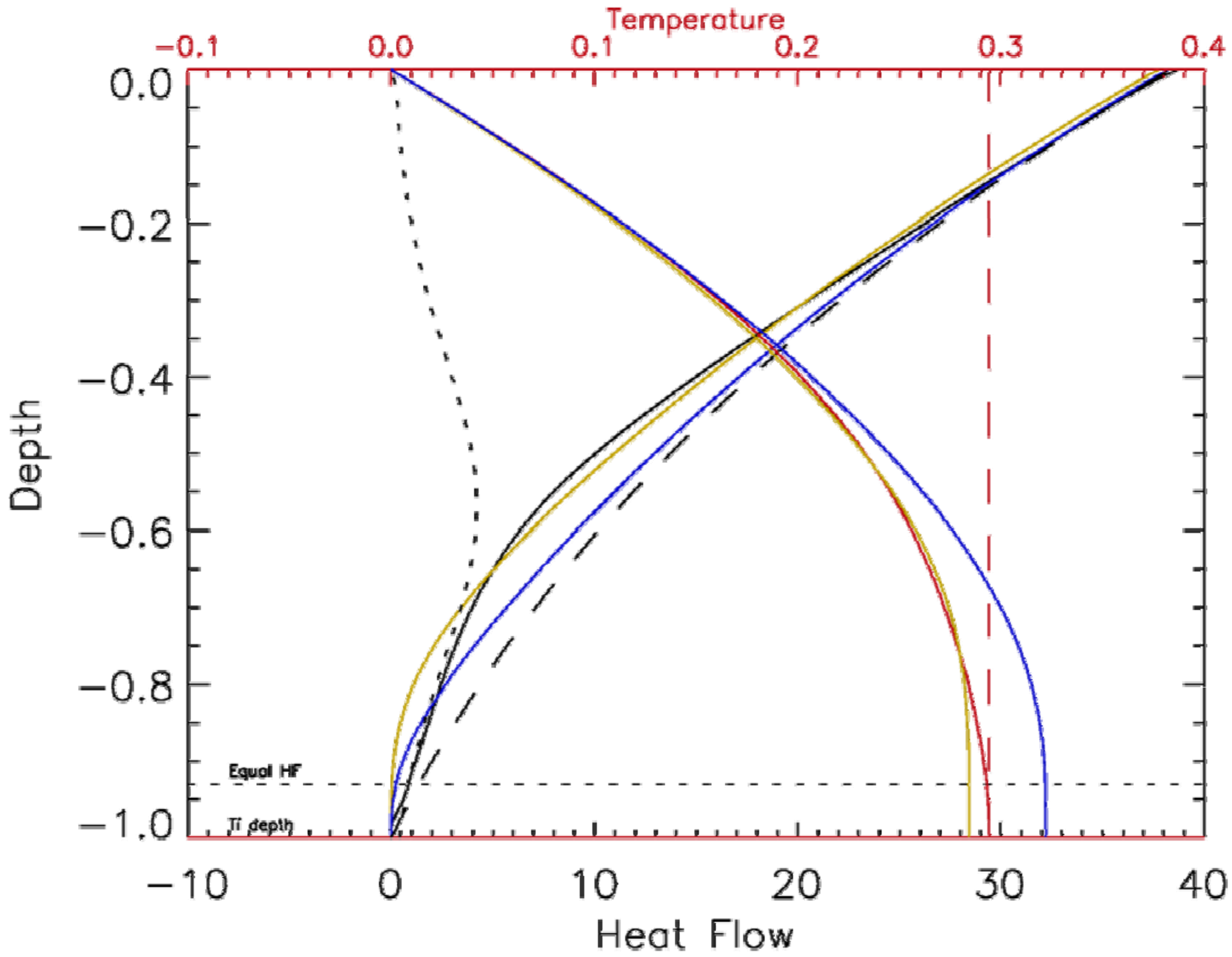
HFR expamples



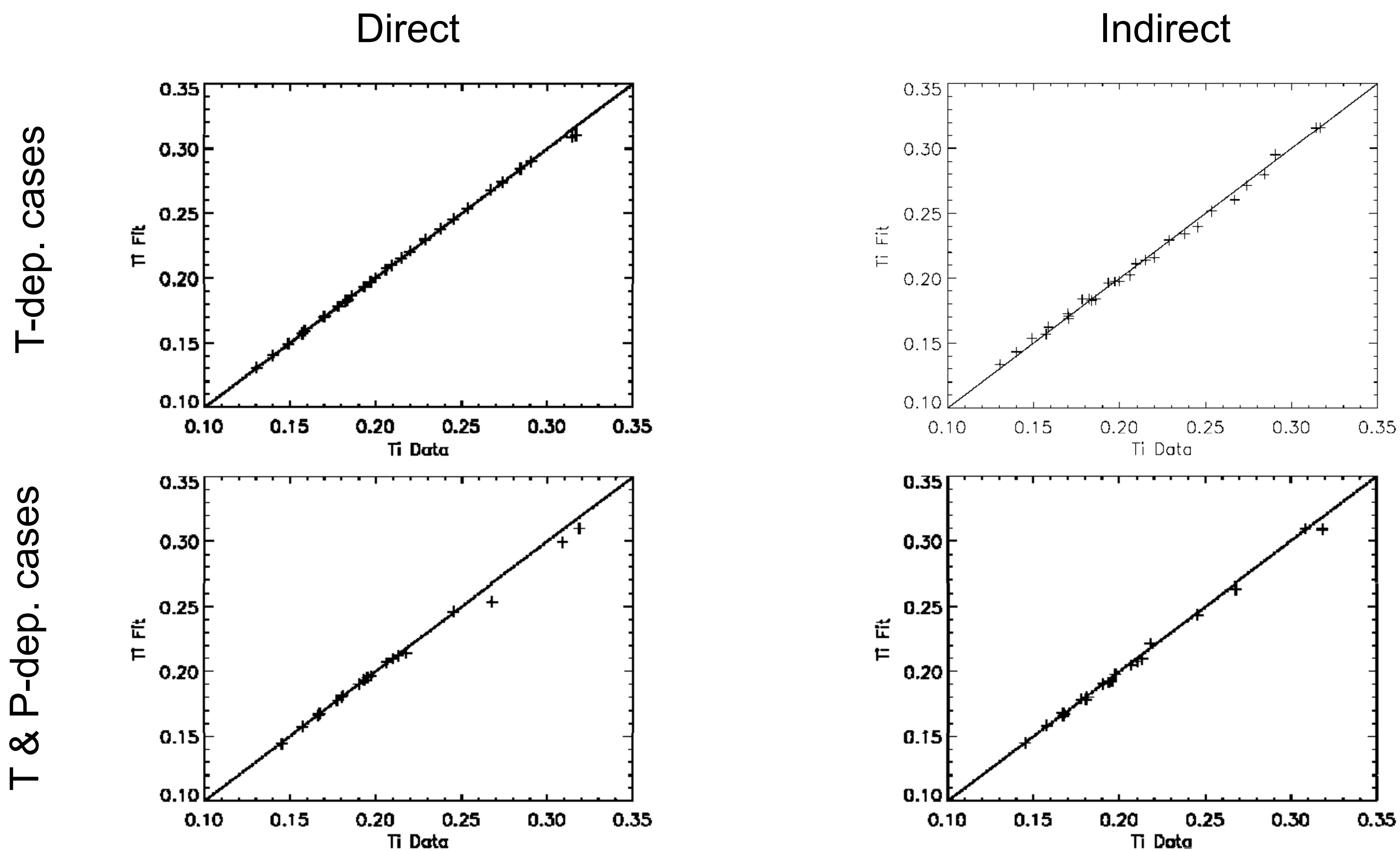
HFR expamples – TP mobile case

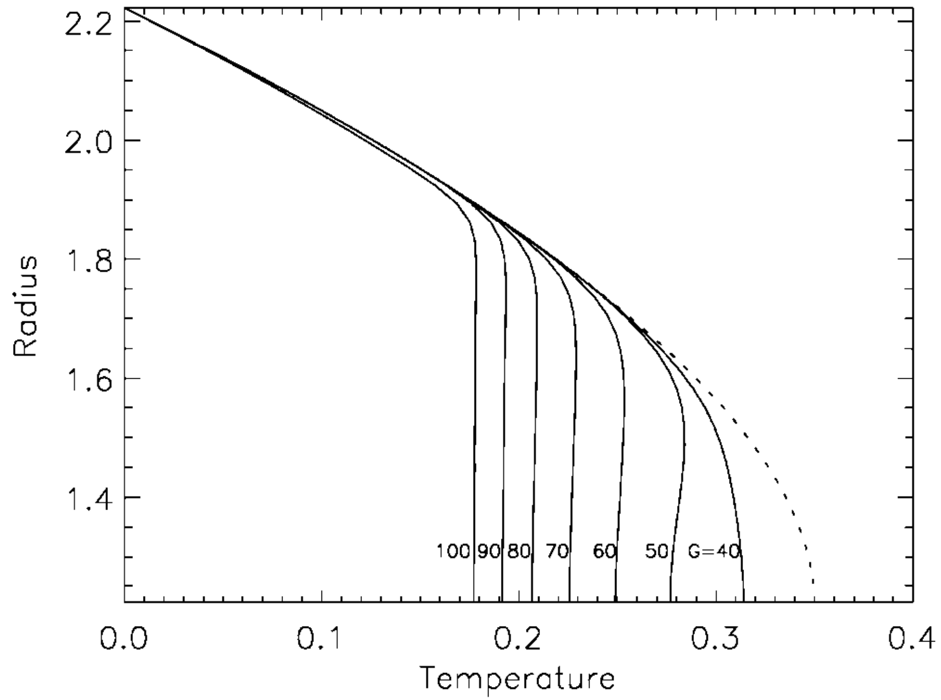


HFR examples: complete failure on weak convection

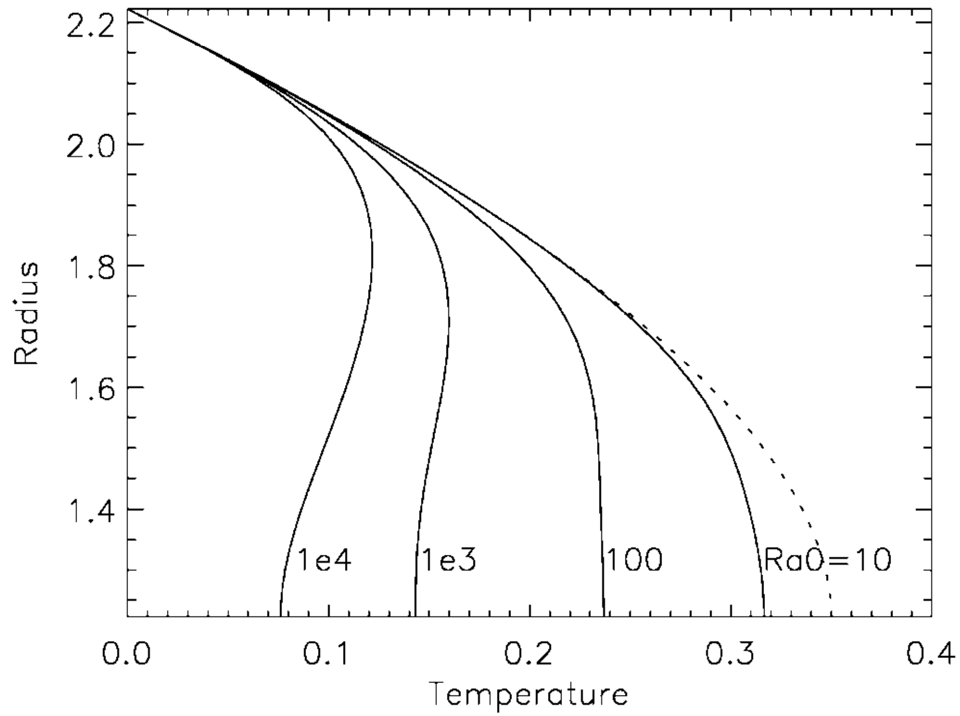


Fit of internal temperature with HFR method





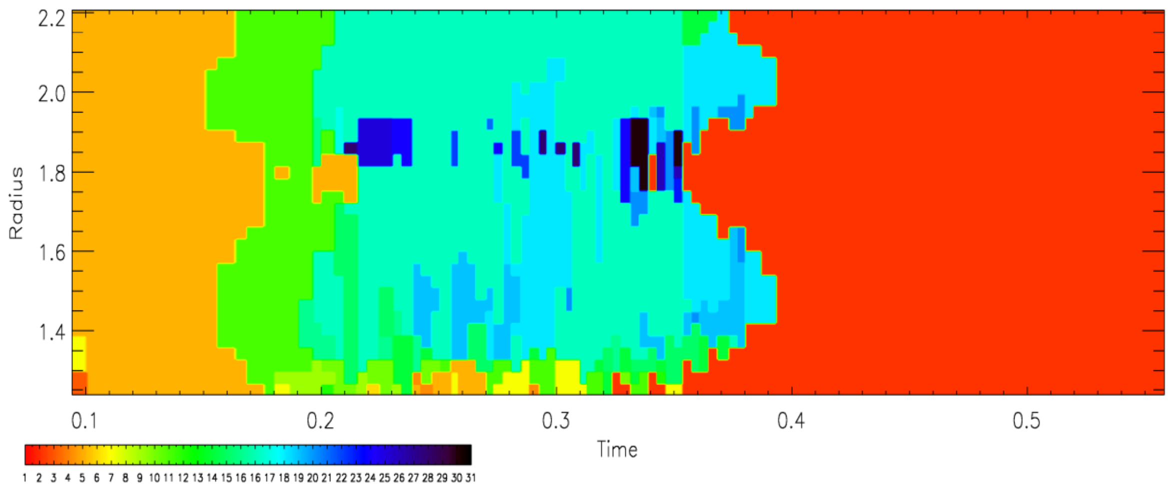
2.2
2.0
1.8
1.6
1.4
Radius
100 90 80 70 60 50 G=40
0.0
0.1
0.2
0.3
0.4
Temperature



2.2
2.0
1.8
1.6
1.4
Radius
1e4
1e3
100
Ra0=10
0.0
0.1
0.2
0.3
0.4
Temperature

Anomalies

- One case with too high viscosity contrast and without a stagnant lid
- Confirmed in 2D with high resolution
- Ra0=1000, γ=100, $\Delta\eta_P$=100



The Frank-Kamenetskii approximation

- From chemical combustion theory: approximates Arrhenius laws to linear exponential laws
- Mixed definition of the DAFK parameter, sometimes just γ and sometimes γ*Ti

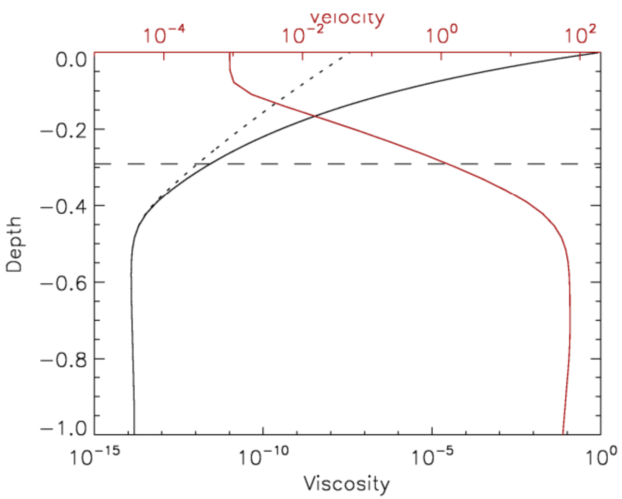
Arrhenius

$$\eta = \eta_{ref}\,\exp\left(\frac{A}{T + T_0}\right)$$

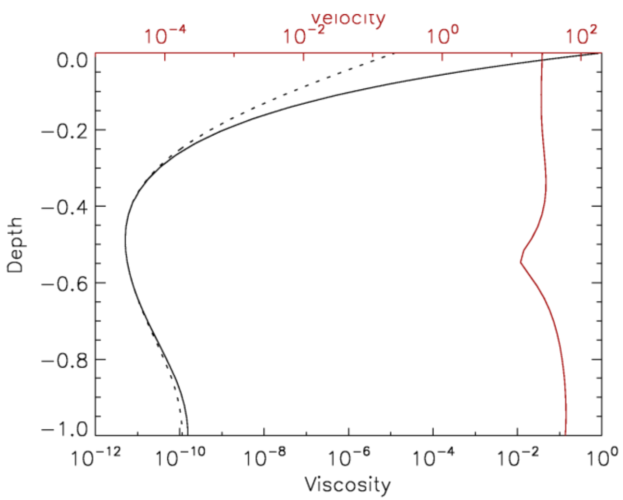
Linearised

$$\eta = \frac{\mu}{\mu_0} = \eta_S\,\exp(-\gamma\,T)$$

Stagnant lid case



Degree-One case



Reconstruction from simulated T profile: Dotted line from linear, black line Arrhenius

OK for SL cases, others?