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Mantle convection modeling with depth-dependent viscosity – Effects on the convective planform and plume behavior

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Motivation

- Plume debate: spatial and temporal dynamics of mantle plumes
- Influenced by rheology of mantle material
  - Dependence of the viscosity on temperature and pressure
- Depth-dependent viscosity increases the wavelength of mantle flow
- Greater viscosity contrast due to pressure than previously expected (Forte and Mitrovica, 2001)
- High viscosity at the CMB: to high for instabilities to trigger plumes?

Basic equations

- Equation of State

$$\rho = \rho_0 \left(1 - \alpha \left(T - T_0\right)\right)$$

- Equation Heat Transport

$$\frac{\partial T}{\partial t} + \vec{\nabla}\left(\vec{v}T\right) - \vec{\nabla}^2 T = 0$$

- Continuity Equation

$$\vec{\nabla} \cdot \vec{v} = 0$$

- Equation of Motion

$$-\vec{\nabla} p + \vec{\nabla} \underline{\underline{\sigma}} + Ra \cdot T \vec{e}_z = 0$$

ρ: density	p: pressure
α: thermal expansivity coefficient	$\underline{\underline{\sigma}}$: stress tensor
T: temperature	Ra: Rayleigh number
$\vec{v}$: velocity	

Equation of Motion

$$-\vec{\nabla} p + \vec{\nabla} \underline{\underline{\sigma}} + Ra \cdot T \vec{e}_z = 0$$

- Rayleigh number

$$Ra = \frac{\alpha g \Delta T d^3}{\kappa \nu}$$

α: thermal expansivity coefficient

g: gravitational acceleration

T: temperature κ: thermal diffusivity

d: box thickness ν: kinematic viscosity

Equation of Motion

$$-\vec{\nabla} p + \vec{\nabla}\,\underline{\underline{\sigma}} + Ra \cdot T \vec{e}_z = 0$$

- Rayleigh number

$$Ra = \frac{\alpha g \Delta T d^3}{\kappa \nu}$$

- Stress tensor

$$\underline{\underline{\sigma}} = \eta\left(\vec{\nabla}\vec{v} + \left(\vec{\nabla}\vec{v}\right)^T\right)$$

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- Stress tensor

$$\underline{\underline{\sigma}} = \eta \left(\vec{\nabla} \vec{v} + \left(\vec{\nabla} \vec{v} \right)^T \right)$$

- Viscosity (Arrhenius law)

$$\eta(p,T) =: \eta_{p,T} = C \cdot \exp\left[\frac{E + pV}{\tilde{R} T} \right]$$

Considered parameter: depth-dependent viscosity

$$\eta\left(p,T\right)=:\eta_{p,T}=C\cdot\exp\left[\frac{E+pV}{\tilde{R}T}\right]$$

- Frank-Kamenetskii simplification:

$$\eta_{p,T}=\exp\left[-rT+c\cdot\frac{(1-z)}{d}\right]$$

$$r=\ln\left(\Delta\eta_T\right),\ \Delta\eta_T=1$$

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$$\eta_{p,T} = \exp\left[-rT + c \cdot \frac{(1-z)}{d}\right]$$

$$r = \ln(\Delta\eta_T),\ \Delta\eta_T = 1$$

- Depth-dependent viscosity:

$$\eta_p = \exp\left[c \cdot \frac{(1-z)}{d}\right]$$

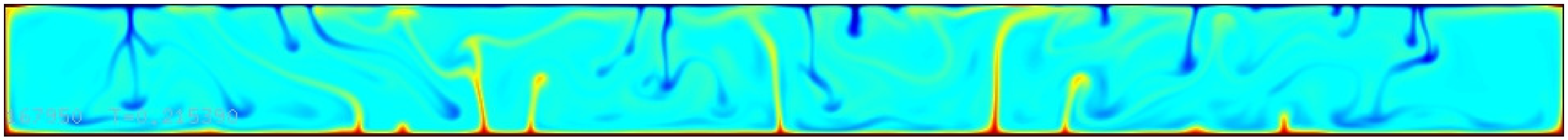
$$c = \ln(\Delta\eta_p)$$

Numerical method

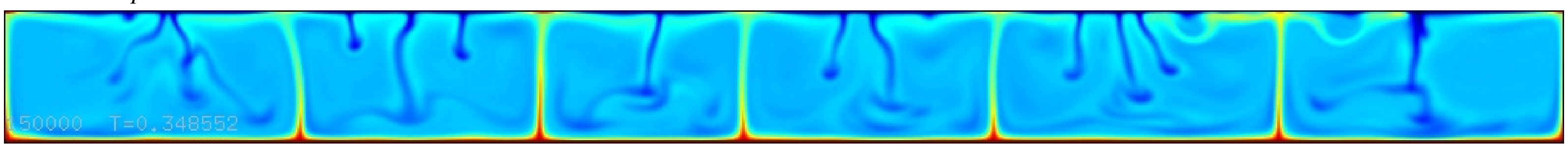
- 2D model of mantle convection (Trompert and Hansen, 1996)
  - based on finite volume discretization, implizit procedure, multigrid procedure
- Aspect ratio of 12 (1024 x 128), grid refinement in z-direction
- Boundary conditions:
  - temperature: Dirichlet for top and bottom boundary, Neuman for side walls
  - Velocity: stress-free
- Surface Rayleigh number $Ra_{surf} = 10^7$
- $\Delta\eta_p$ is increased from 3 to 1000

Results: qualitative analysis

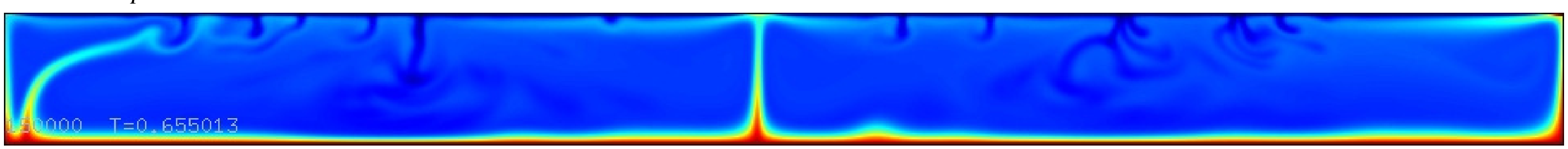
$\Delta\eta_p = 3$



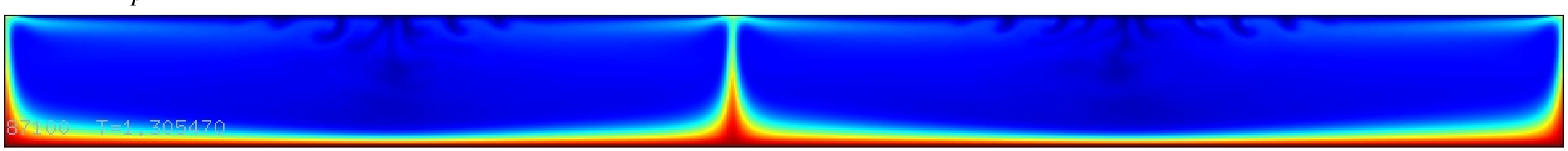
$\Delta\eta_p = 10$

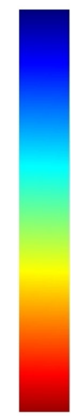


$\Delta\eta_p = 100$

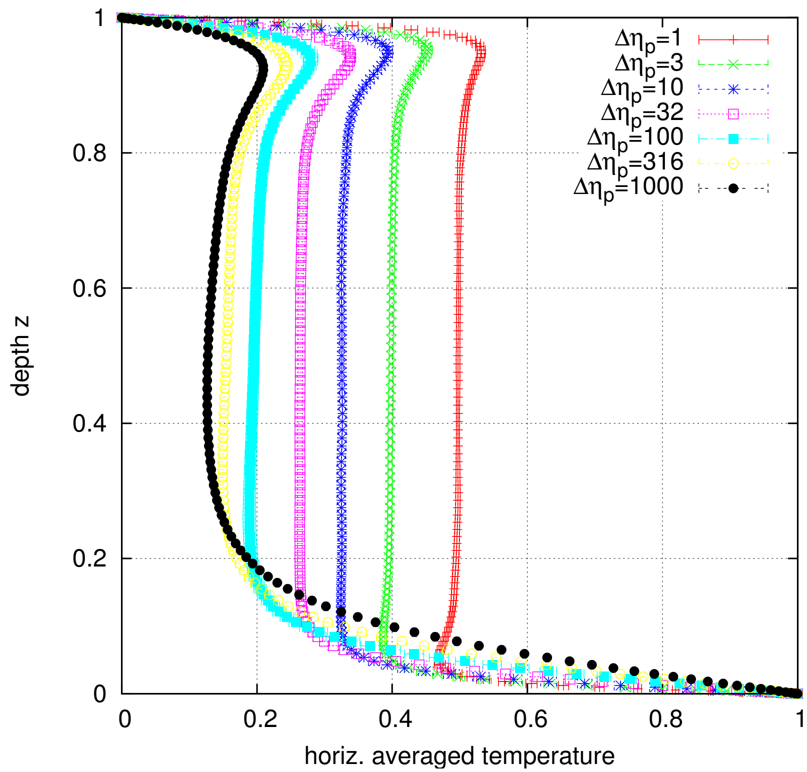


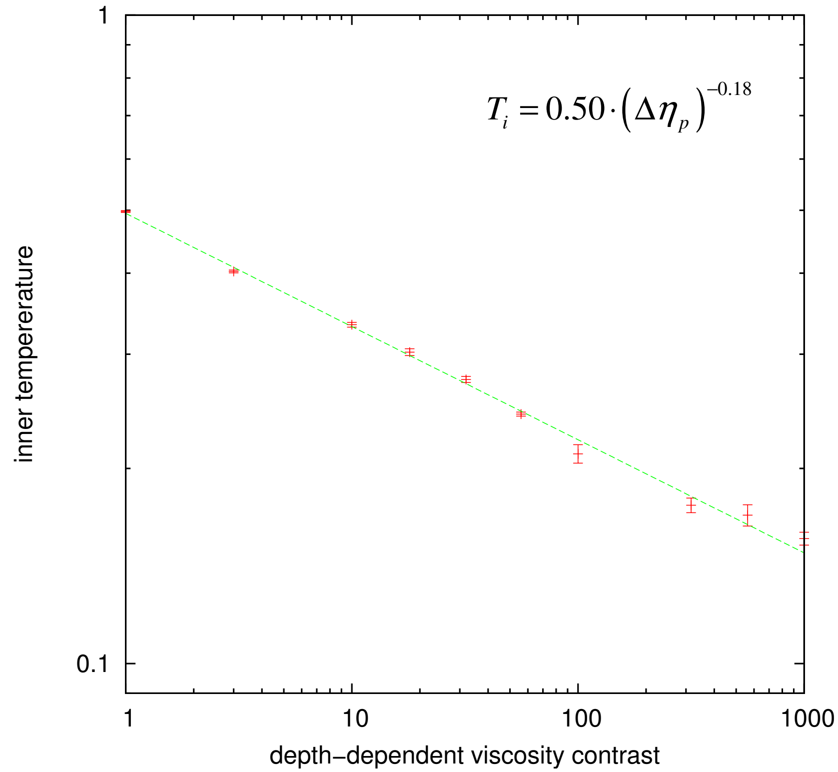
$\Delta\eta_p = 1000$





Temperature profiles





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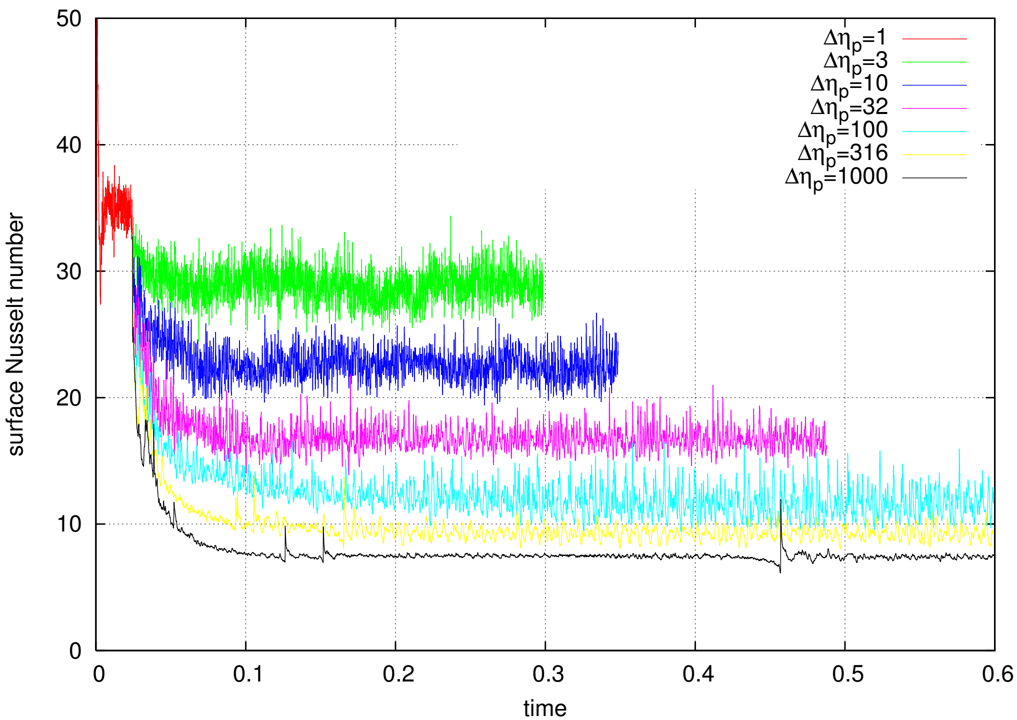
Surface Nusselt number



Nu: measure of the efficiency of heat transport

$$Nu = \frac{q_{total}}{q_*}$$

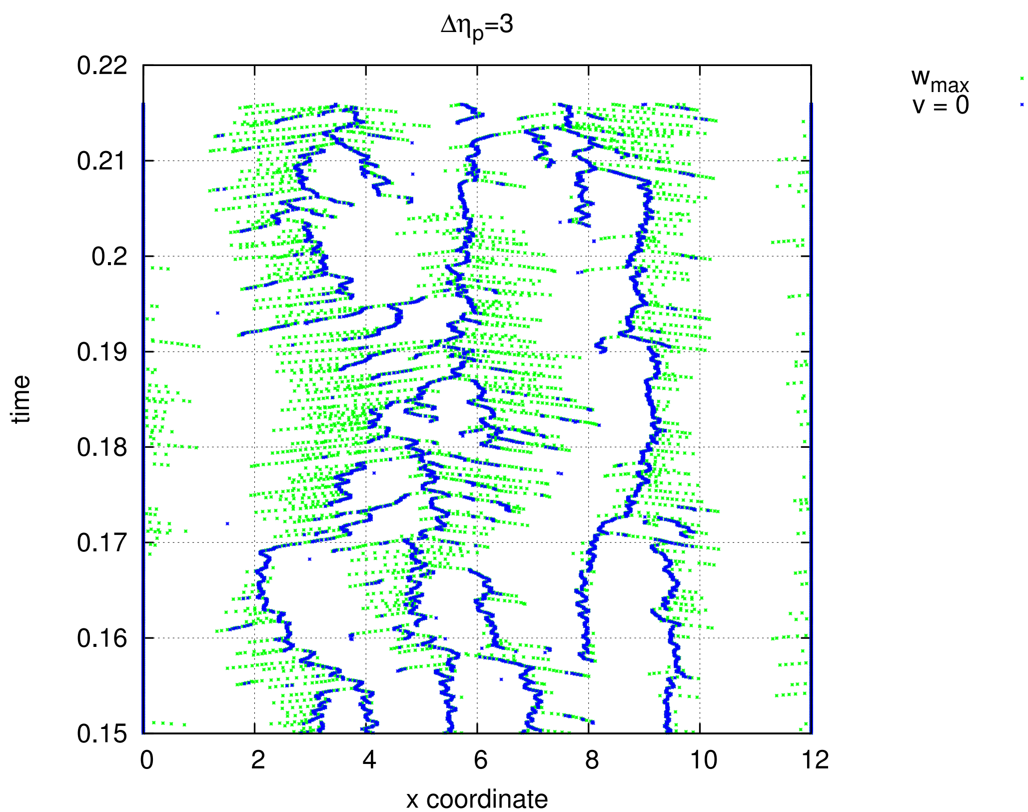
$$q_{total} = q_{cond} + q_{conv}$$

$$q_* = \left(\frac{k \Delta T}{d} \right)$$

Surface Nusselt number:

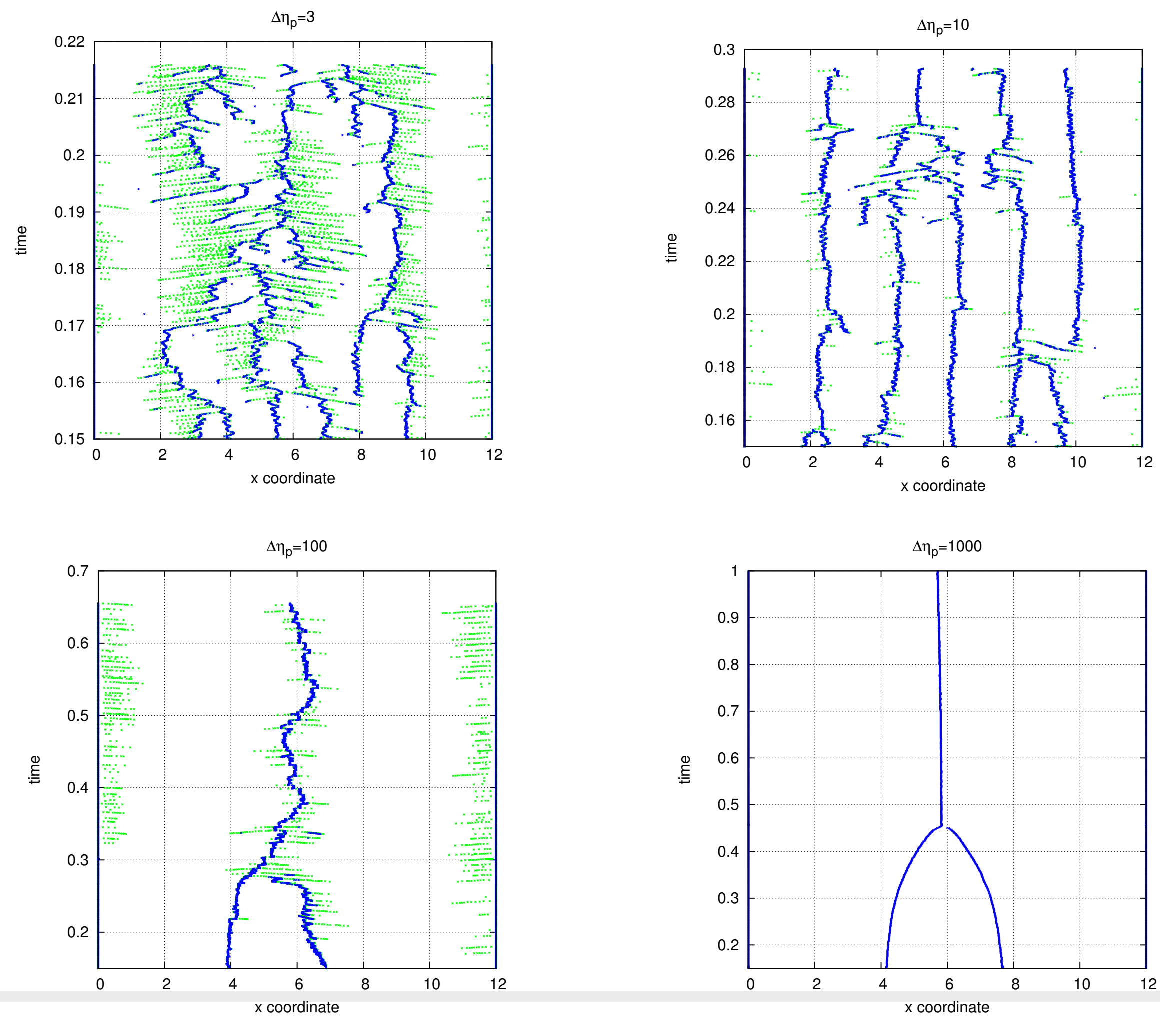
averaged heat flux through surface in z-direction

Nodal lines



$\Delta\eta_p$=3
time
x coordinate
0.22
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0
2
4
6
8
10
12
w_{max}
v = 0

Nodal lines


w_{max}
v = 0
$\Delta\eta_p$=3
$\Delta\eta_p$=10
$\Delta\eta_p$=100
$\Delta\eta_p$=1000
time
x coordinate

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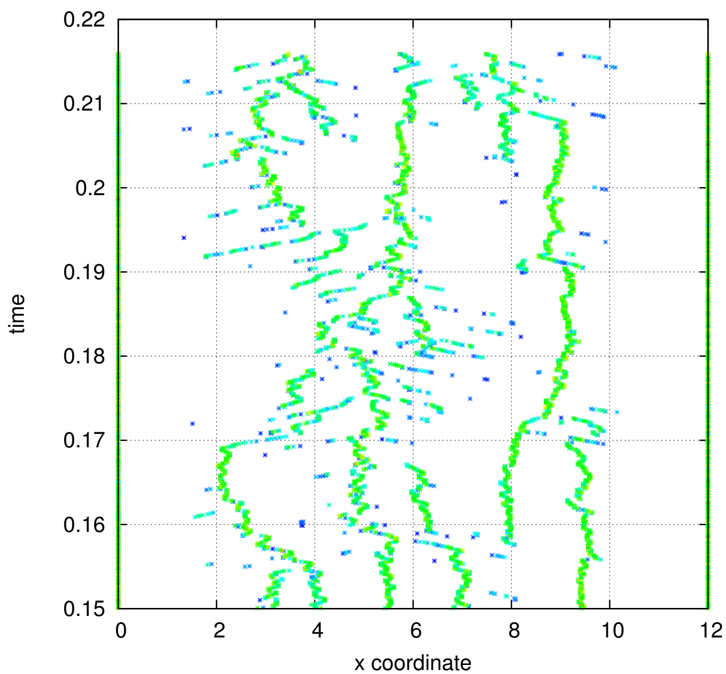
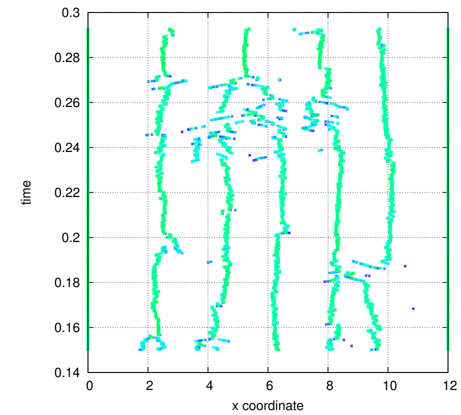
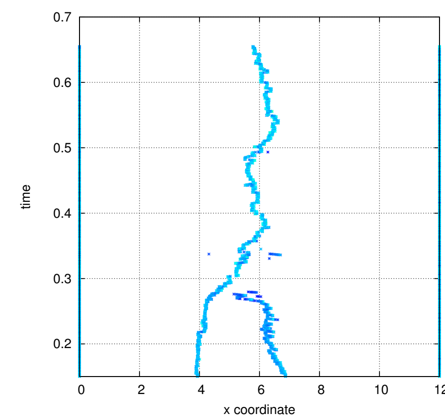
Vertical velocity component



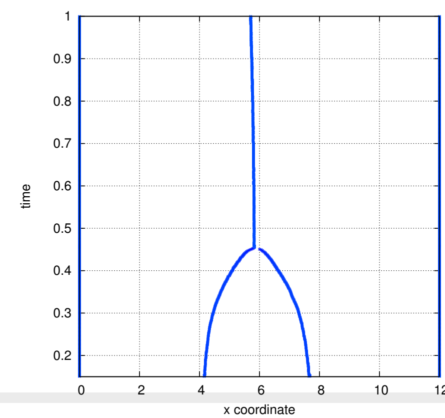
$\Delta\eta_p = 3$



$\Delta\eta_p = 10$



$\Delta\eta_p = 100$



$\Delta\eta_p = 1000$

Conclusion

With increasing viscosity contrast:

- Internal temperature and surface Nusselt number decreases
- Longer aspect ratio cells
- Buoyancy focused into a few upwellings
- Smaller instabilities travel within the boundary layer
- Upwellings become more stationary
- Roots of plume tails broaden
- Vertical velocity decreases

Outlook

- Have a closer look at velocity components
- Increase aspect ratio for great viscosity contrasts

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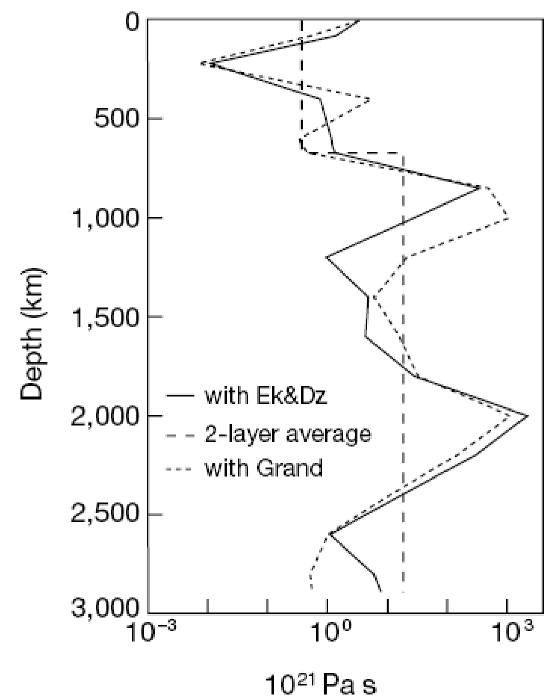
- Have a closer look at velocity components
- Increase aspect ratio for great viscosity contrasts

- Add temperature dependence of viscosity
  - Examine viscosity profiles



Taken from Forte and Mitrovica, 2001

References

- Forte, A.M. and J.X. Mitrovica, 2001. Deep-mantle high-viscosity flow and thermochemical structure inferred from seismic and geodynamic data, *Nature*, 410, 1049-1056.
- Trompert, R. and U. Hansen, 1996. The application of a finite volume multigrid method to three-dimensional flow problems in a highly viscous fluid with variable viscosity. *Geophys. Astrophys. Fluid Dynamics*, 83, 261-291.

Thank you very much for your attention!