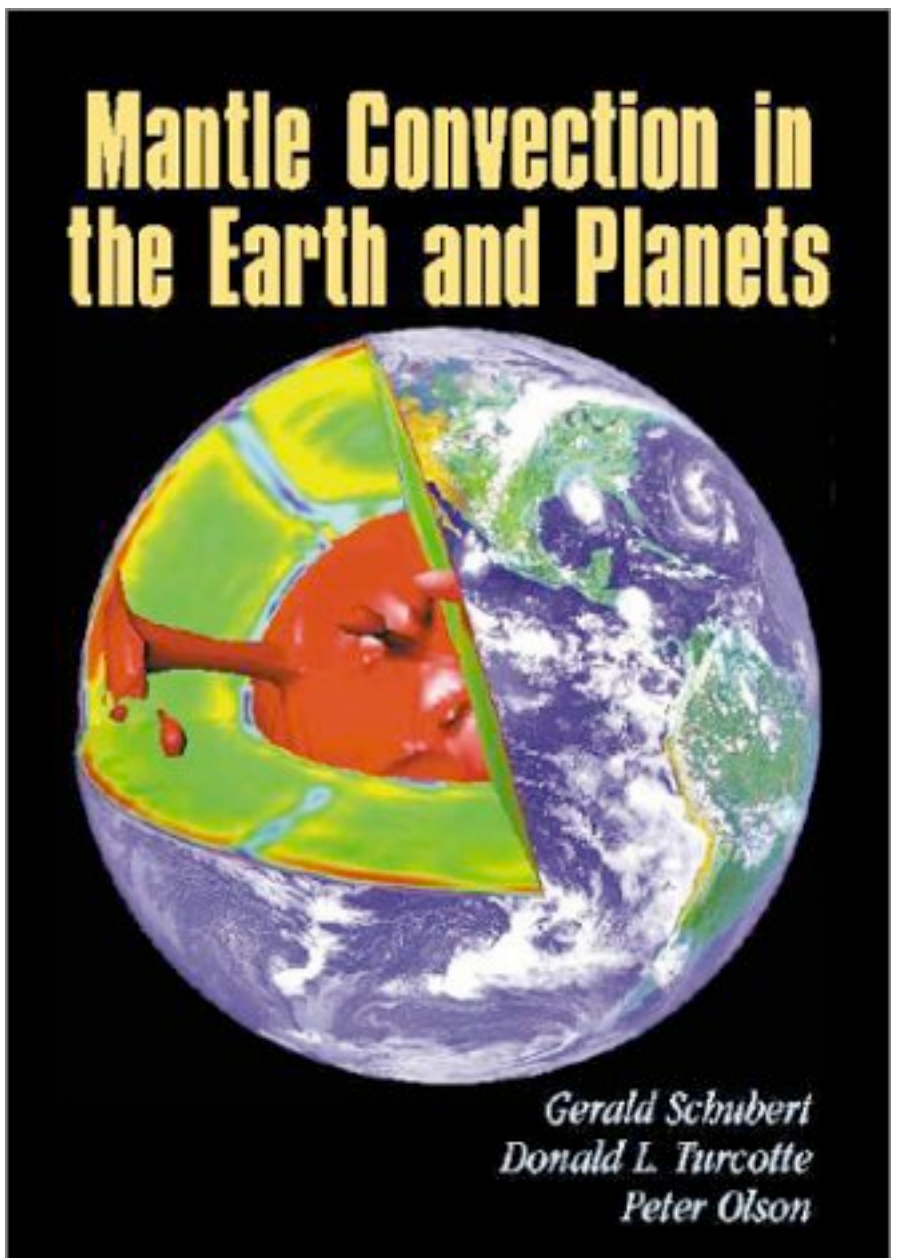
The dynamics of magma in a convecting mantle (a biased survey)

Richard F. Katz
Department of Earth Sciences, University of Oxford

with M. Spiegelman, P. Kelemen, B. Holtzman, C. Manning, M.G. Worster

Range of length-scales for mantle dynamics

Planetary scale
V e r y l o n g !



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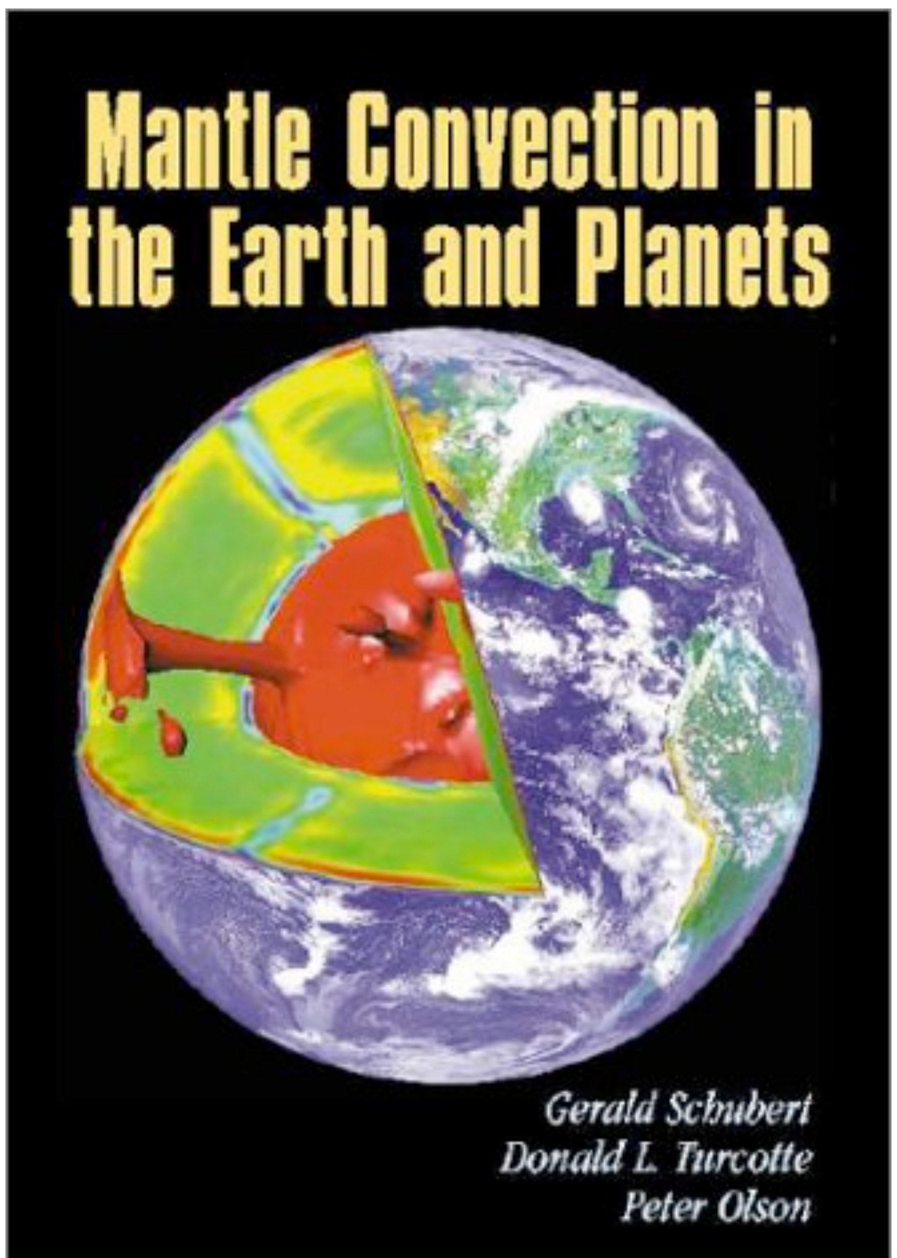
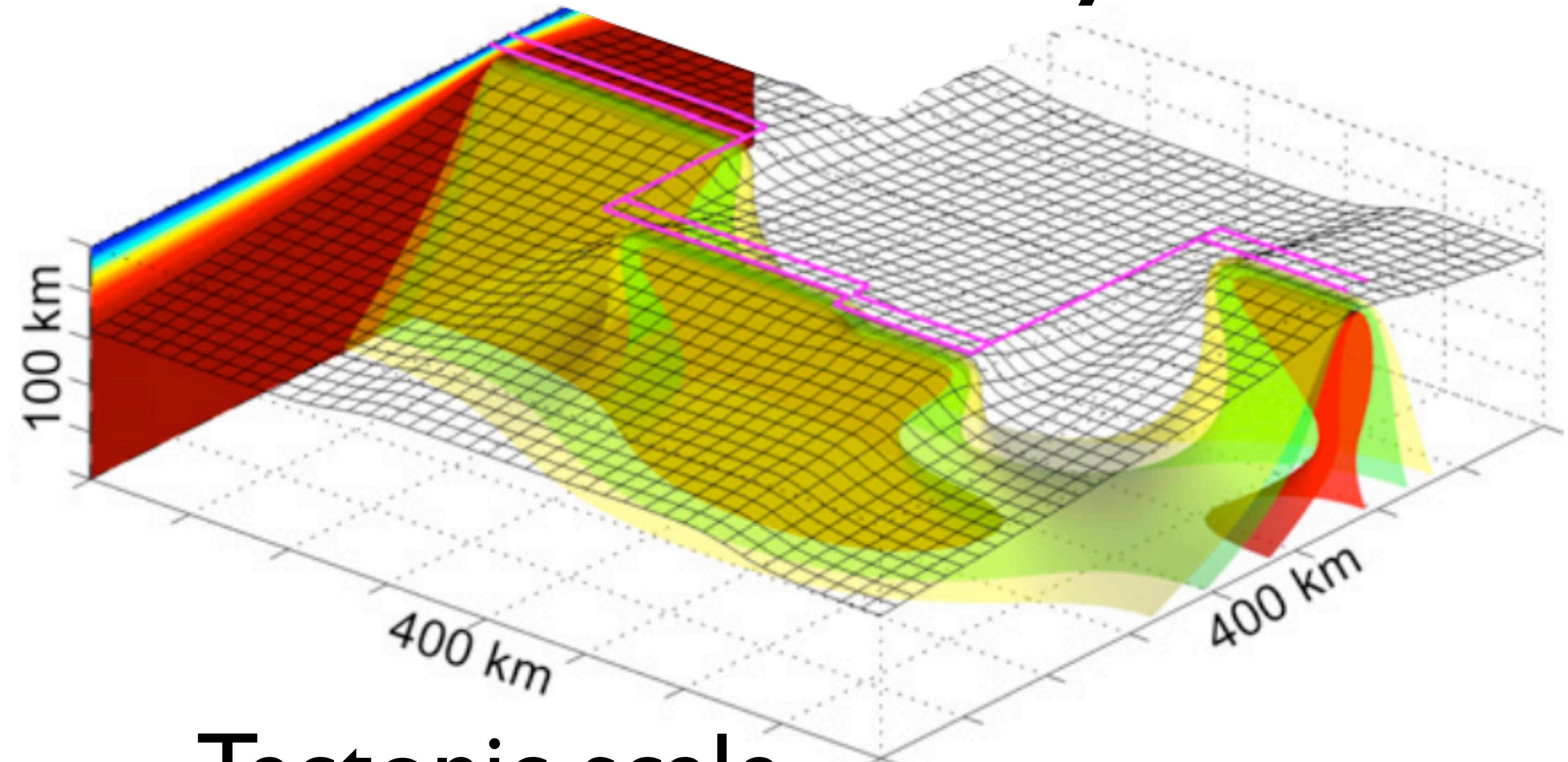
Very long!





Tectonic scale
~100-1000 km

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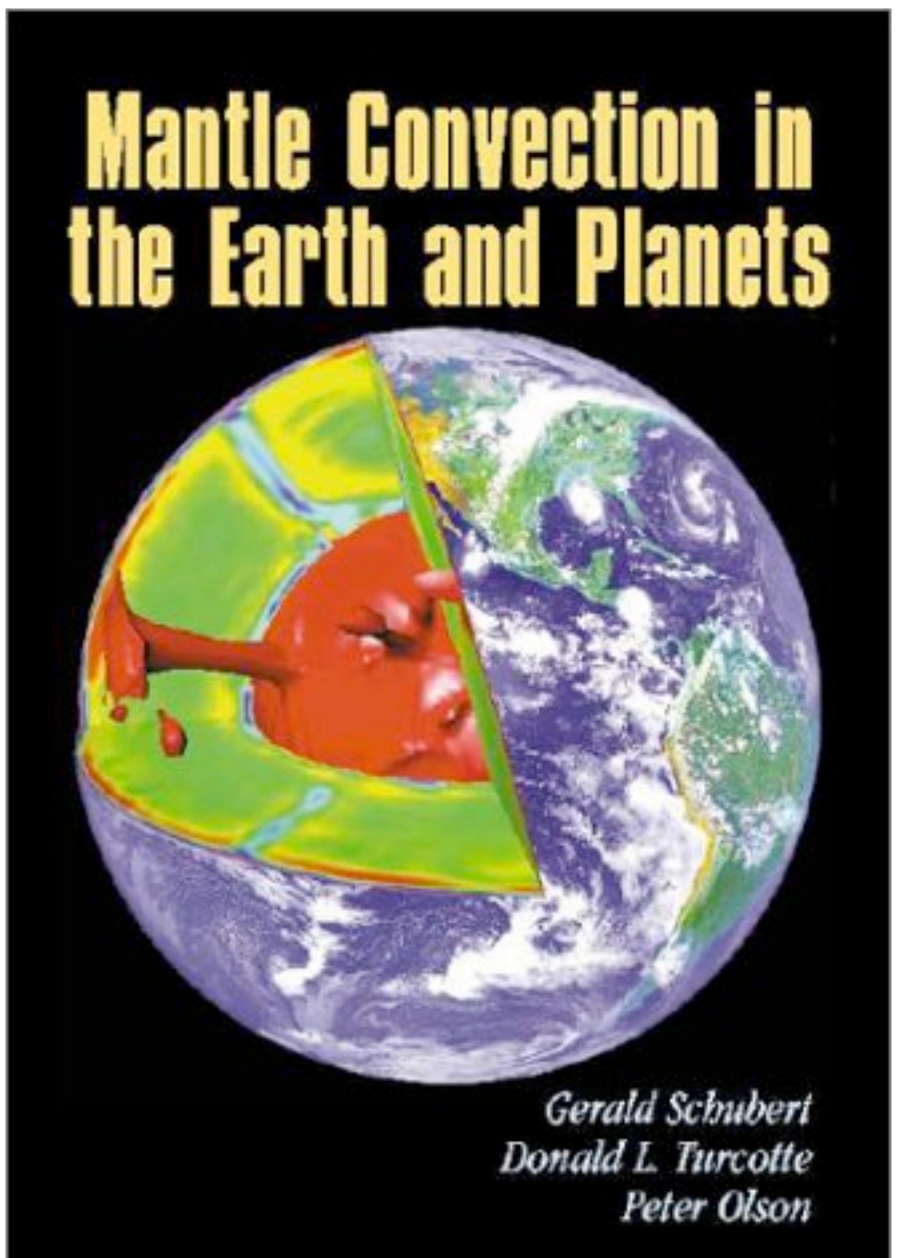
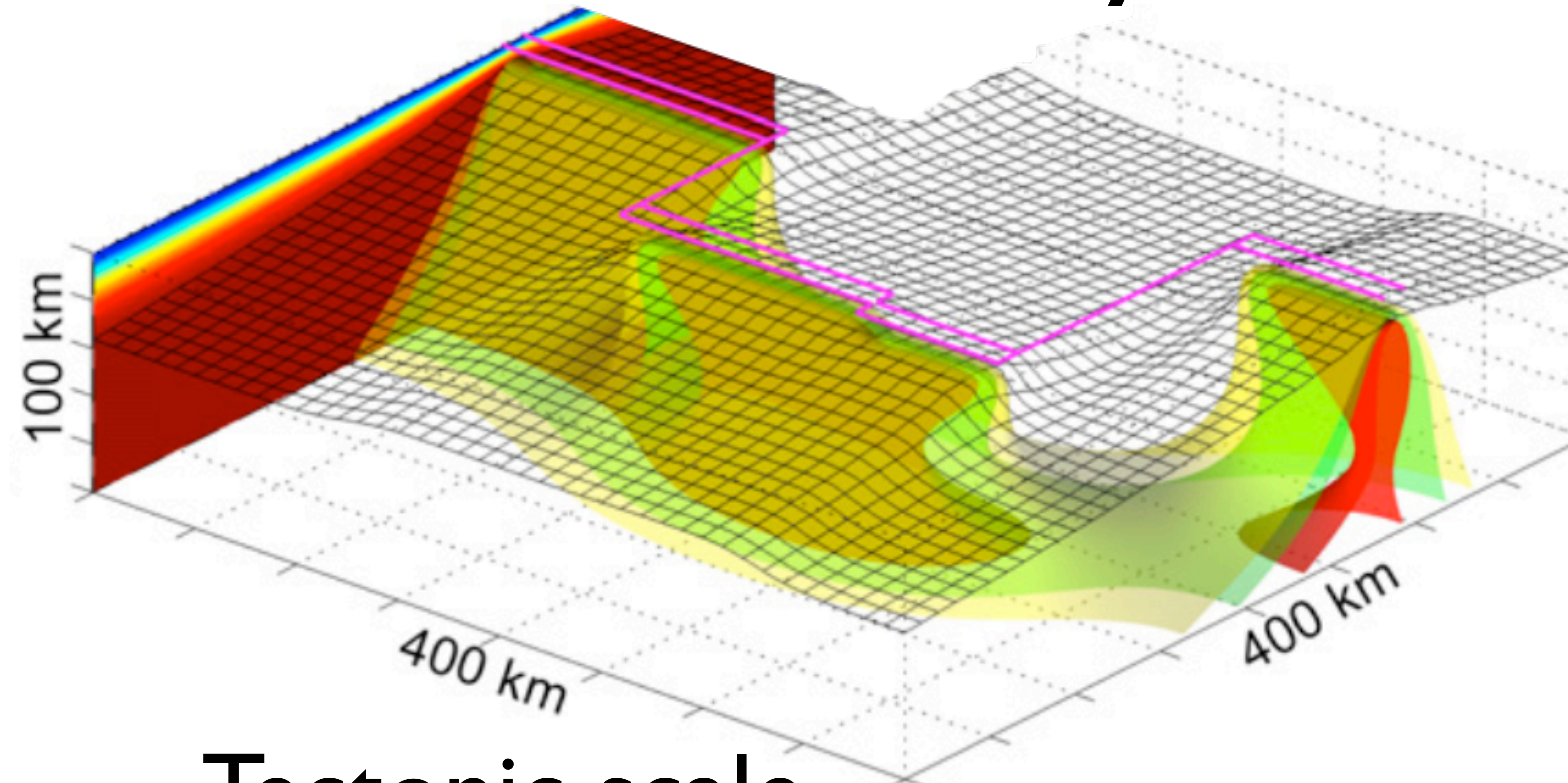
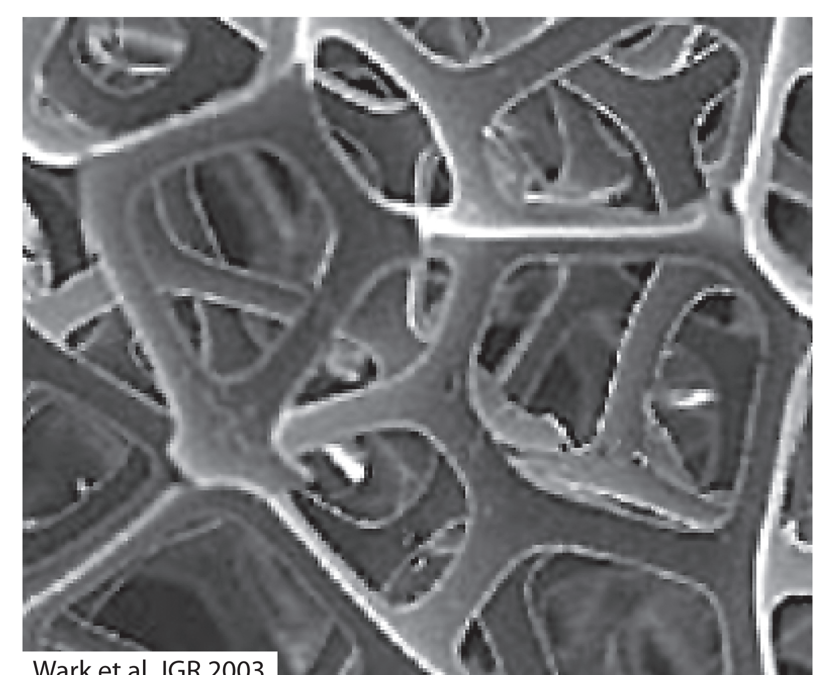




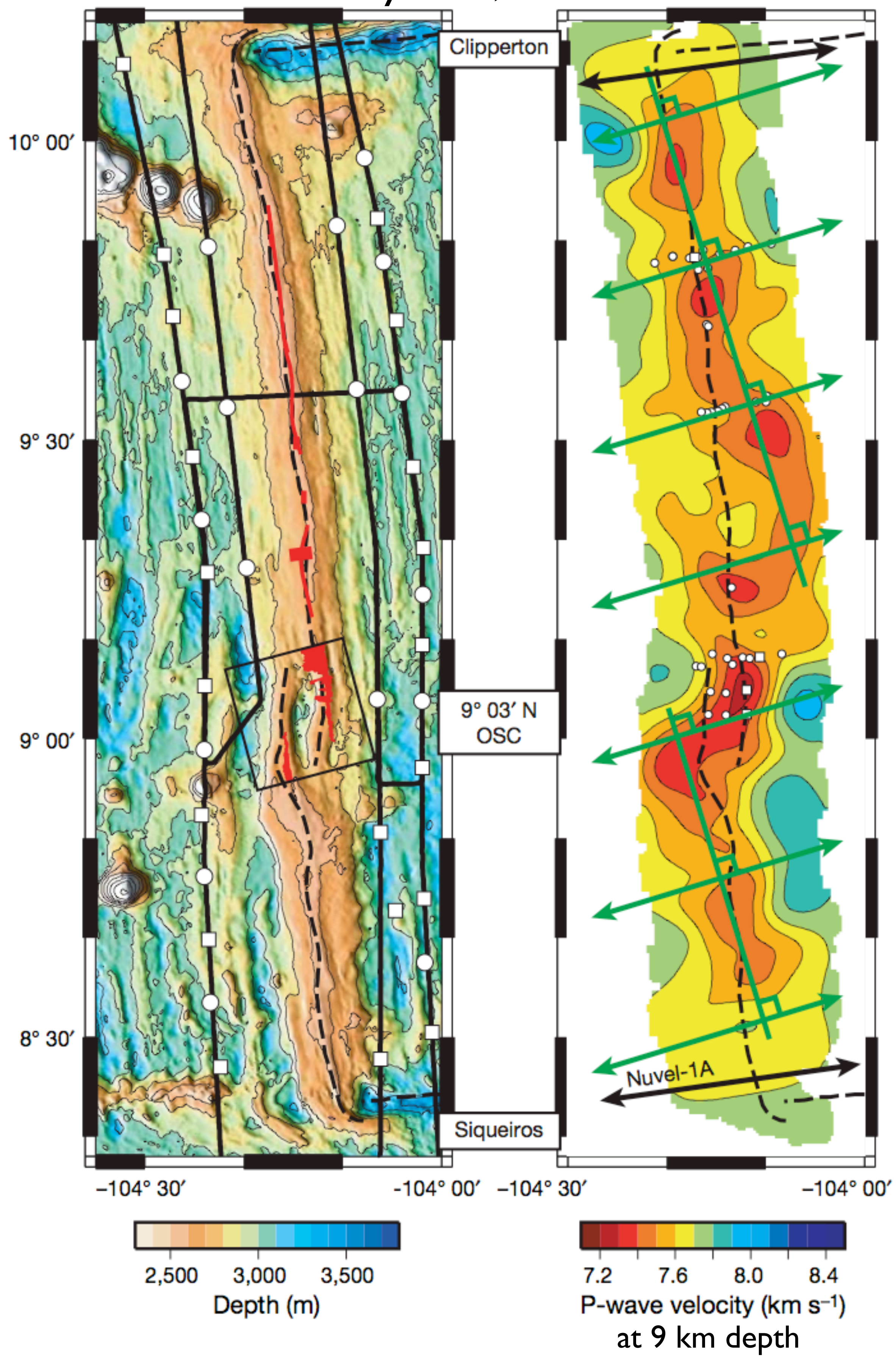
Tectonic scale
~100-1000 km

Grain scale
~1-10 mm



Seismic tomography of plate boundaries

Toomey et al, *Nature* 2007



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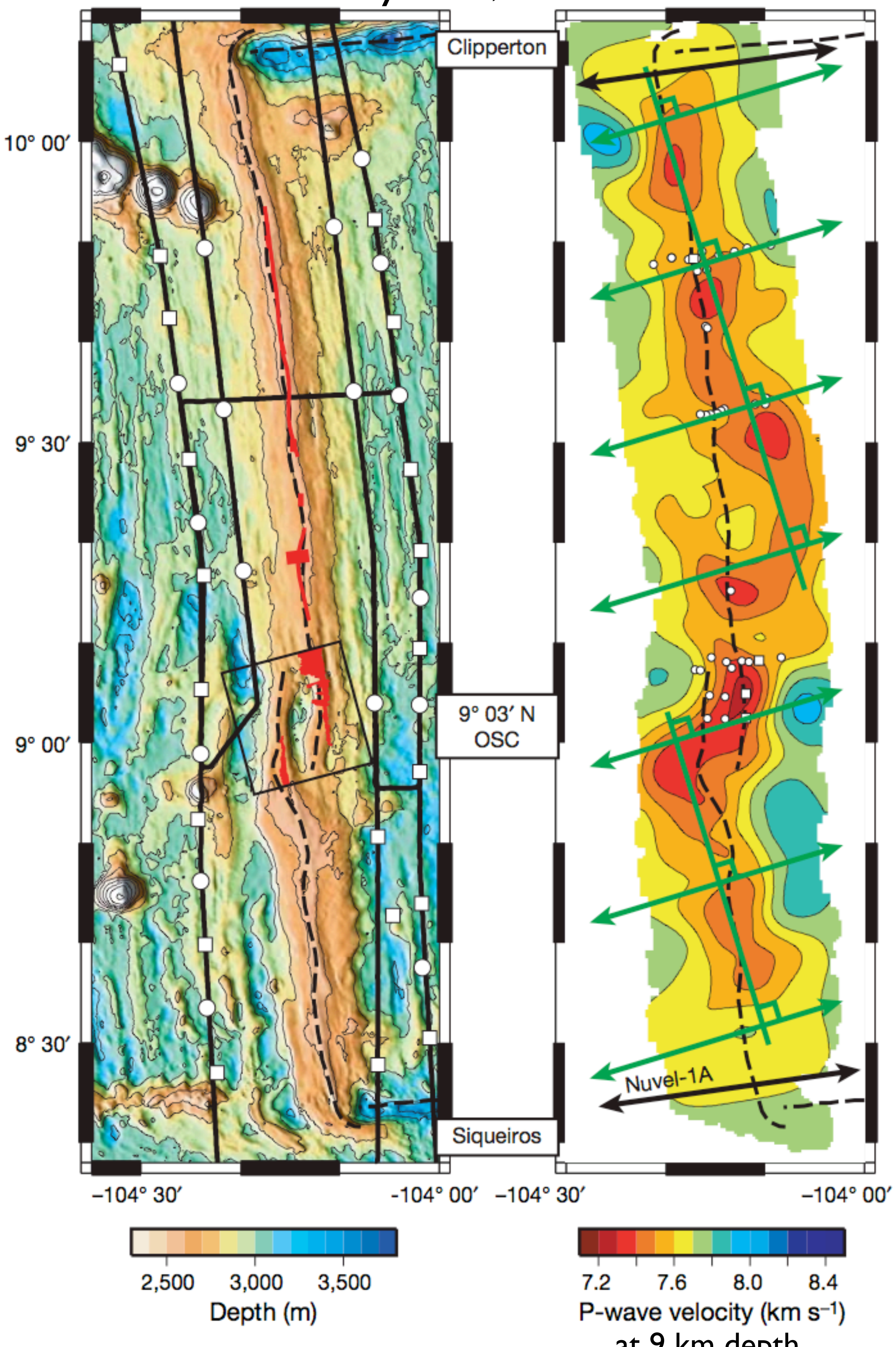
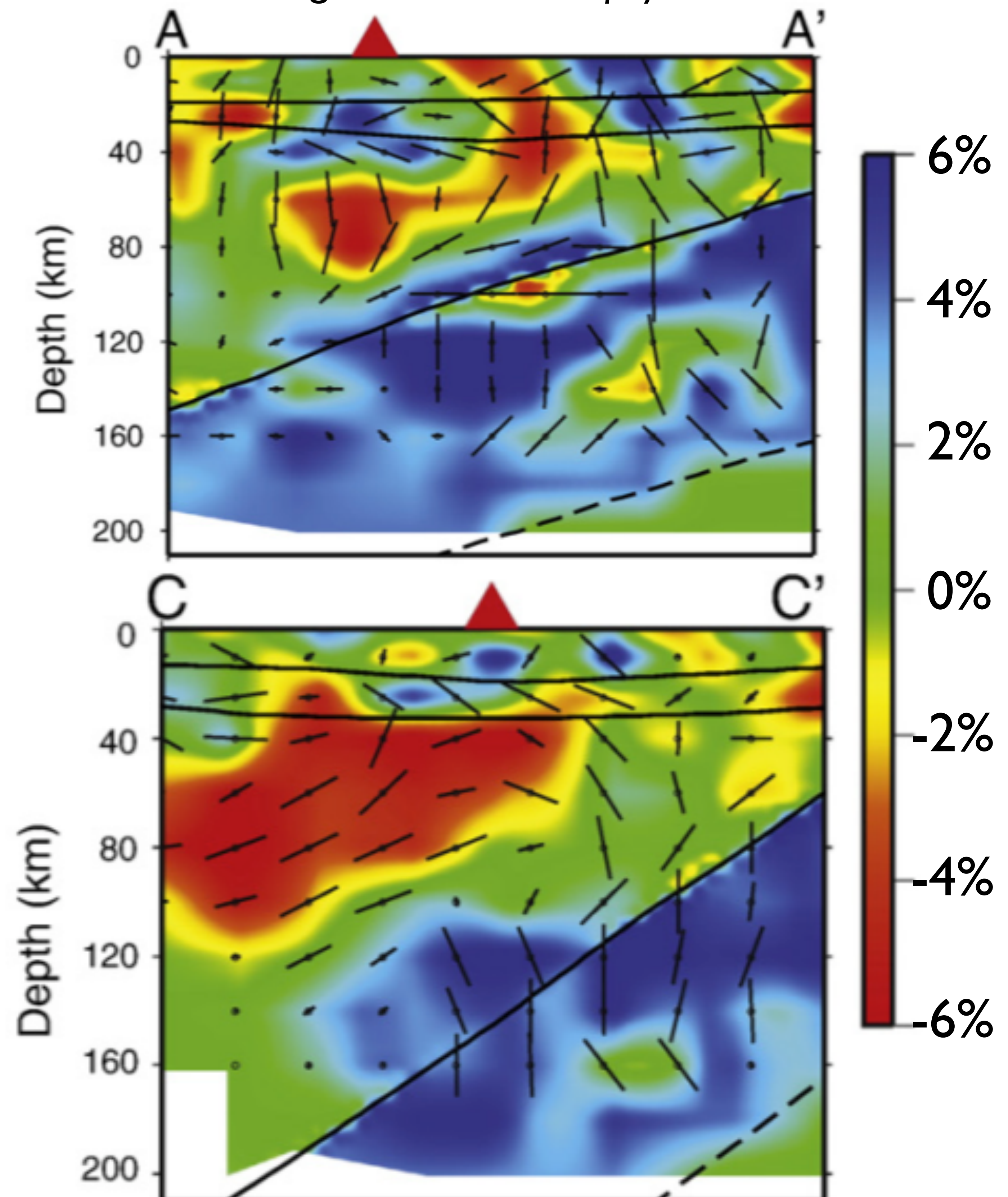
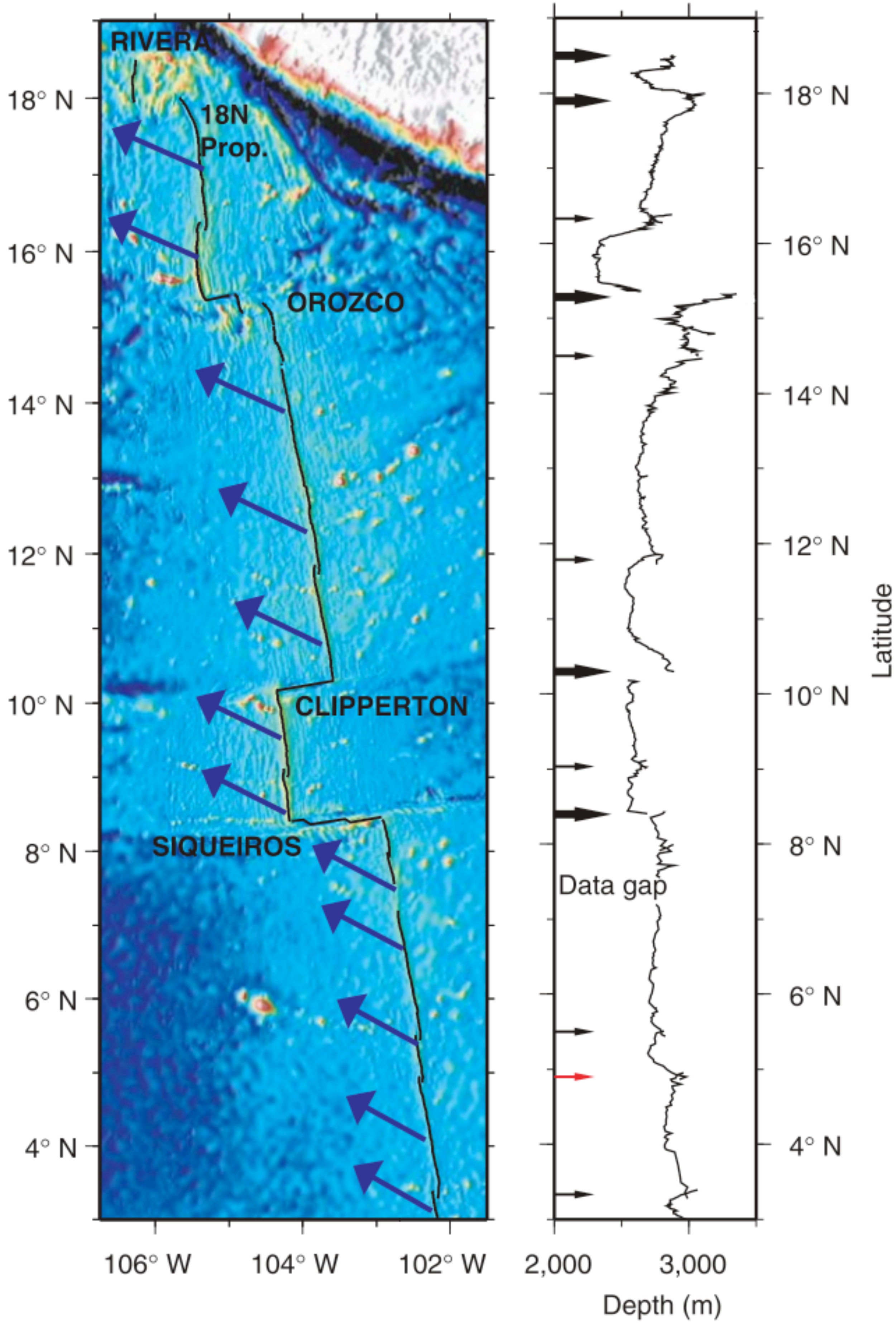


Hokkaido P-wave tomography
Wang & Zhao, *Tectonophys* 2009



Bathymetric asymmetry at mid-ocean ridges

Carbotte, Small & Donnelly, *Nature* 2004



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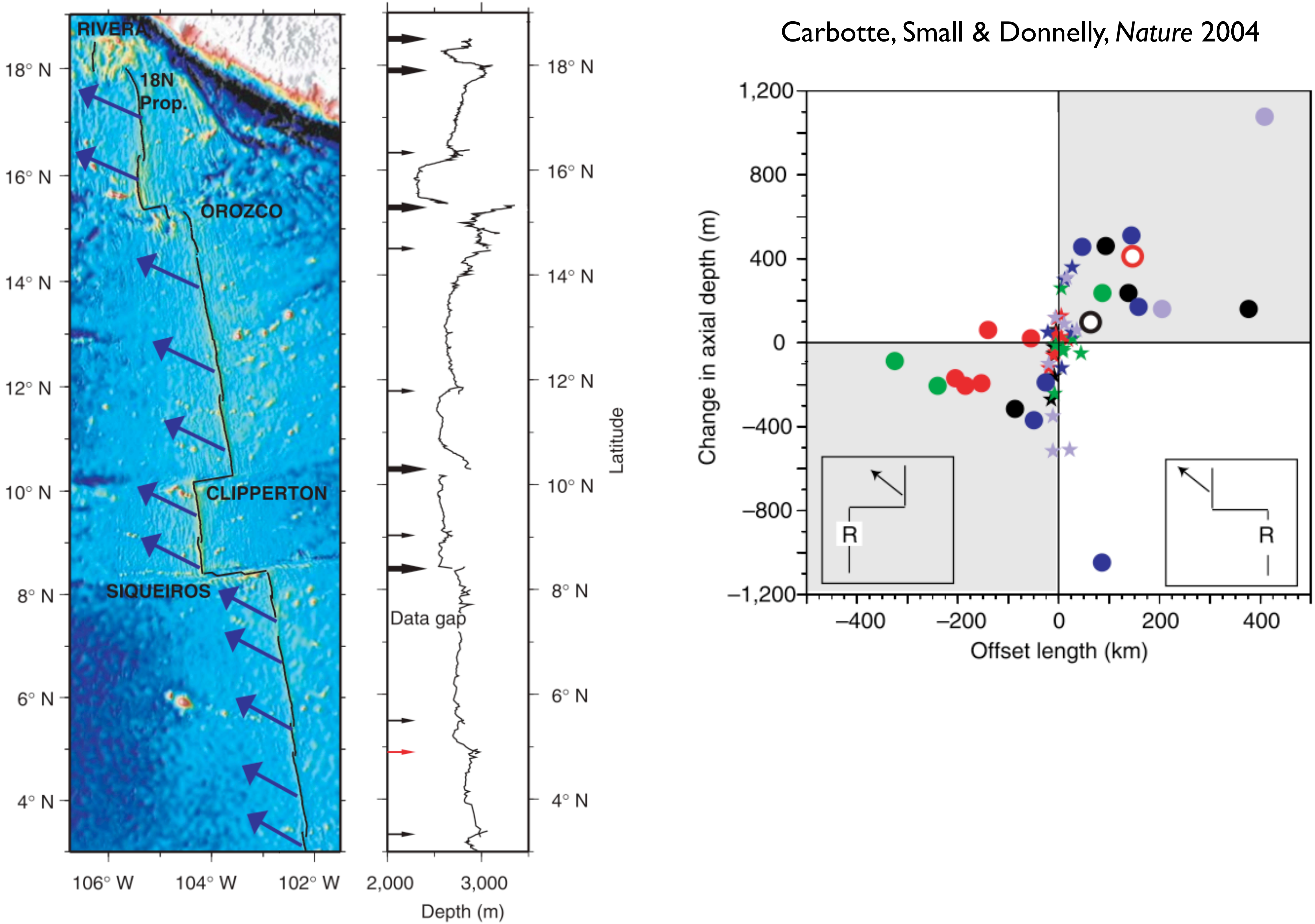

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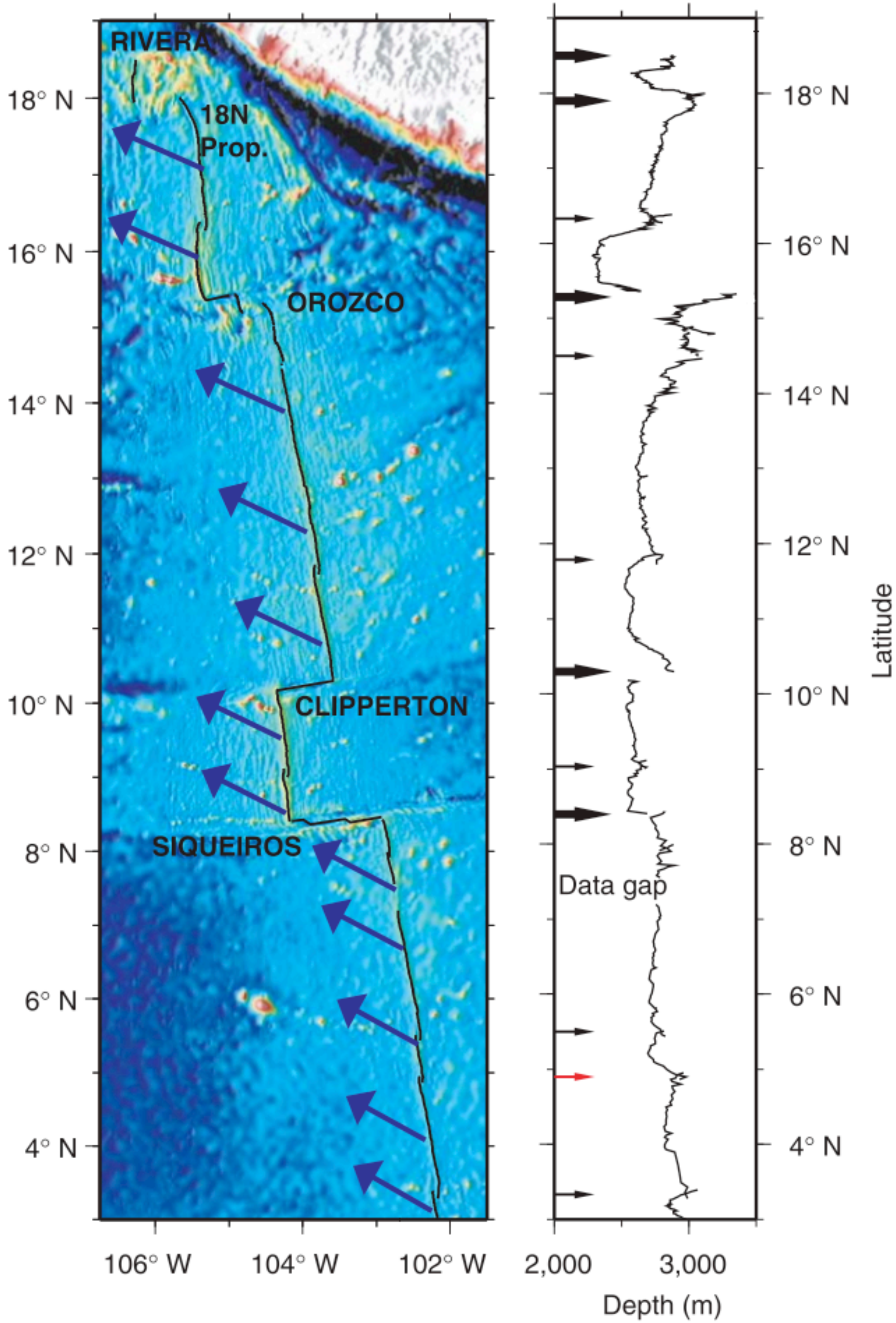

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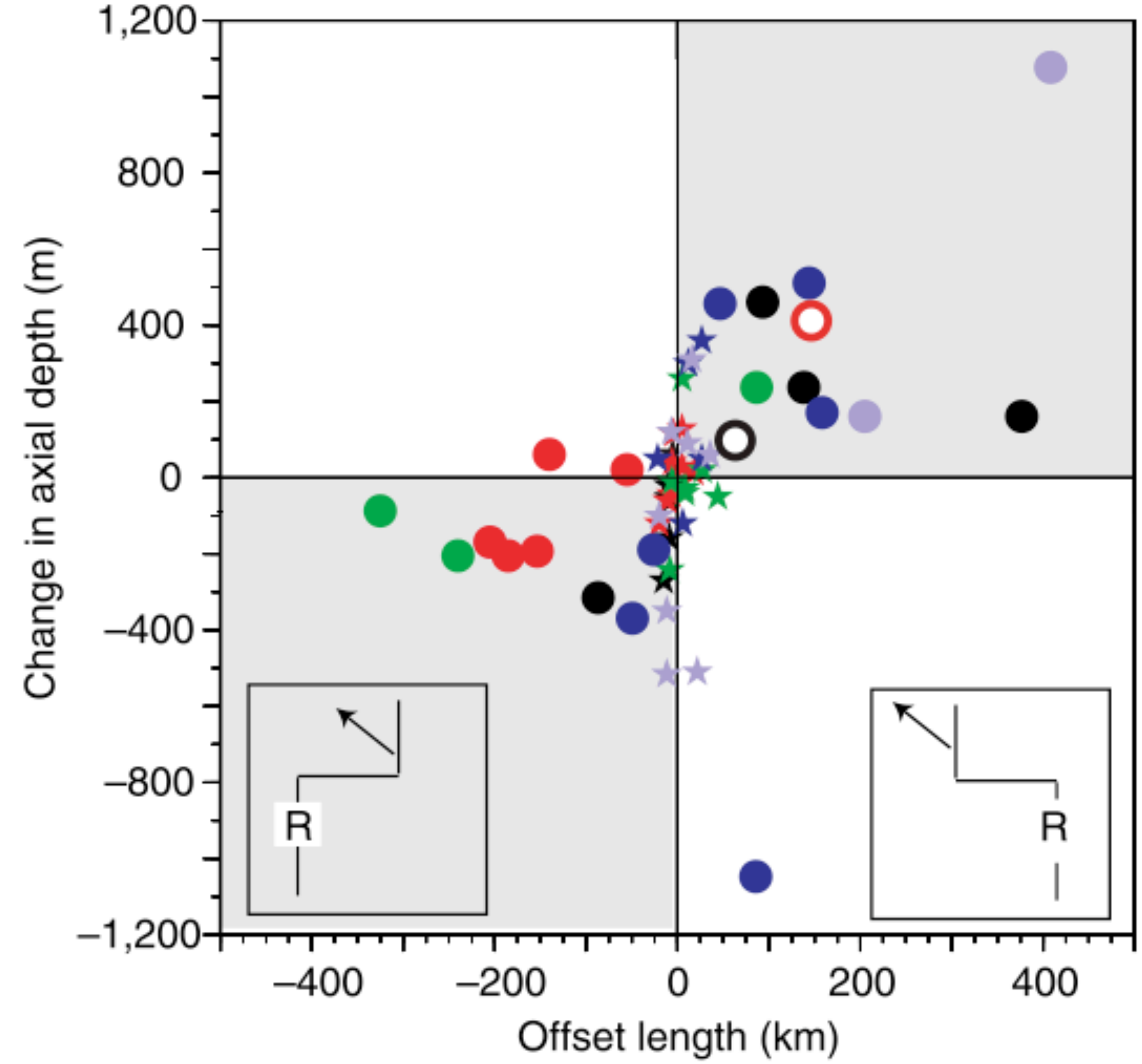
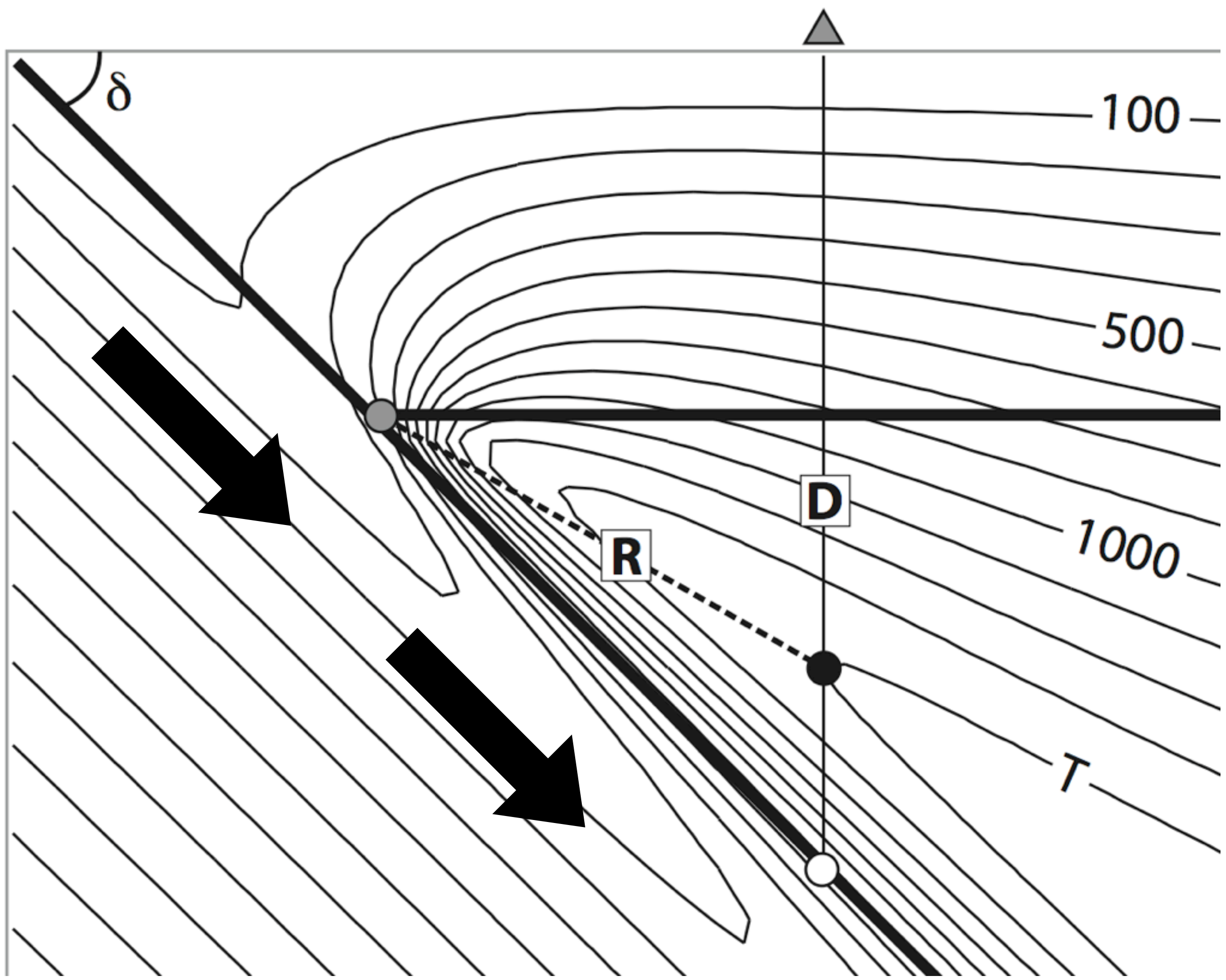
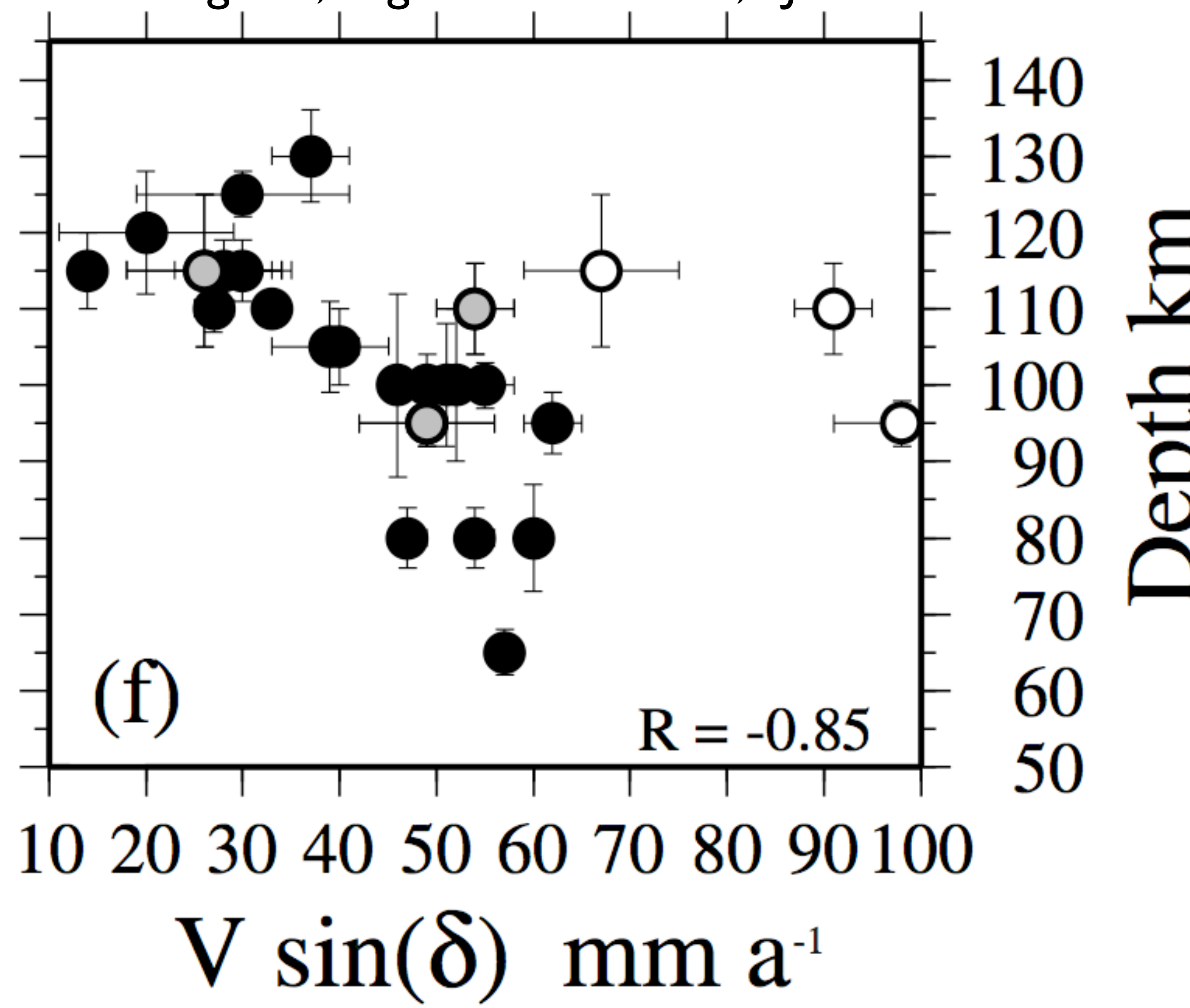


[3D models by Sam Weatherley]

Distribution of volcanoes in subduction zones

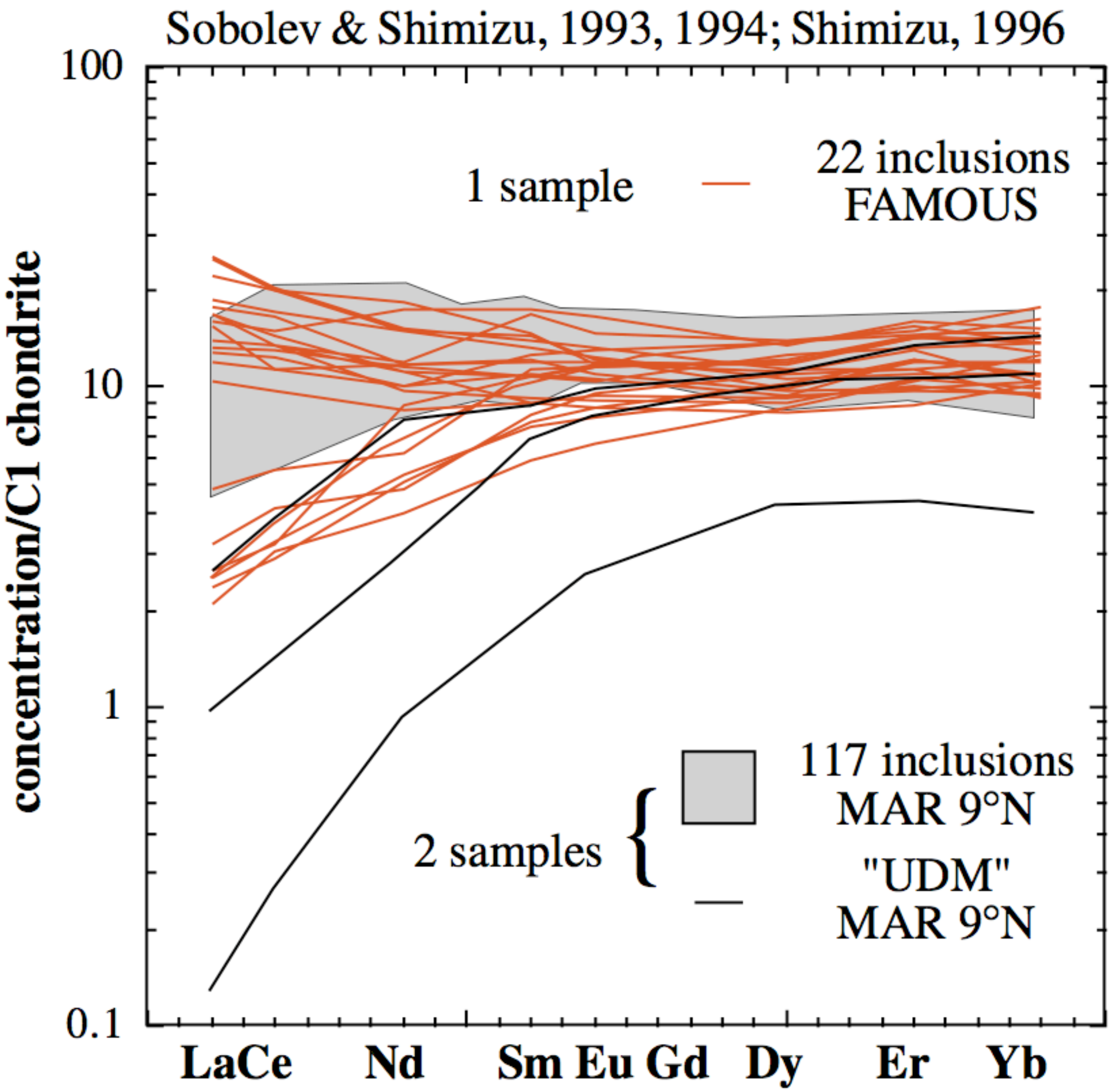


England, Engdahl & Thatcher, *GJI* 2004

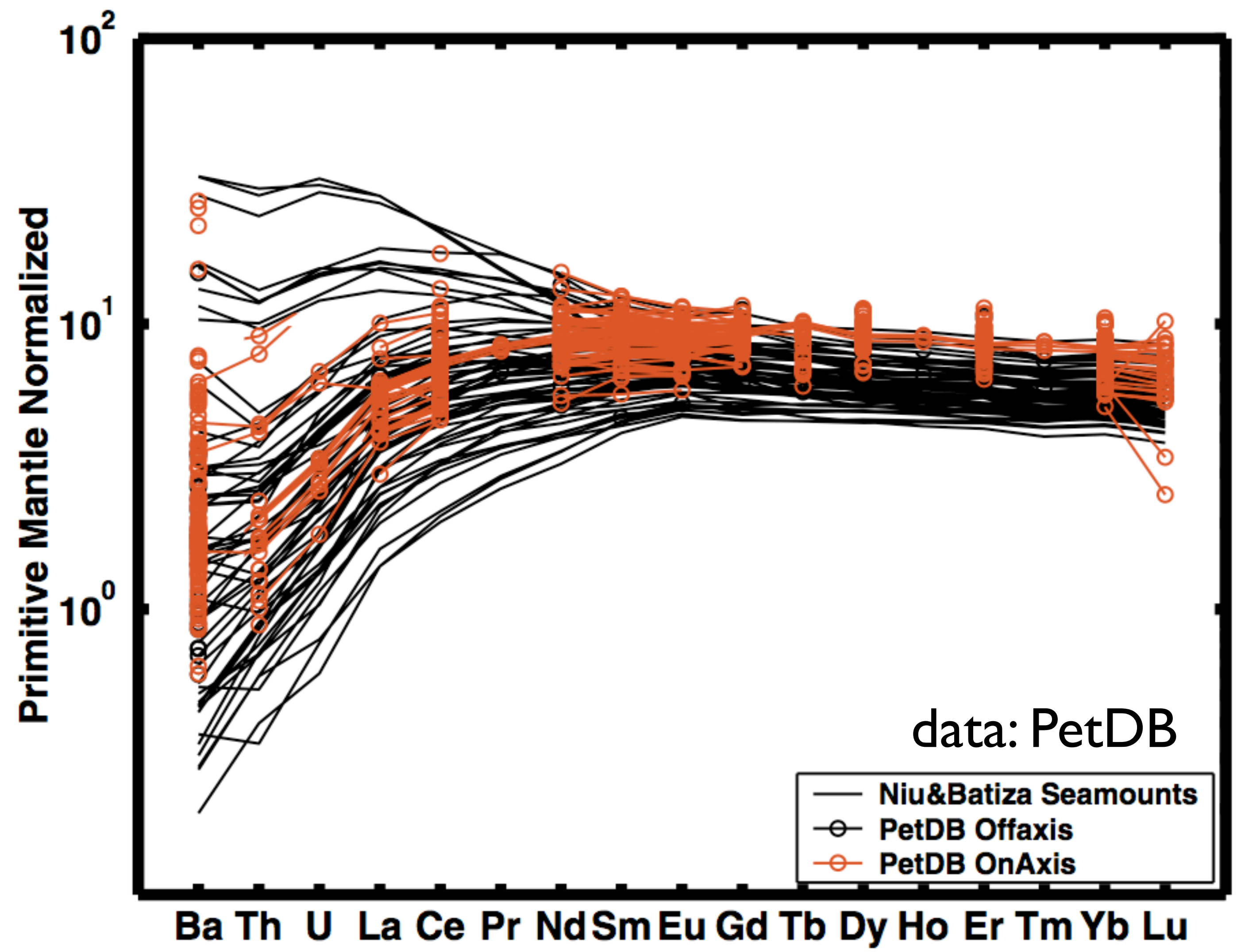


MORB geochemistry: trace elements

Melt inclusions



Whole rocks



Spiegelman & Kelemen, *G-cubed* 2003

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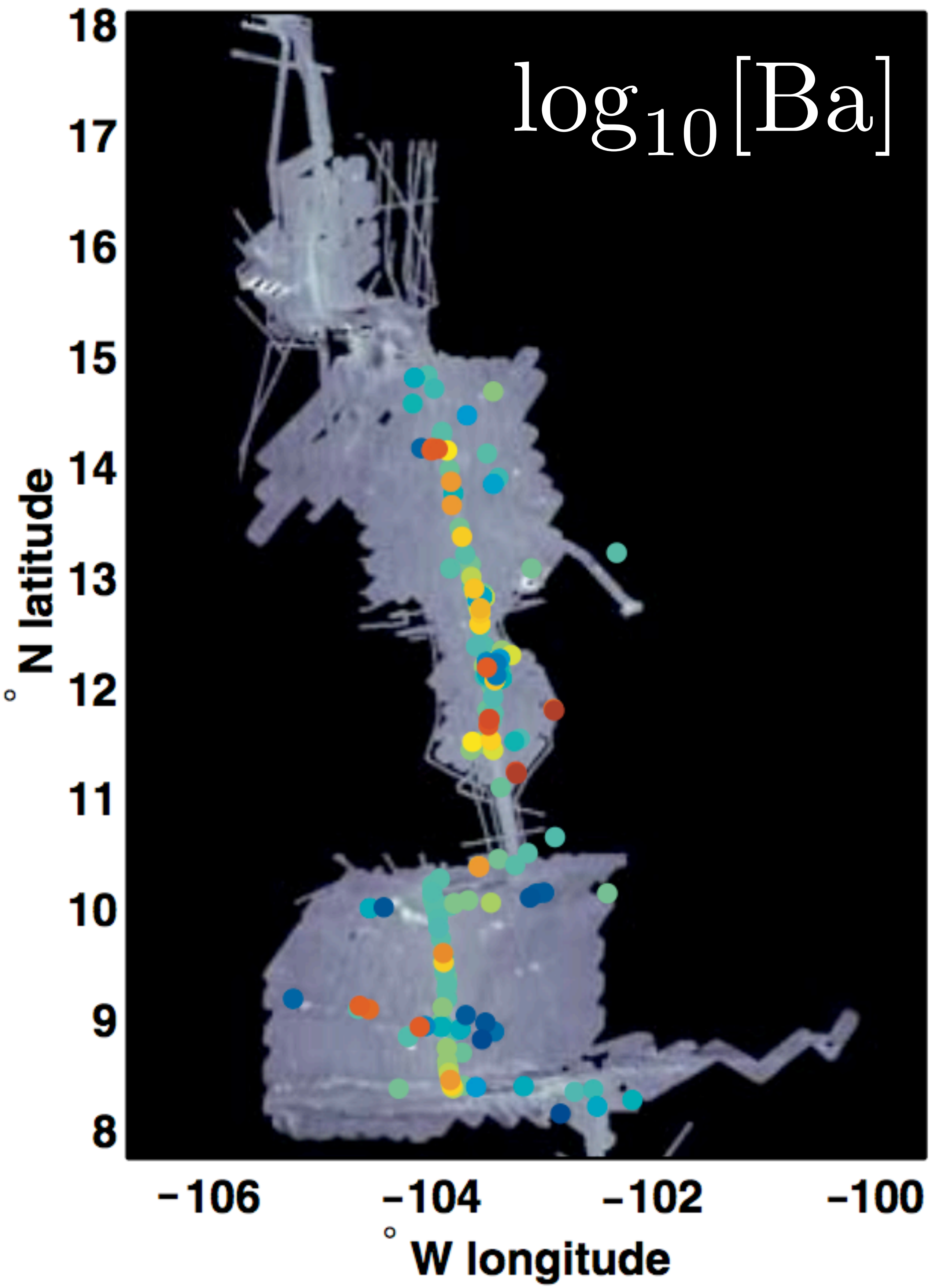

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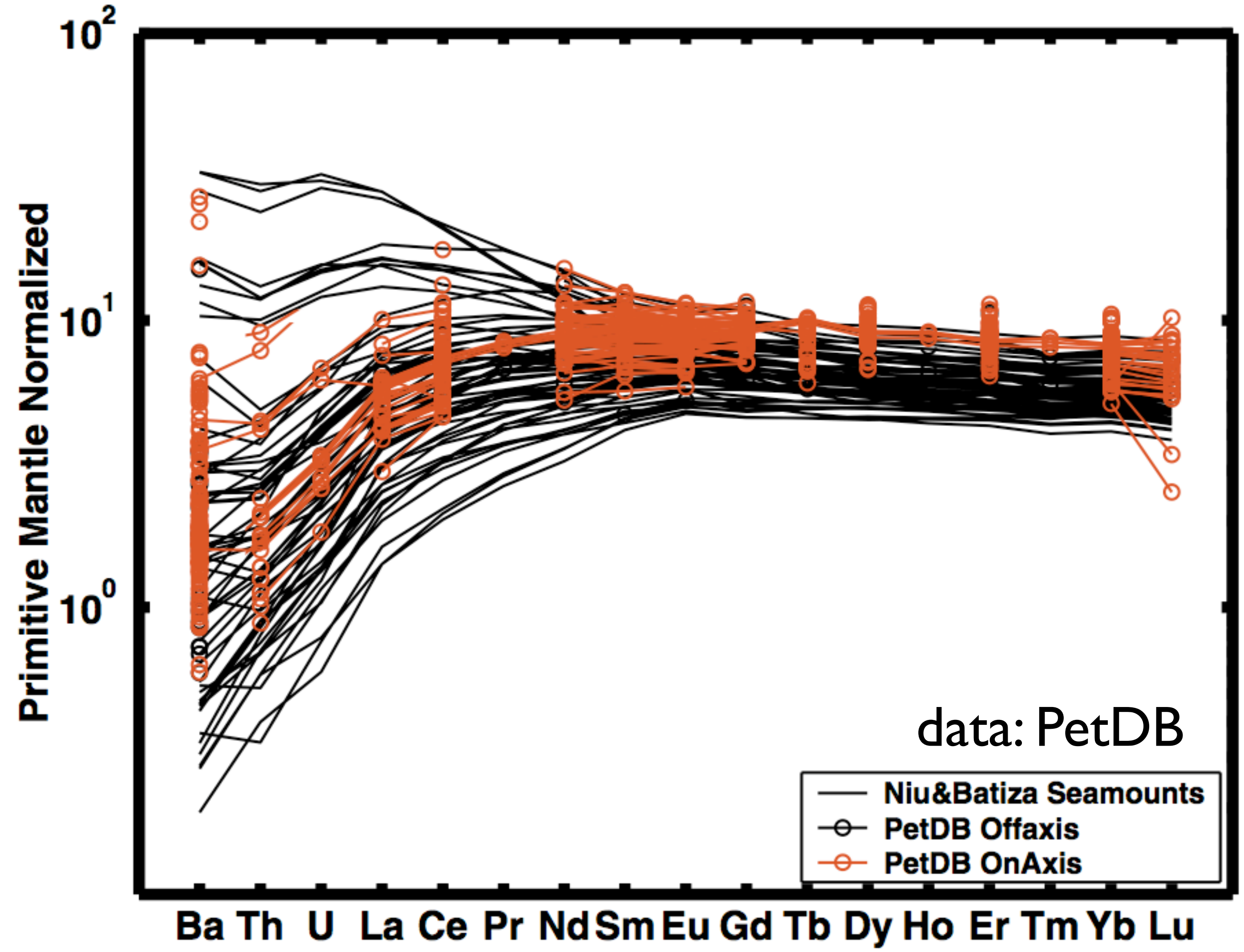
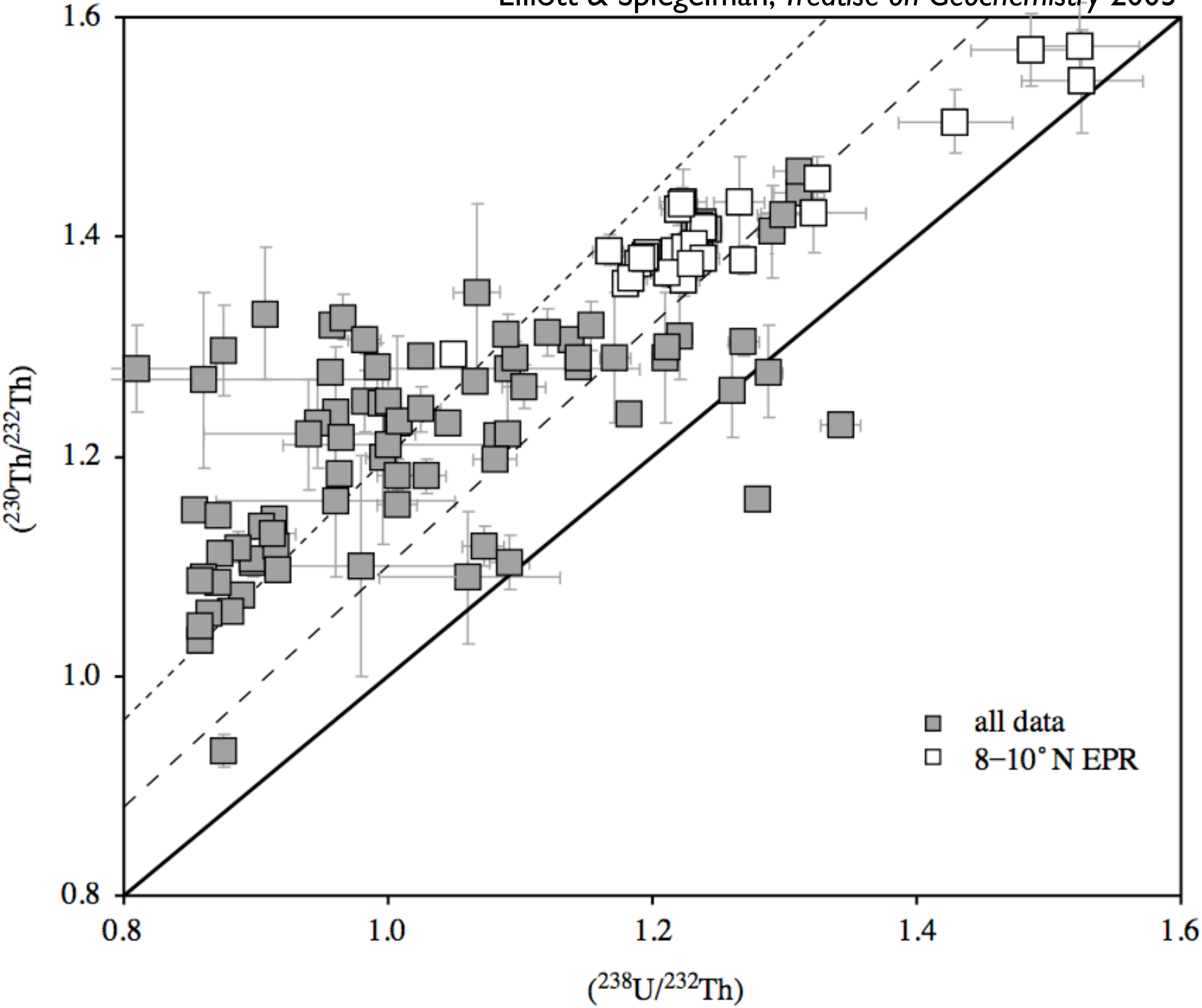


Spiegelman & Kelemen, *G-cubed* 2003

MORB geochemistry: isotopes

Elliott & Spiegelman, *Treatise on Geochemistry* 2003



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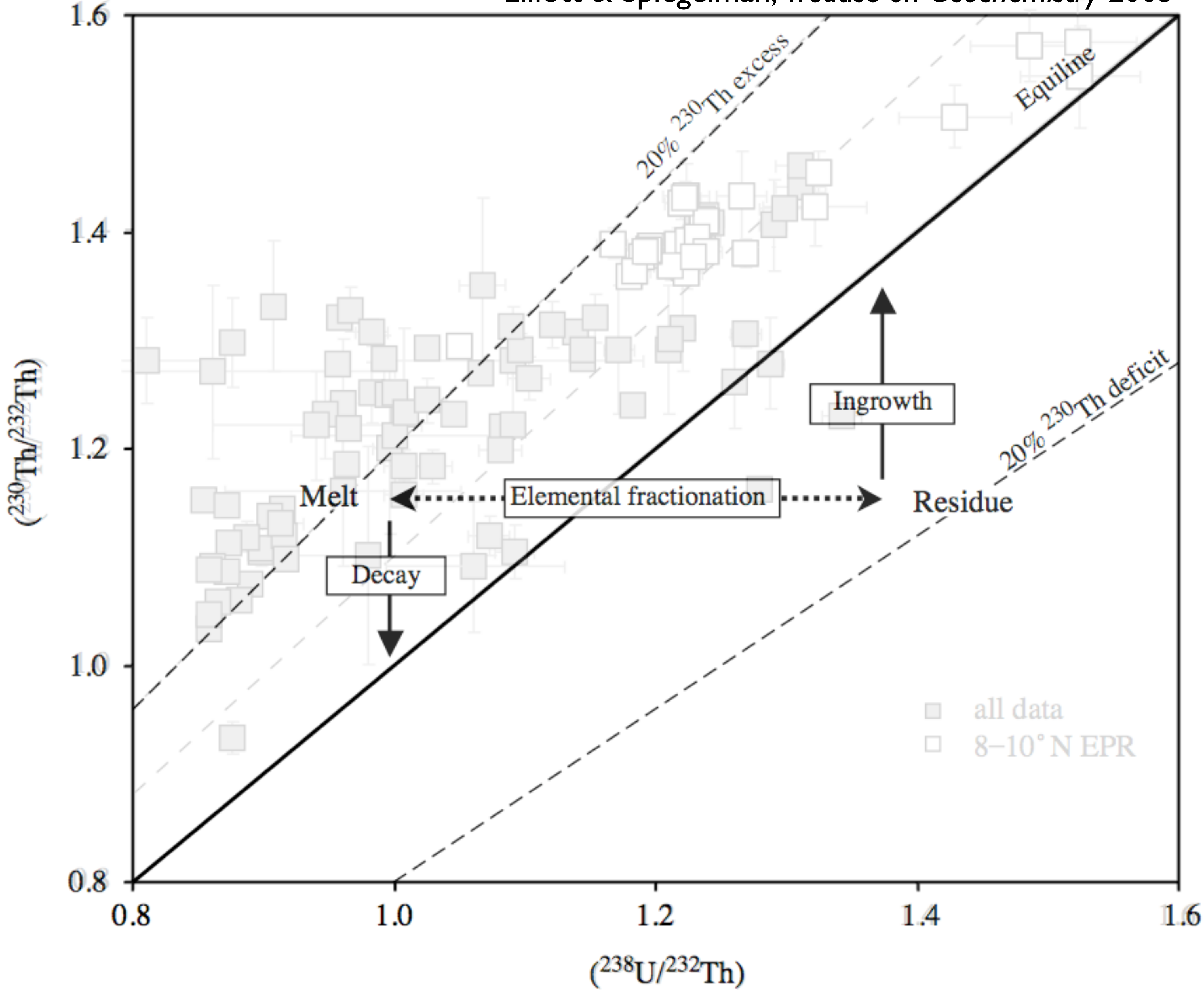

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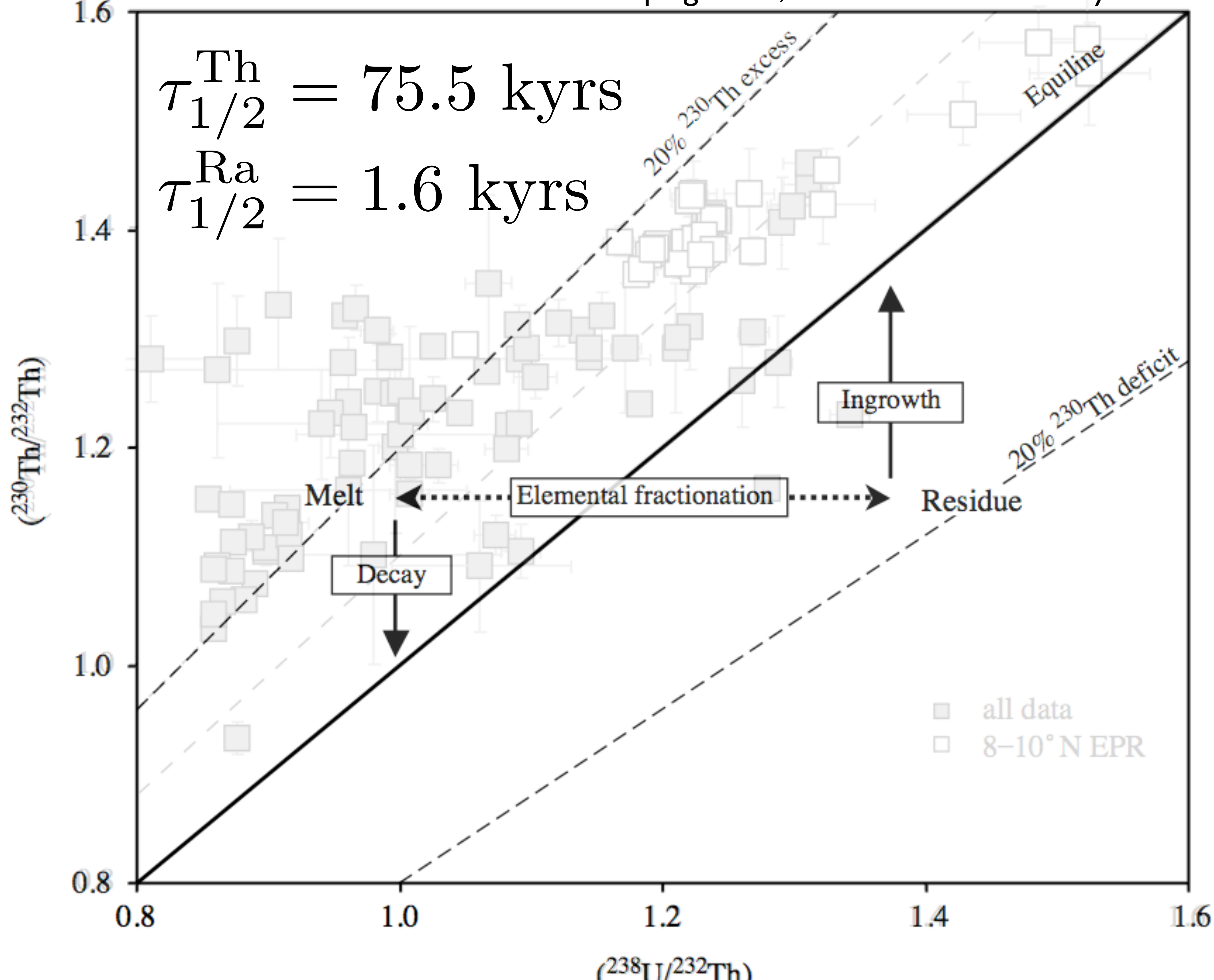
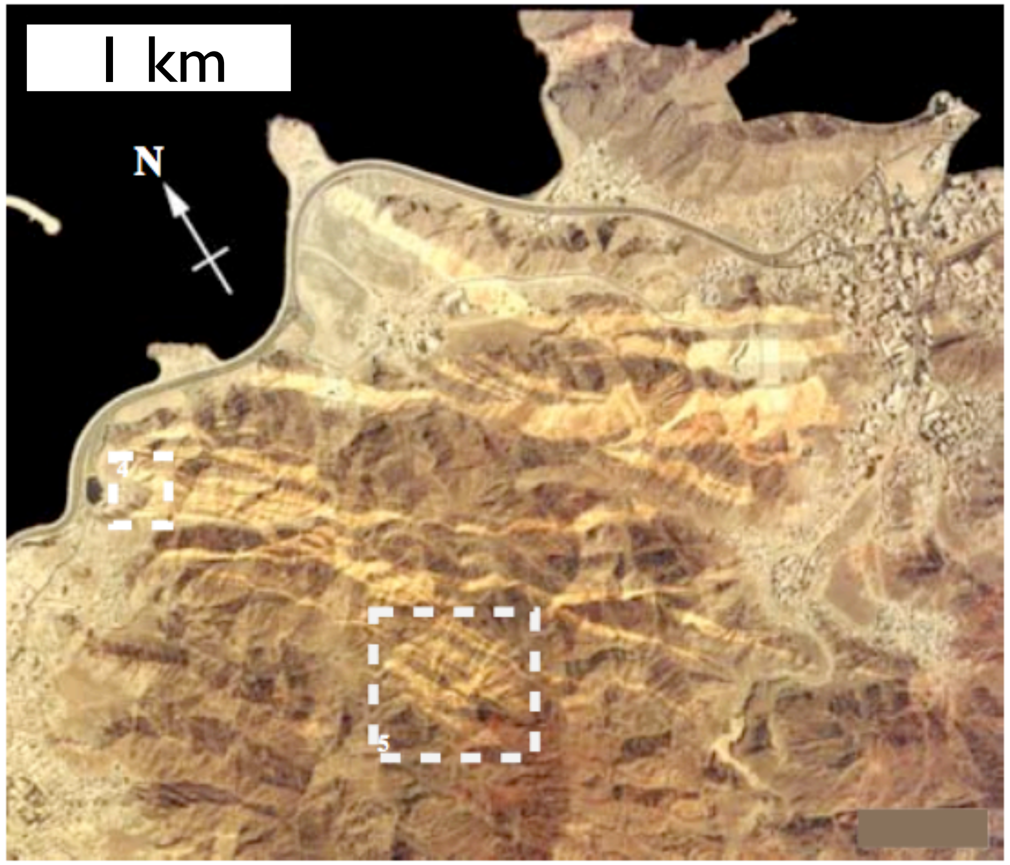
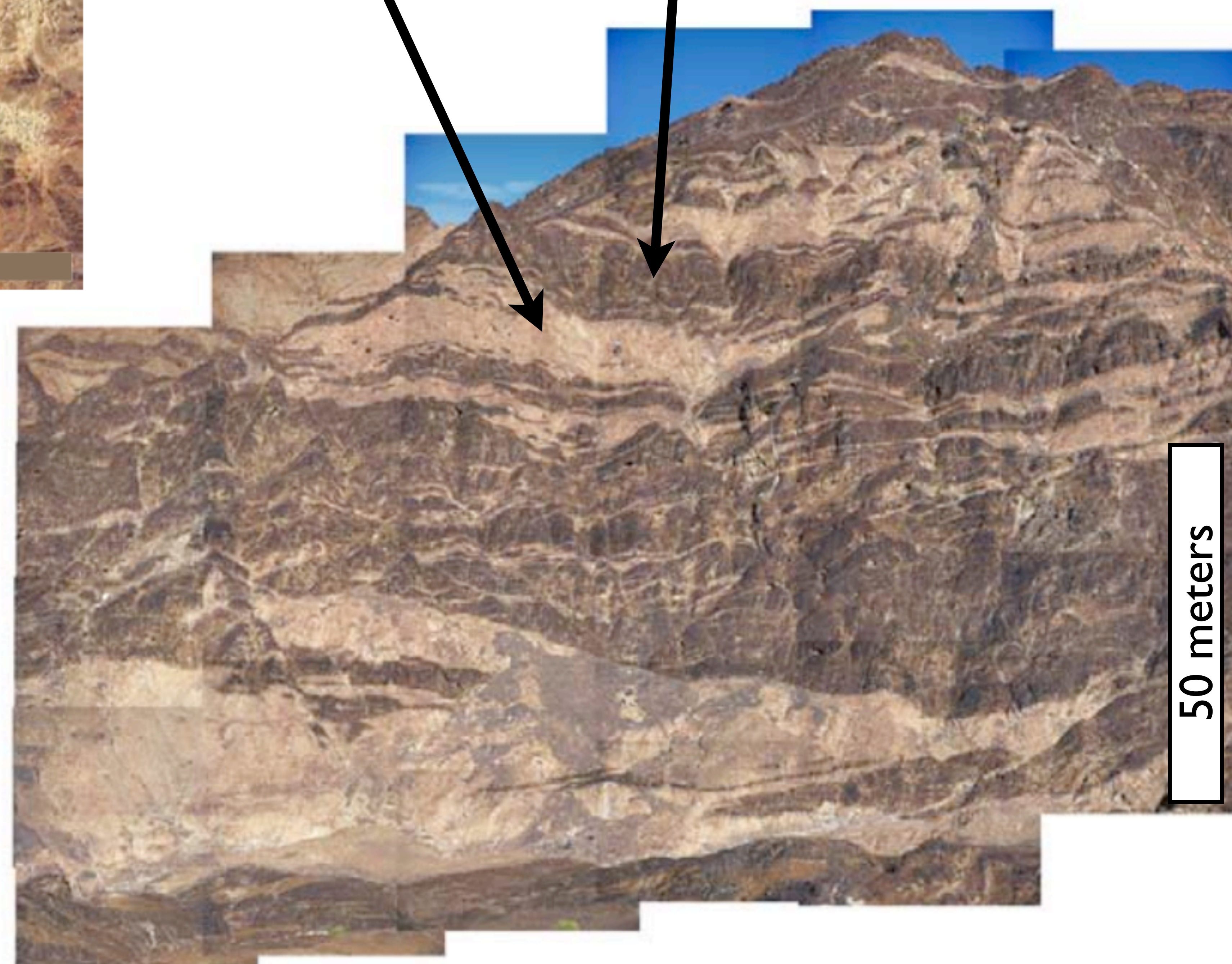


Geology of ophiolites



Harzburgite (OPX + Olivine)

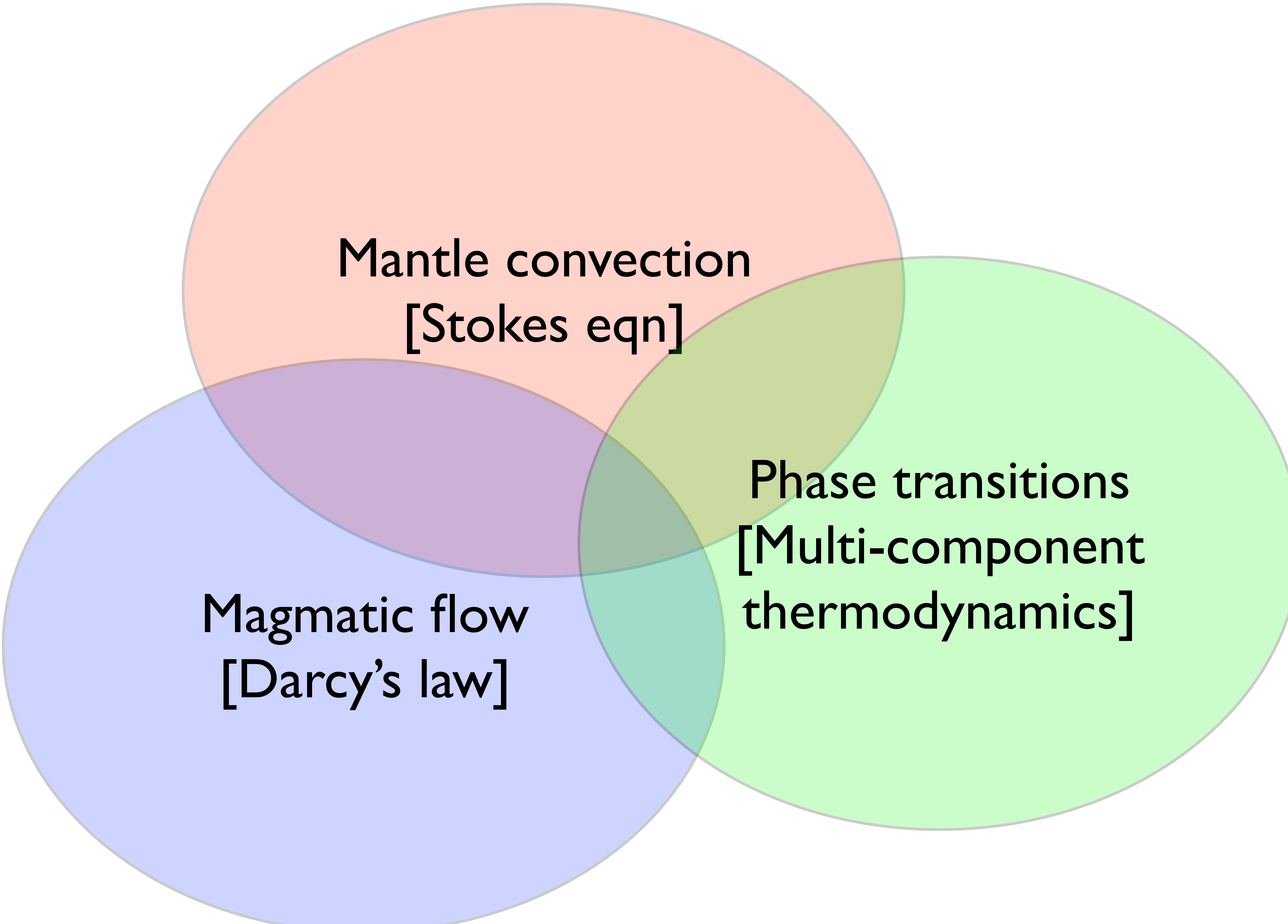
Dunite (Olivine)



Oman ophiolite

Braun & Kelemen, *G-cubed* 2002

A continuum approach to magma dynamics



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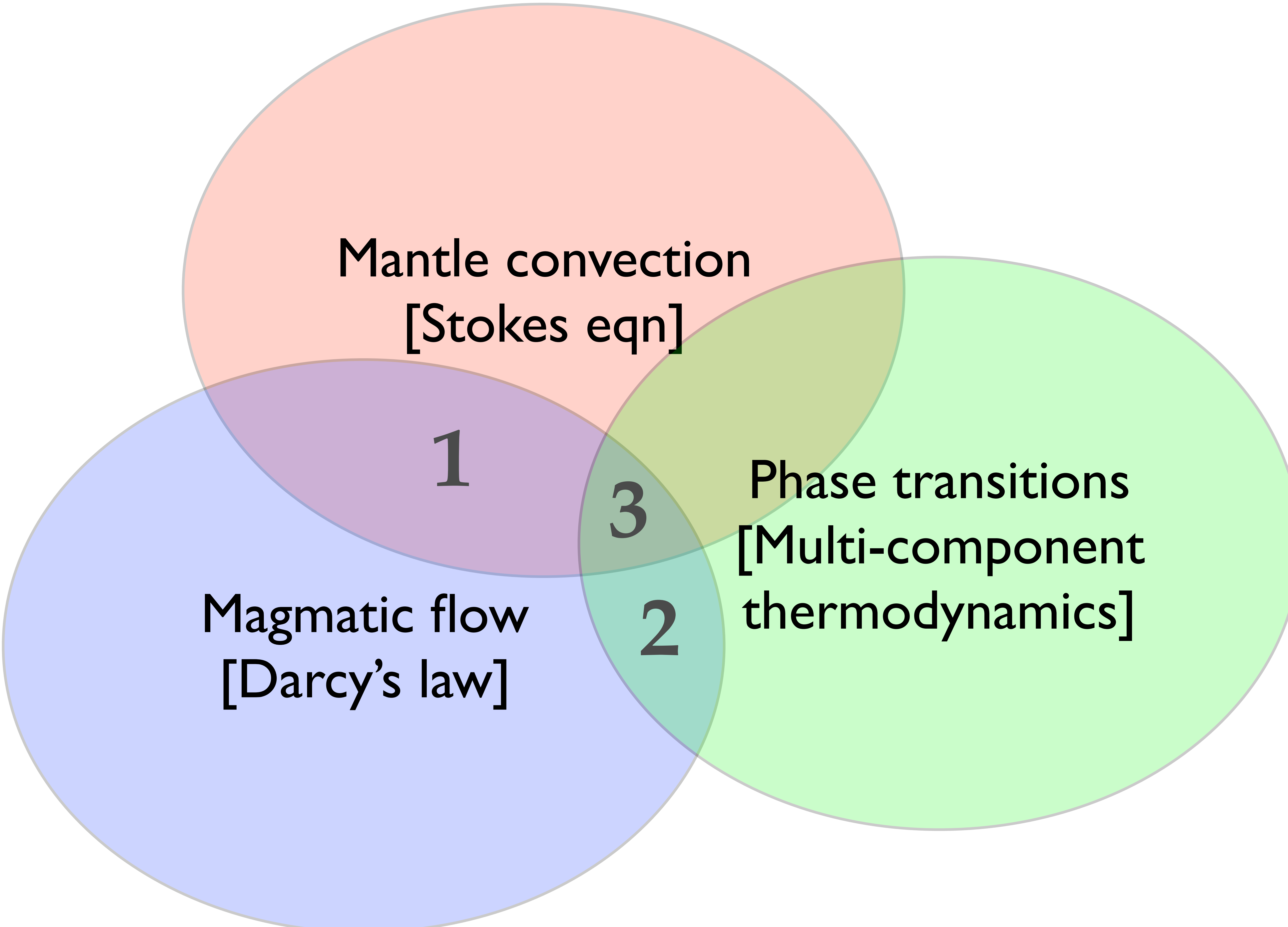

An introduction to magma dynamics

1. Mechanics
   1.1. Conservation equations
   1.2. Solitary waves & the compaction length
   1.3. Shear-band instabilities

2. Thermochemistry
   2.1. Conservation equations
   2.2. 1D, one-component upwelling columns
   2.3. Reactive infiltration instability

3. Putting it together: tectonic-scale models
   3.1. Problems and solutions
   3.2. Melt focusing at mid-ocean ridges
   3.3. Active convection at mid-ocean ridges

1.1 Mechanics: conservation equations

- D.P. McKenzie, *J. Petrology* 1984
- A. Fowler, *Geophys. Astrophys. Fluid Dyn.* 1985
- D.R. Scott & D.J. Stevenson, *GRL* 1986
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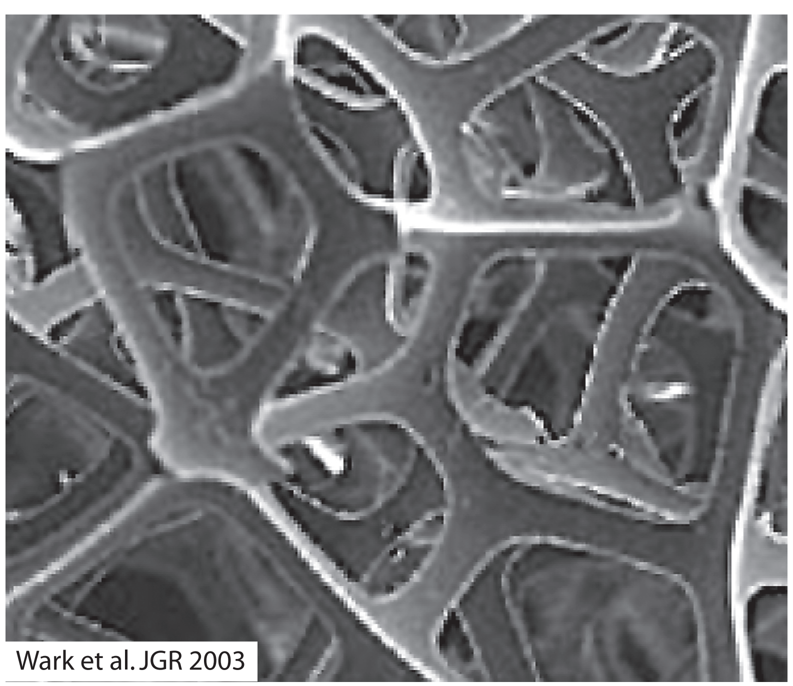
RVE



Potato

Gauss

1.1

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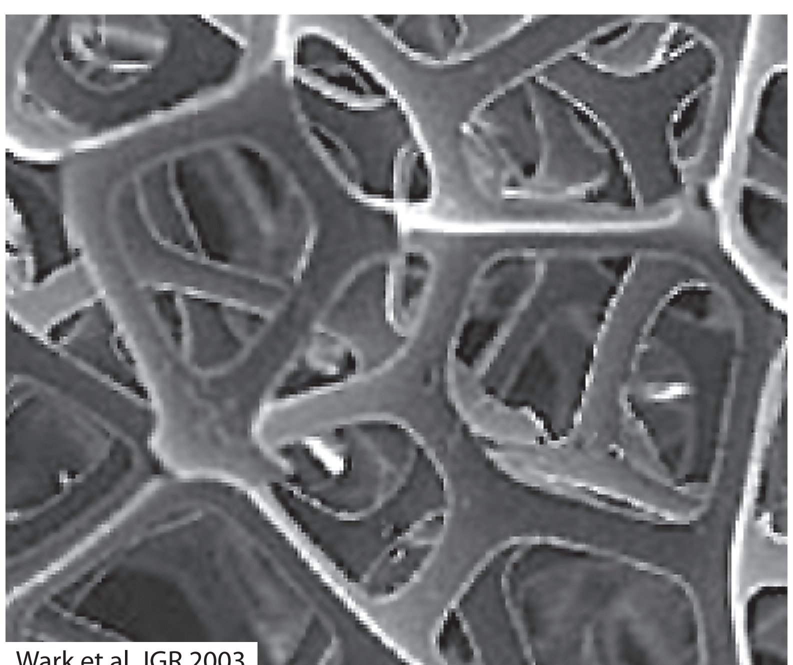

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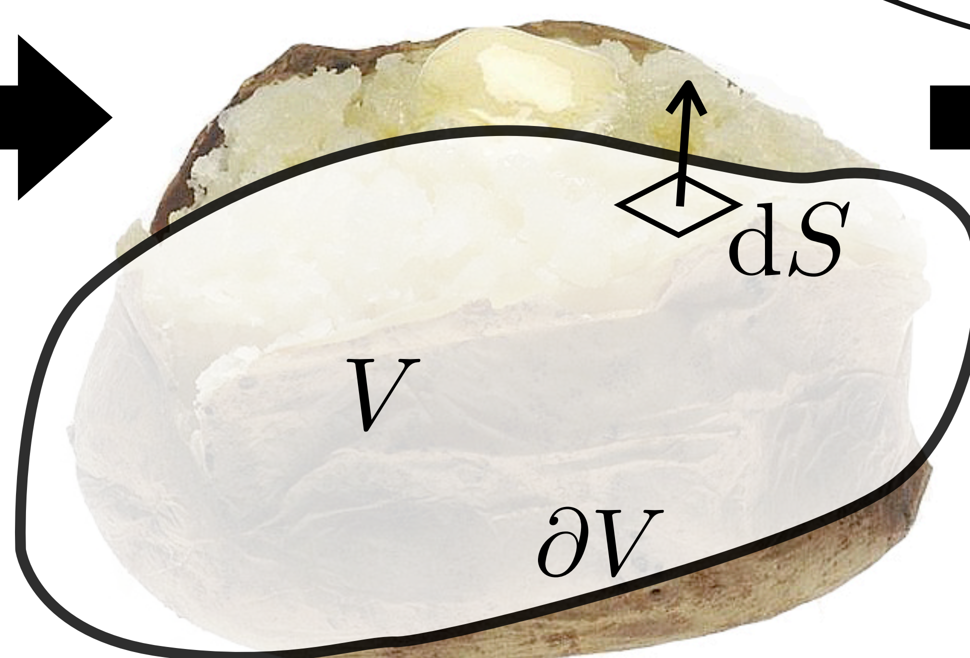

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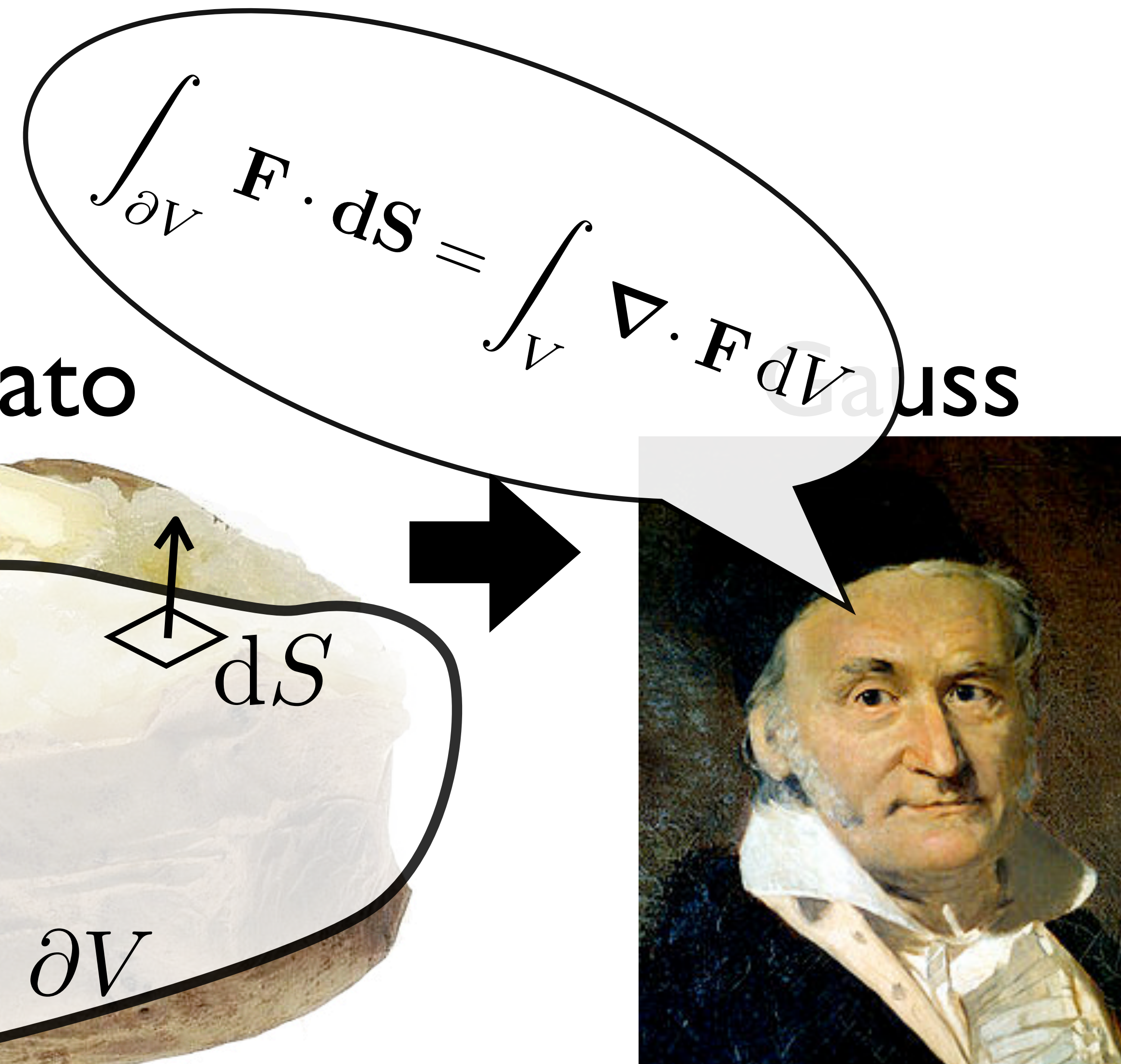

1.1 Mechanics: conservation equations

Conservation of mass

$$\frac{\partial\phi}{\partial t} + \boldsymbol{\nabla}\cdot\phi\mathbf{v}_f = \frac{\Gamma}{\rho_f}$$

$$\frac{\partial(1-\phi)}{\partial t} + \boldsymbol{\nabla}\cdot(1-\phi)\mathbf{v}_m = -\frac{\Gamma}{\rho_m}$$

McKenzie ’84, Bercovici & Ricard *GJI* ’03, sec 4.1

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McKenzie '84, Bercovici & Ricard *GJI* '03, sec 4.1

Mechanics: compaction & viscosity

Matrix stress (McKenzie '84):

$$\sigma_{ij}^m = -P_f\delta_{ij} + \zeta\delta_{ij}\frac{\partial v_k^m}{\partial x_k} + \eta\left(\frac{\partial v_i^m}{\partial x_j} + \frac{\partial v_j^m}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_k^m}{\partial x_k}\right),$$

Matrix dynamic pressure:

$$P_m = -\frac{1}{3}\mathrm{Tr}(\sigma_{ij}^m) = P_f - \zeta\frac{\partial v_k^m}{\partial x_k}$$

Rearrange to give:

$$\boxed{P_f - P_m = \zeta\boldsymbol{\nabla}\cdot\mathbf{v}_m}$$

see also:
Bercovici & Ricard *GJI* '03

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Viscosities:

$$\eta = \eta_0\exp\left(\frac{E^*}{RT}\right)\exp(-\alpha\phi)$$

(e.g. Karato & Wu '93)

$$\zeta = \zeta_0\exp\left(\frac{E^*}{RT}\right)\phi^{-1}$$

(Batchelor '67, Šrámek et al '07, Hewitt & Fowler '08, Simpson '08)

Solitary waves

$$K = K_0 \phi^n$$ (permeability law)

non-dimensional, 1D, no melting, ϕ_0 is background porosity

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z}(1 - \phi_0 \phi) W$$ (mass)

$$\frac{\partial^2 W}{\partial z^2} - \frac{W}{\phi^n} - \frac{1 - \phi_0 \phi}{1 - \phi_0} = 0$$ (momentum)

or, assuming: $\phi_0 \ll 1, \quad n = 3$

$$\phi_t = [\phi^3(\phi_{zt} - 1)]_z$$ (non-linear wave equation)

e.g. Barcilon & Richter, *JFM* 1986

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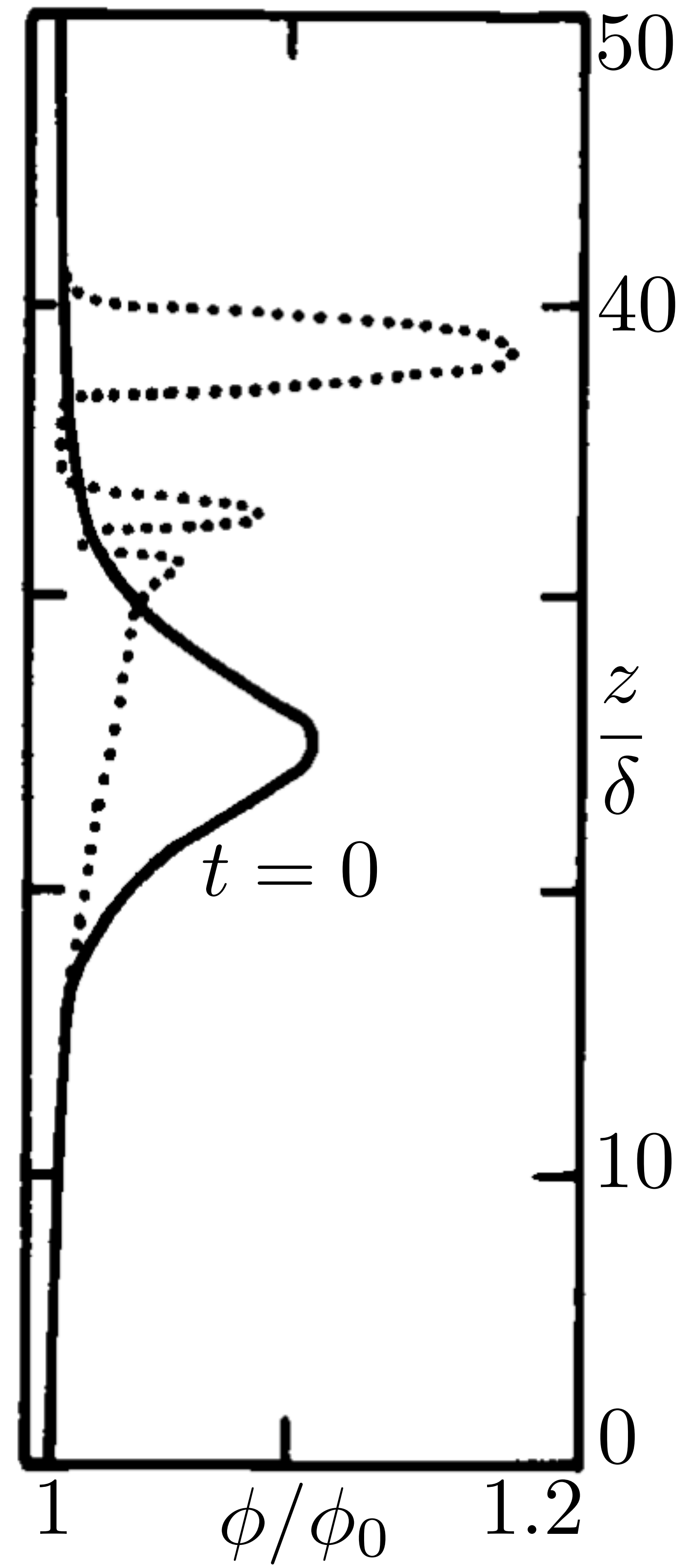

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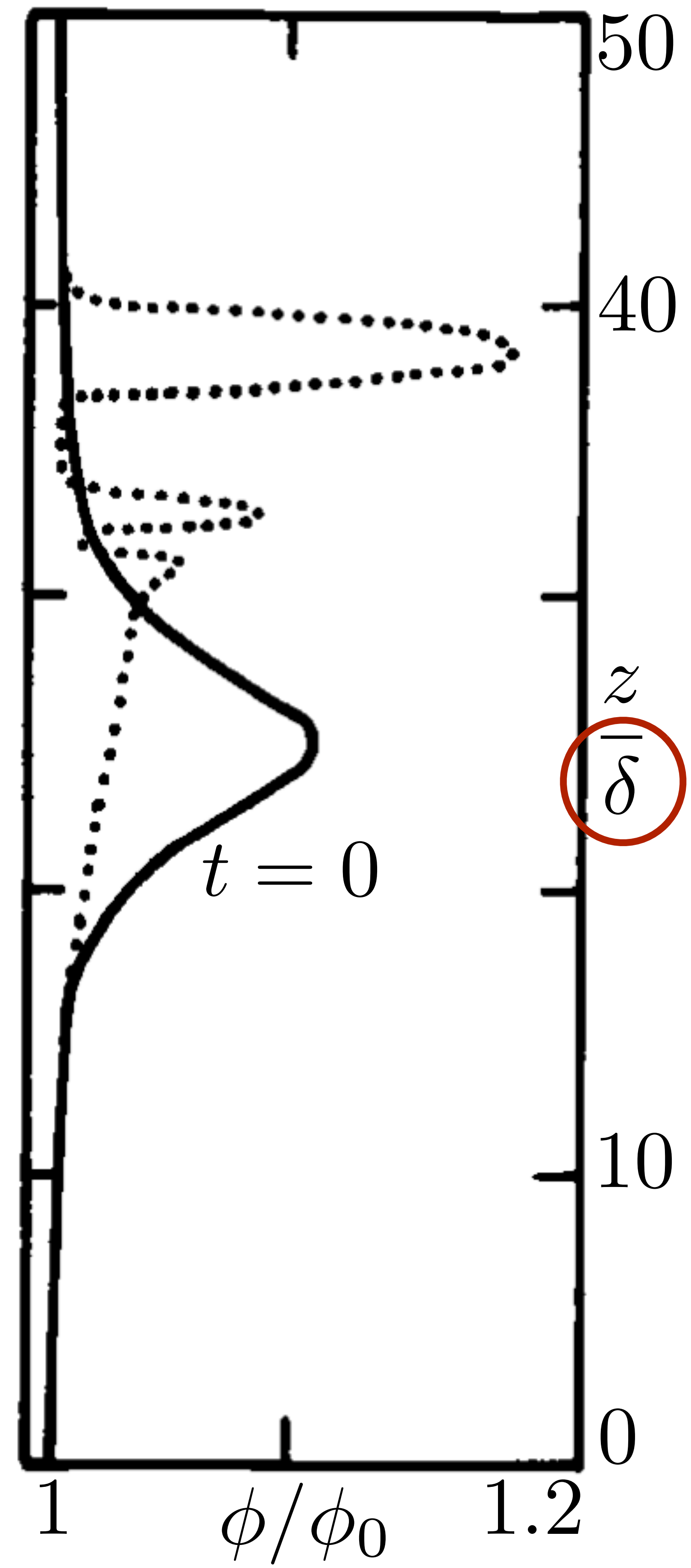

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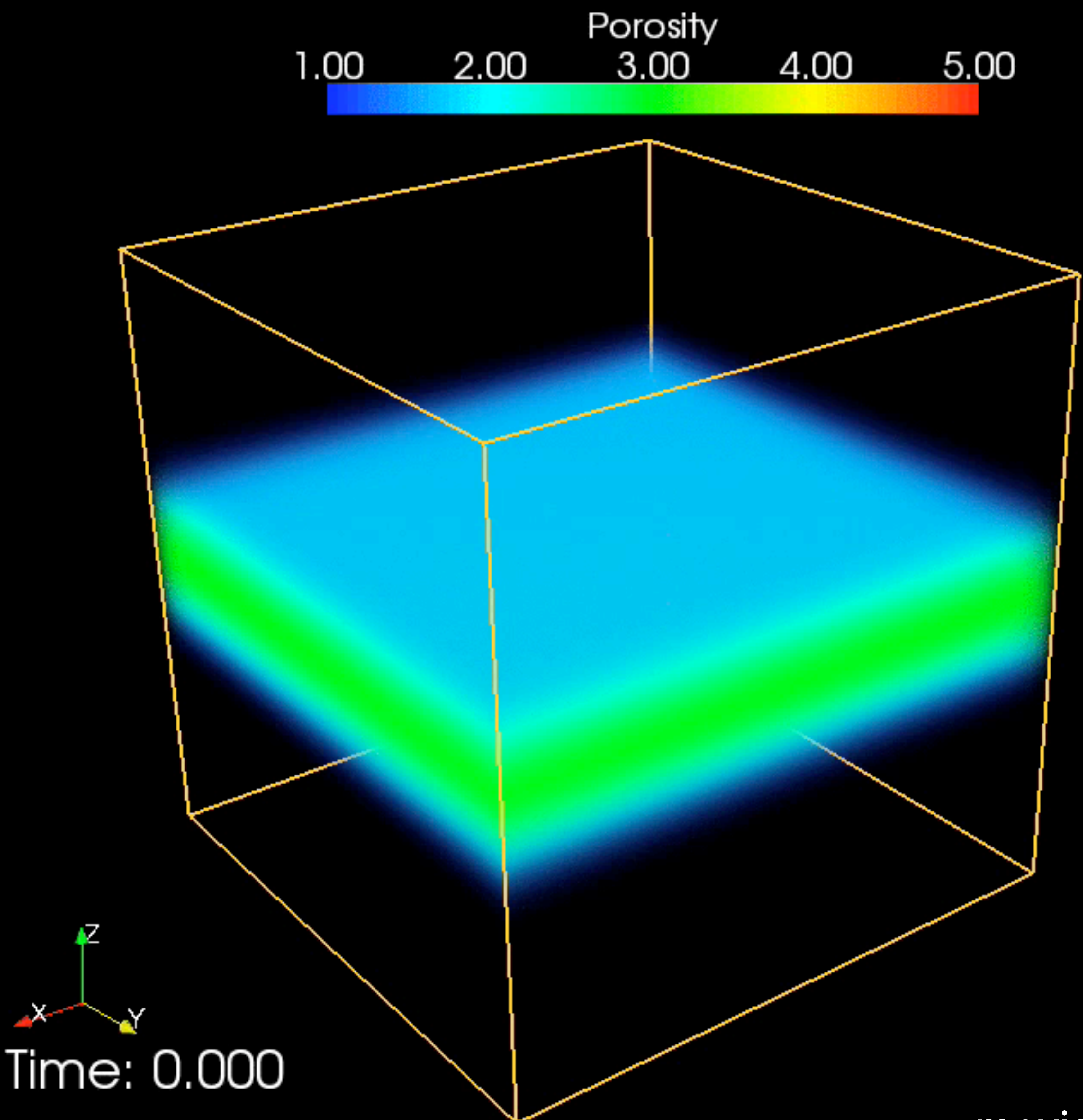
Solitary waves in 3D



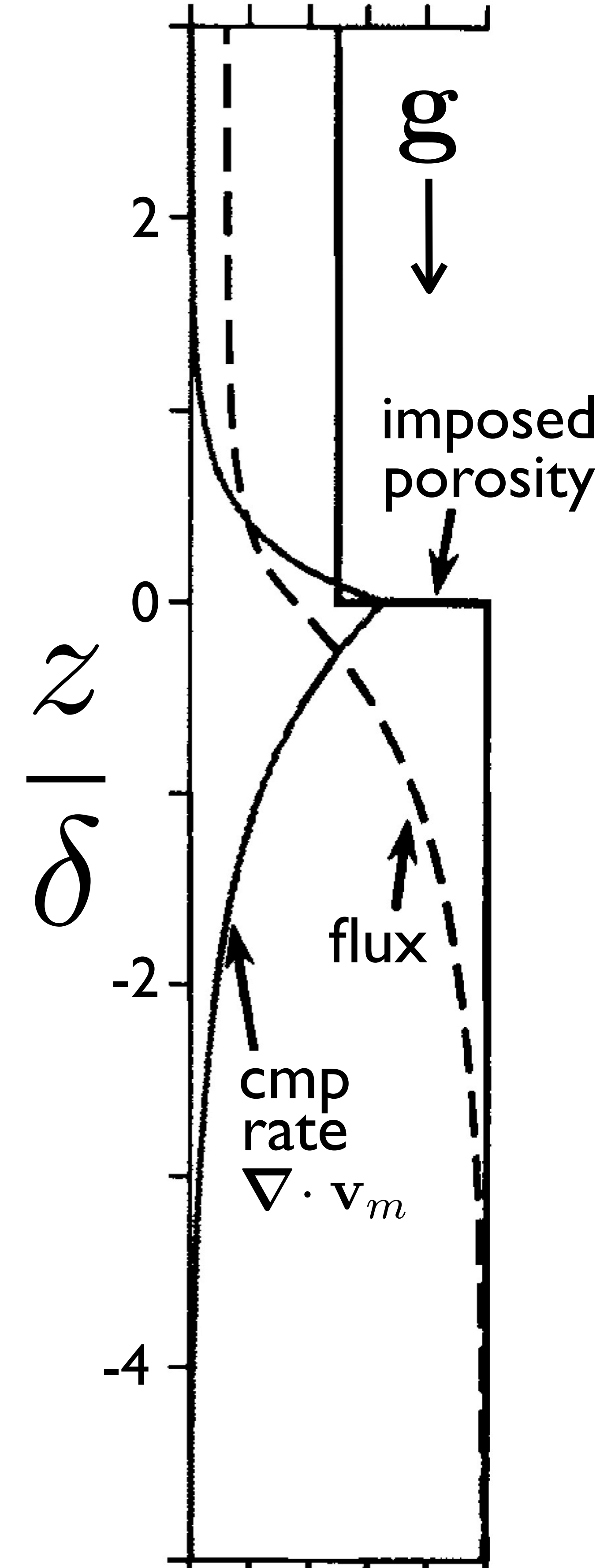
movie by M. Spiegelman

The compaction length

Natural length-scale:
Compaction length

$$\delta = \sqrt{\frac{(\zeta + 4\eta/3)K}{\mu}}$$

matrix bulk viscosity	ζ
matrix shear viscosity	η
permeability	K
melt viscosity	μ

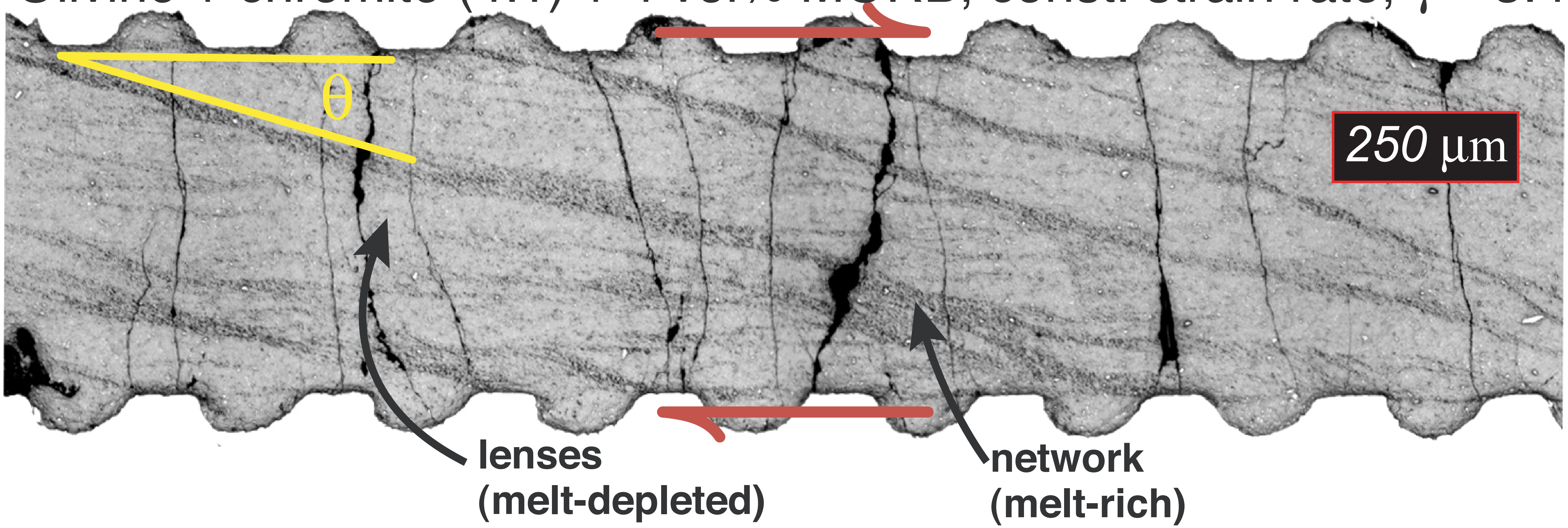


e.g. Spiegelman, *JFM* 1993b

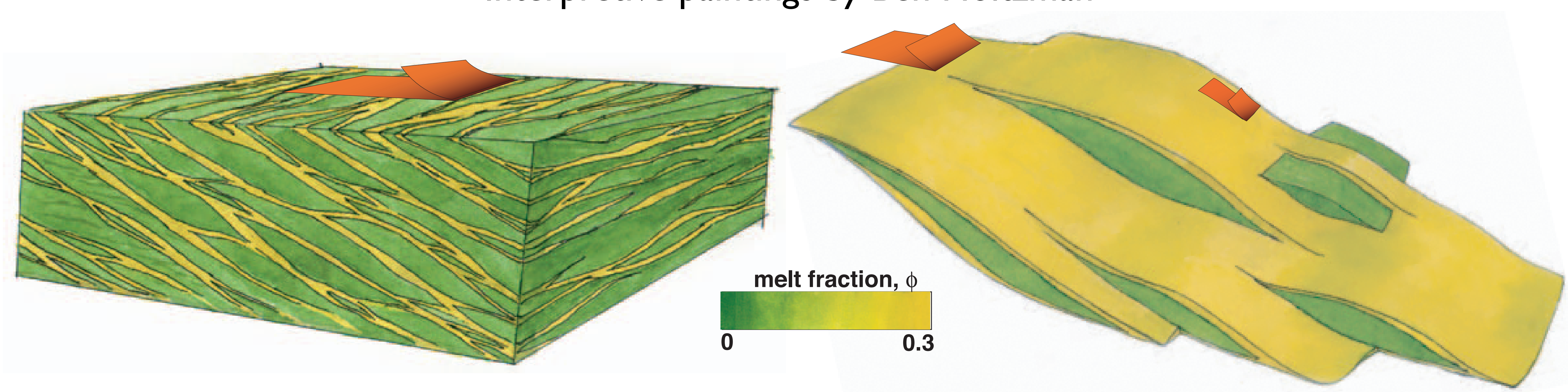
Shear/porosity bands

Laboratory experiment by Holtzman et. al., *G-cubed* 2003

Olivine + chromite (4:1) + 4 vol% MORB, const. strain rate, $\gamma = 3.4$



Interpretive paintings by Ben Holtzman



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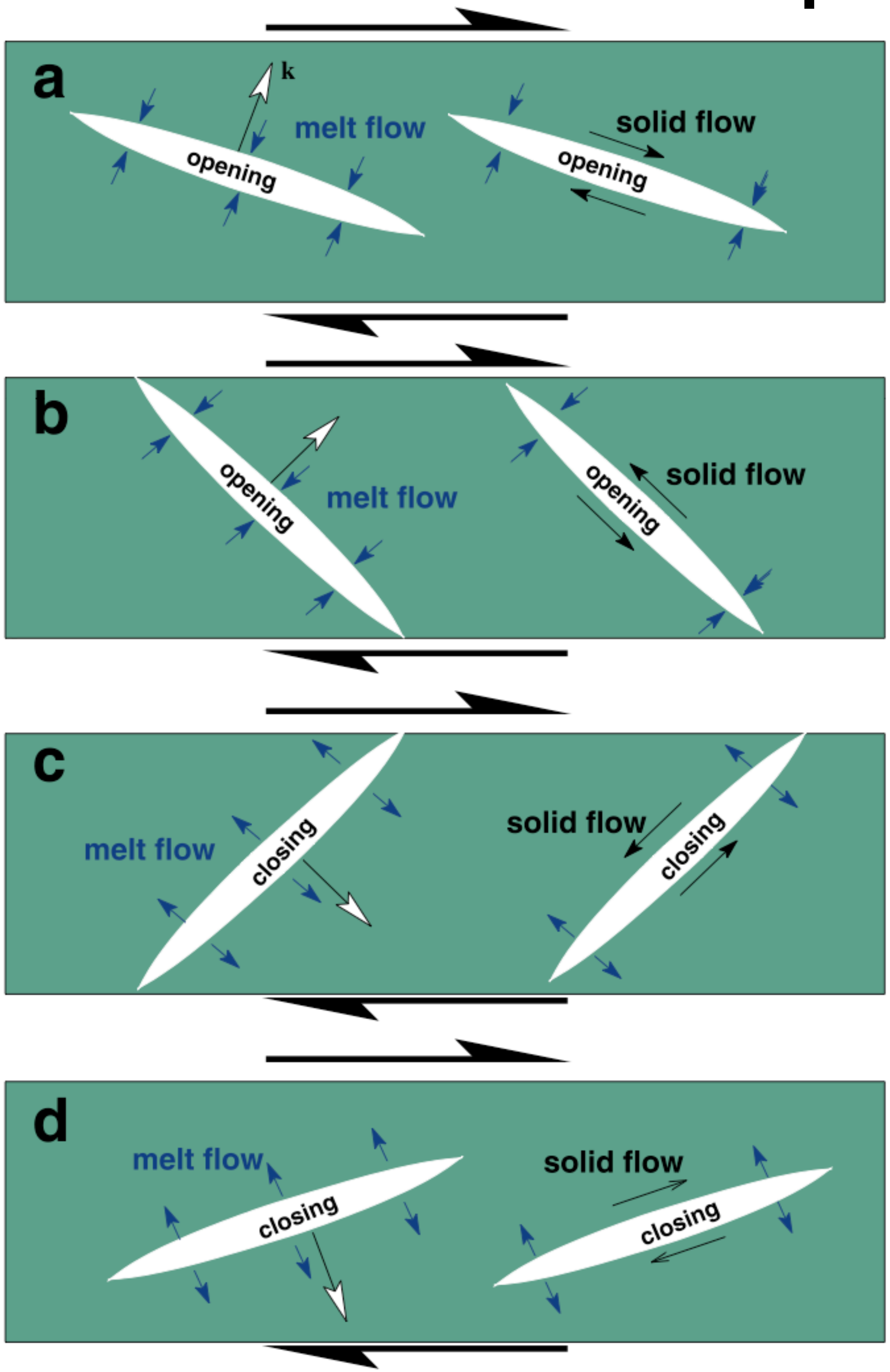
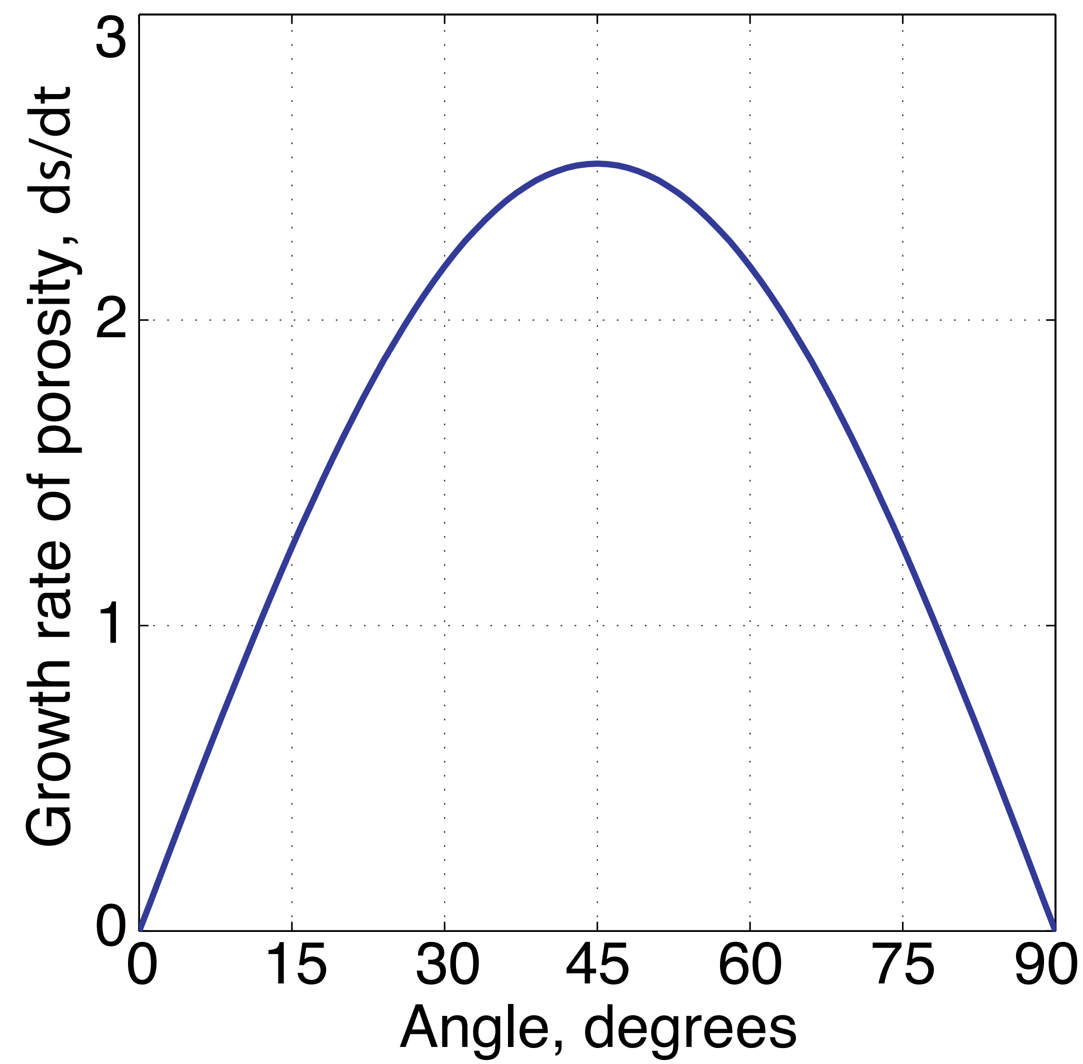


Stevenson, *GRL* '89; Spiegelman, *G^3* '03; Katz et al, *Nature* '07 [& others]

Linear stability analysis



$$\eta = \eta_0 \exp(-\alpha\phi)$$

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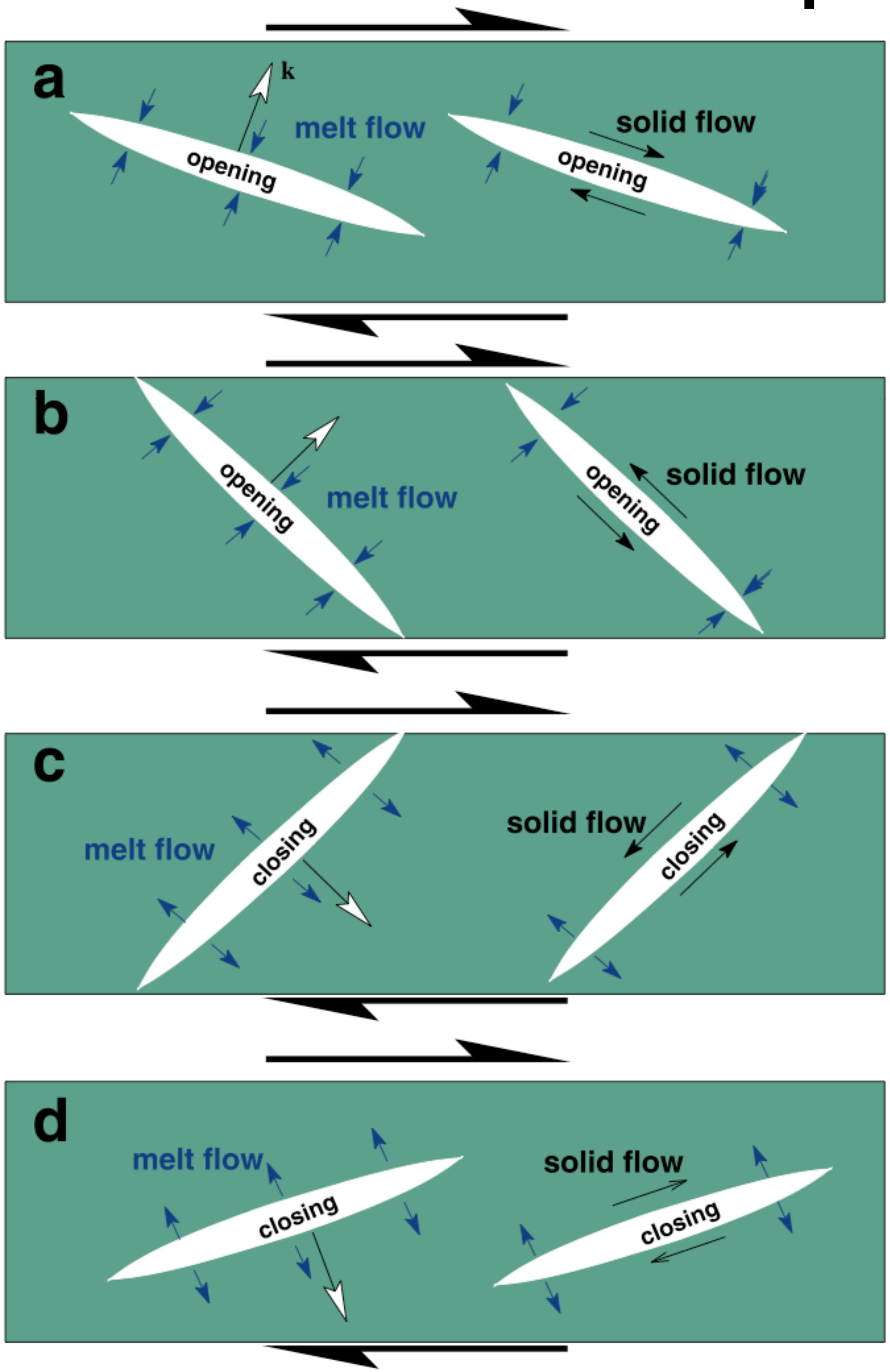

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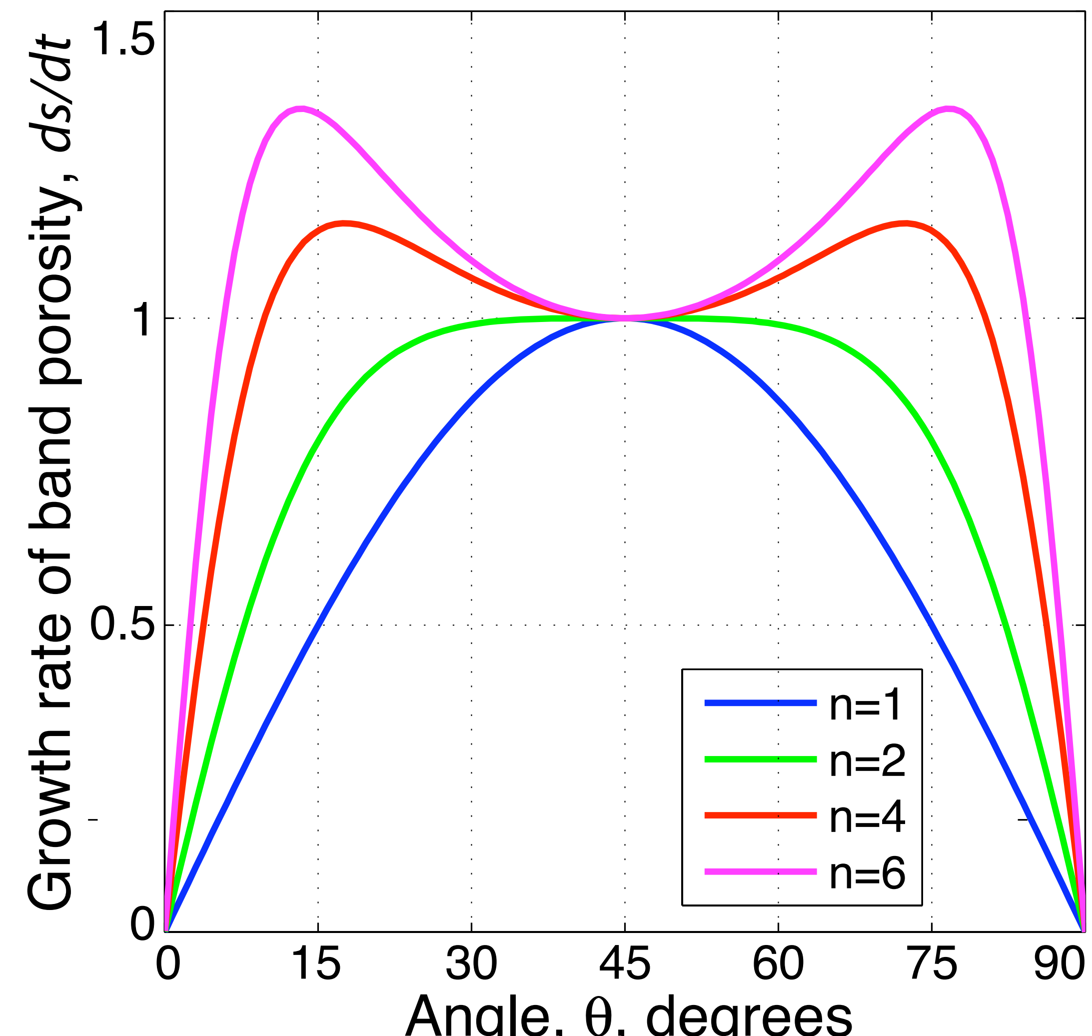
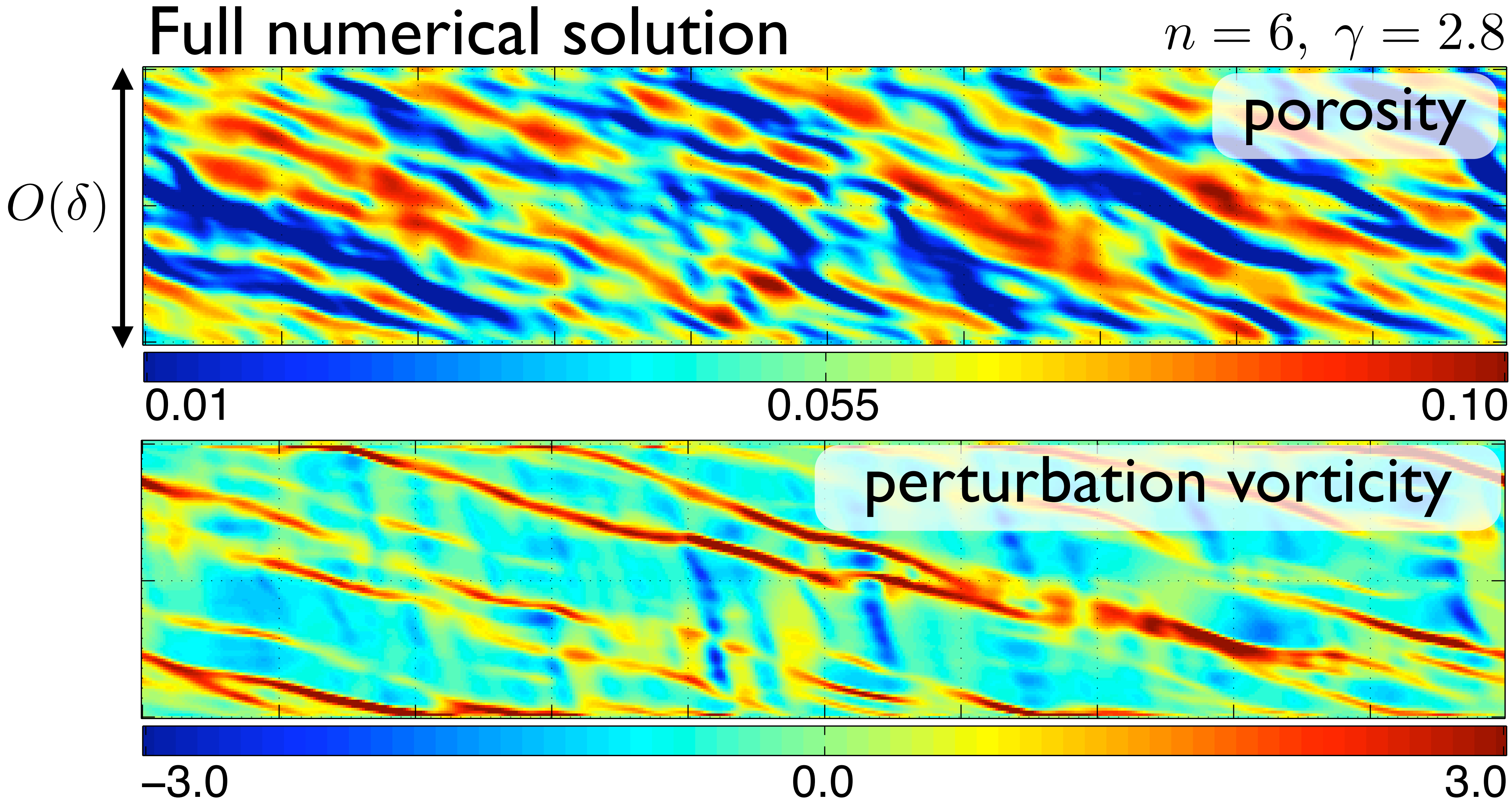


$$\eta = \eta_0 \exp(-\alpha\phi)\dot{\epsilon}_{II}^{\frac{1-n}{n}}$$

Shear/porosity bands



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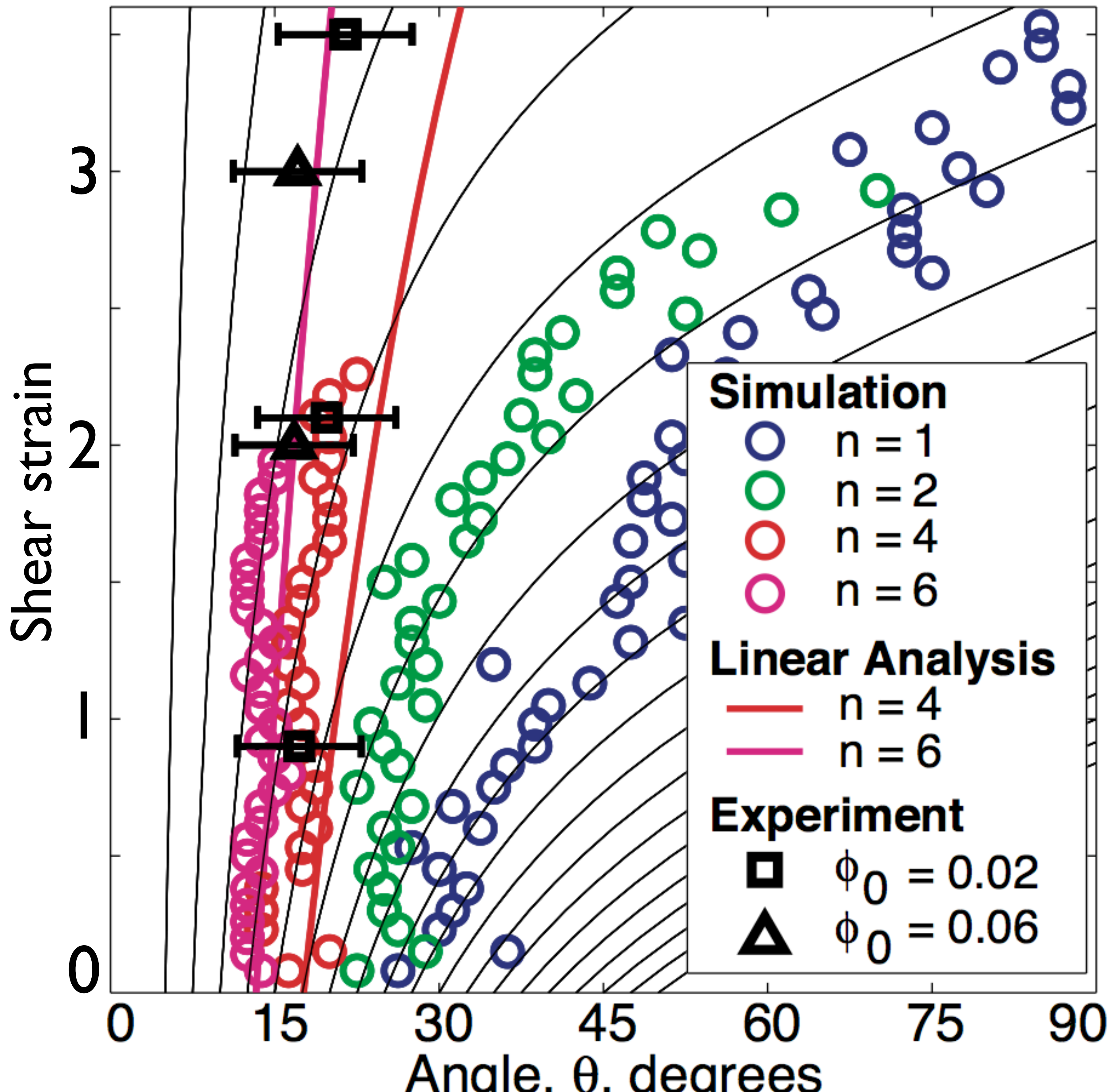
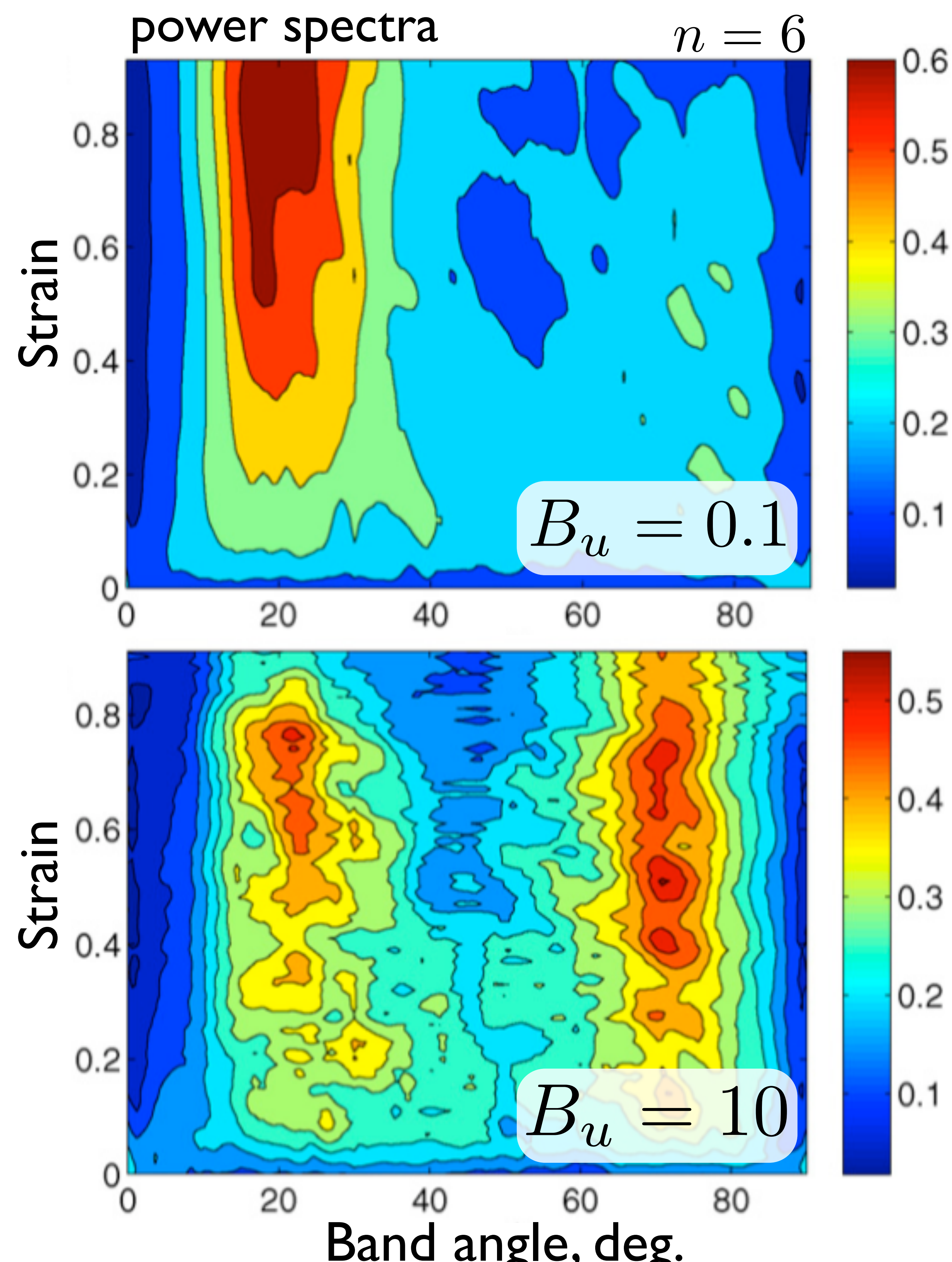


Shear/porosity bands with buoyancy

$$B_u = \frac{g\Delta\rho\delta}{\dot{\gamma}(\zeta_0 + 4/3\eta_0)} = \frac{\text{Buoyancy}}{\text{Shear stress}}$$

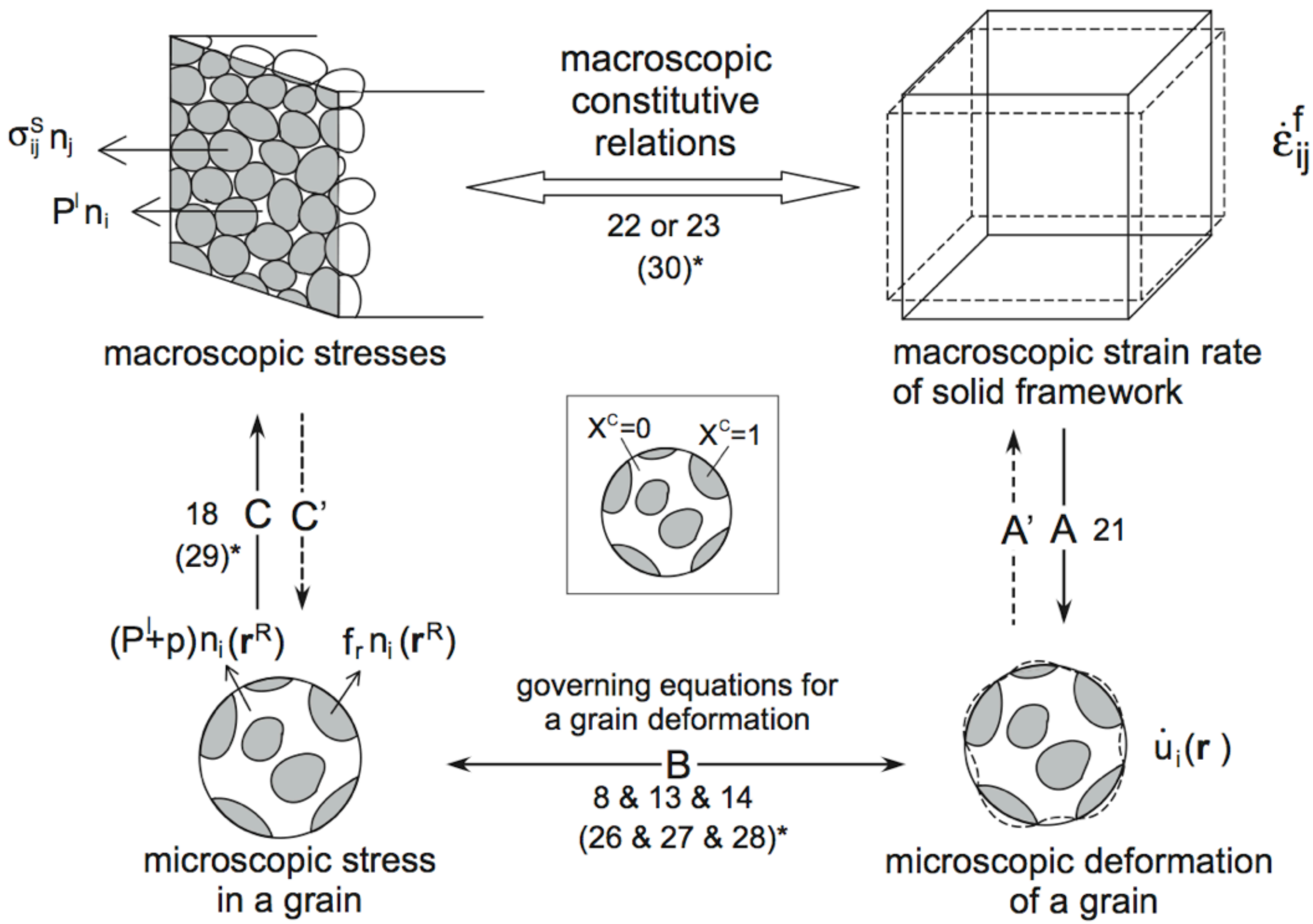
S.L. Butler, *PEPI* 2009



Shear/porosity bands

Melt-pocket alignment & anisotropic viscosity:

- Takei & Holtzman I, II, & III, *JGR* '09 (in press)
- Hier-Majumder, *JGR* '09 (in review)



Homogenization for viscosity tensor based on grain contiguity

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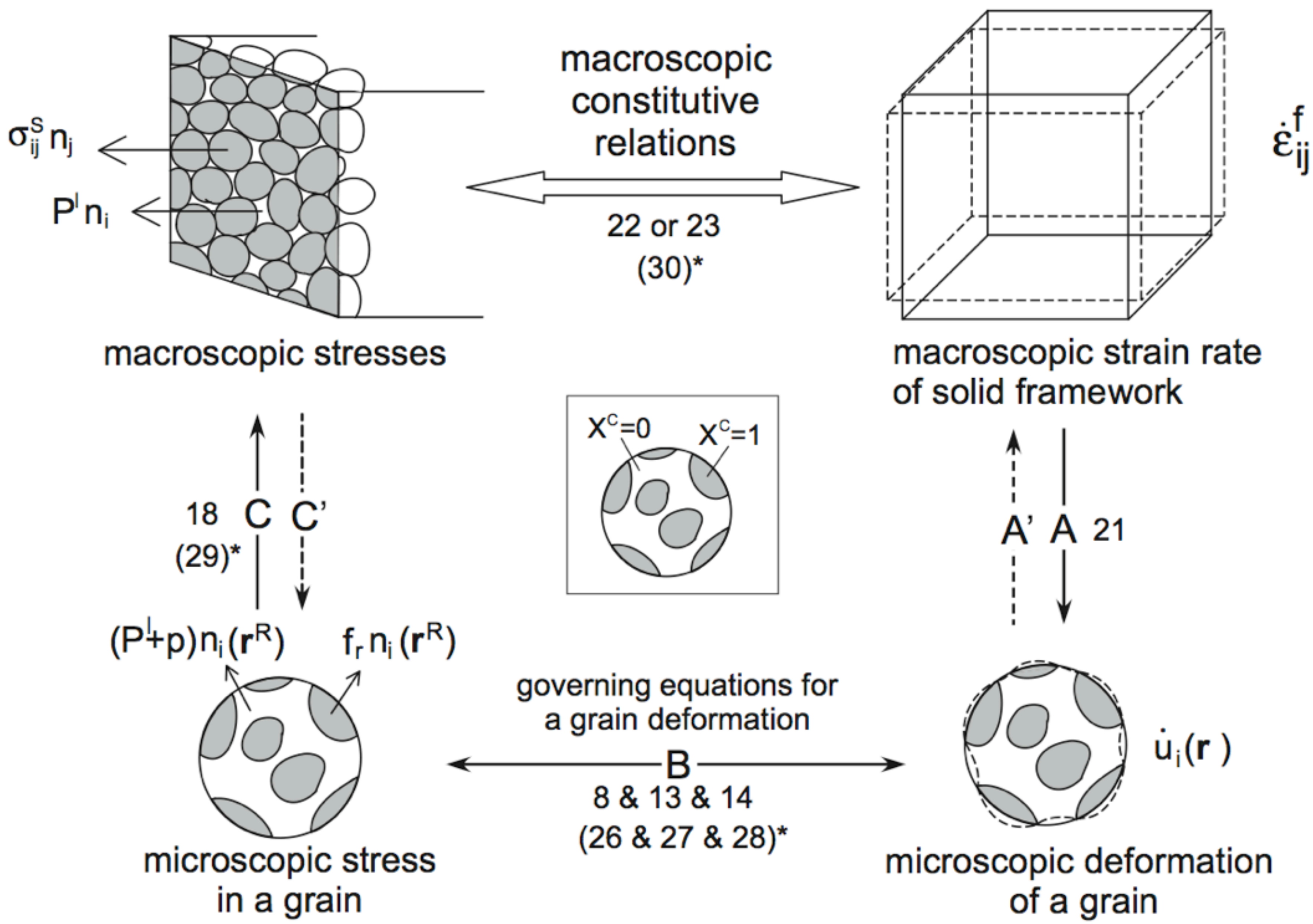

Homogenization for viscosity tensor based on grain contiguity

Also see Sash's poster, today

Mechanics - Summary

- Solitary waves serve as a benchmark for numerical codes.
- Role of solitary waves in the mantle?
- Shear/porosity banding in lab experiments = direct observations of magma dynamics.
- Role of shear/porosity banding in the mantle?

Mechanics - Summary

- Solitary waves serve as a benchmark for numerical codes.
- Role of solitary waves in the mantle?
- Shear/porosity banding in lab experiments = direct observations of magma dynamics.
- Role of shear/porosity banding in the mantle?

To model melting and solidification we need to consider the thermodynamics of the magma/mantle system!

2.1 Thermochemistry: conservation eqns

(e.g. McKenzie ’84, Šrámek et al ’07, Hewitt & Fowler ’08)

Conservation of energy

$$\frac{D_{\mathbf{v}_m}}{Dt}[(1-\phi)\rho_m e_m] + \frac{D_{\mathbf{v}_f}}{Dt}[\phi\rho_f e_f] = \boldsymbol{\nabla}\cdot k\boldsymbol{\nabla}T + \Psi$$

internal energy

diffusion

viscous dissipation

2.1 Thermochemistry: conservation eqns

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internal energy — diffusion — viscous dissipation

(e.g. McKenzie '84, Spiegelman '01)

Conservation of species mass (2 component)

$$\frac{D_m c^m}{Dt} = -\frac{c^R - c^s}{1-\phi}\frac{\Gamma}{\rho_m} \qquad \text{(solid phase)}$$

component mass — reaction

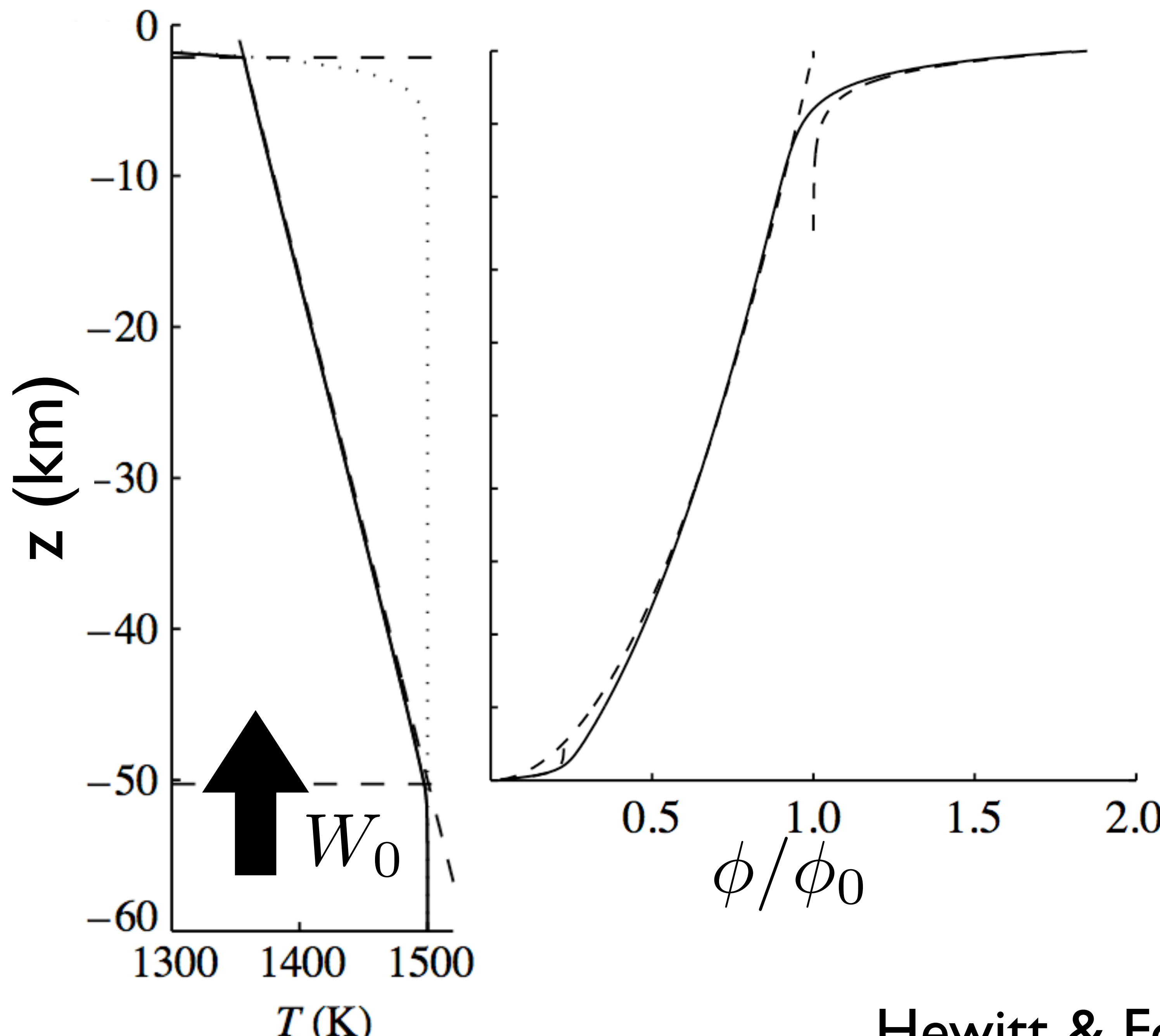
diffusion

$$\frac{D_f c^f}{Dt} = \frac{c^R - c^f}{\phi}\frac{\Gamma}{\rho_f} + \mathcal{D}\nabla^2 c^f \qquad \text{(fluid phase)}$$

2.2

One-component upwelling columns

- Fixed mantle upwelling rate into melting zone.
- Only vertical segregation of melt.
- Conservation of mass, momentum, energy.



Hewitt & Fowler '08, (see also: Šrámek et al '07)

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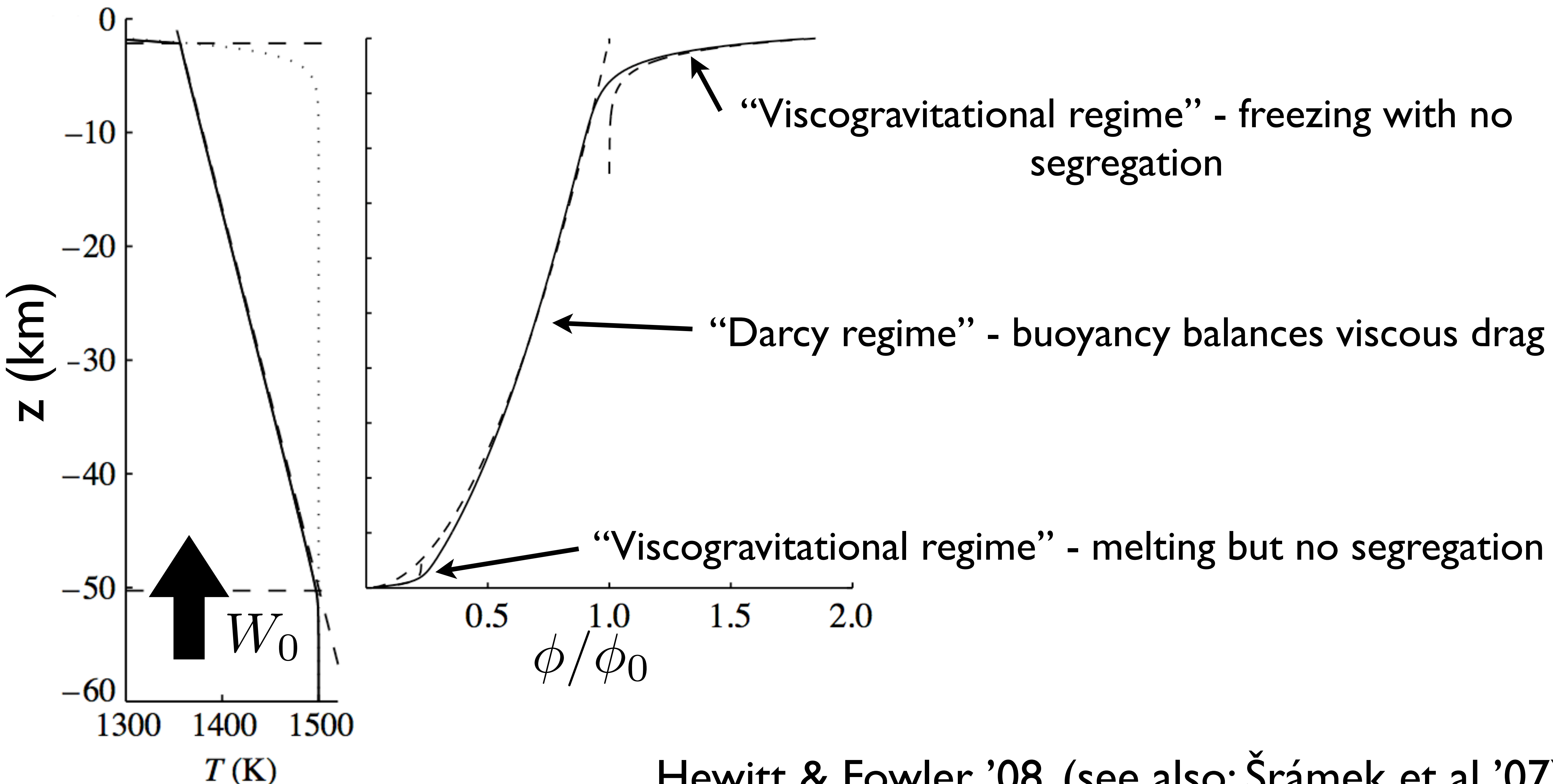
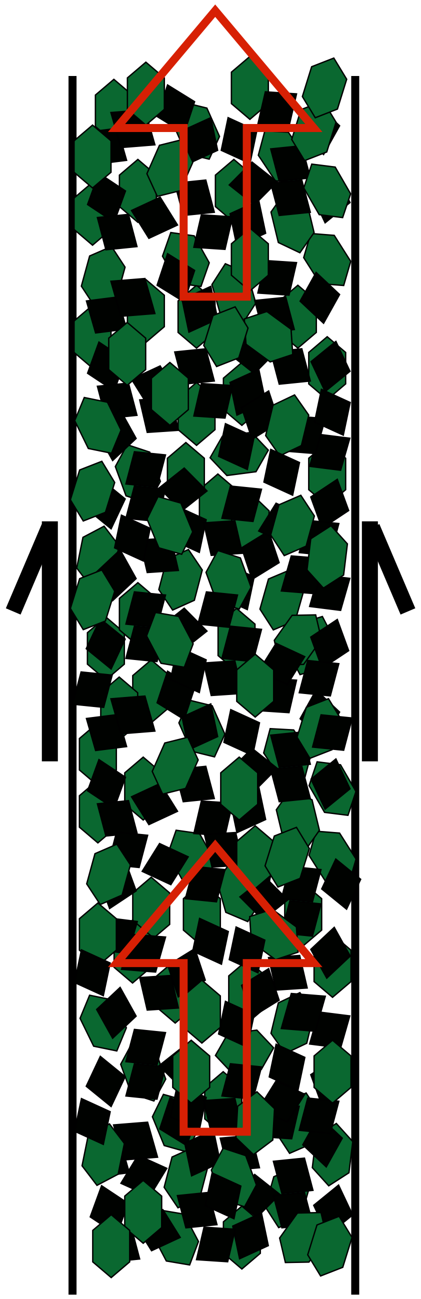


Hewitt & Fowler '08, (see also: Šrámek et al '07)

2.3

Reactive magmatic flow



Melting reaction:
Melt + Pyroxene +
Spinel » Olivine + Melt

Olivine: (Mg,Fe)SiO_2

Pyroxene: (Mg,Fe)Si_2O_6

Mantle flow

Magmatic flow

Aharonov et al, GRL 1995; Spiegelman, *JGR* ‘01

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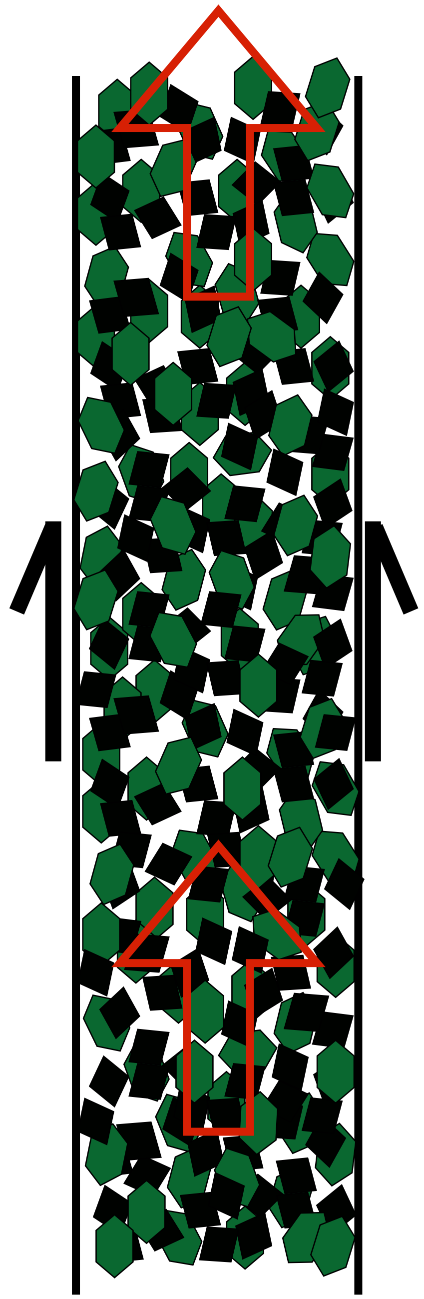

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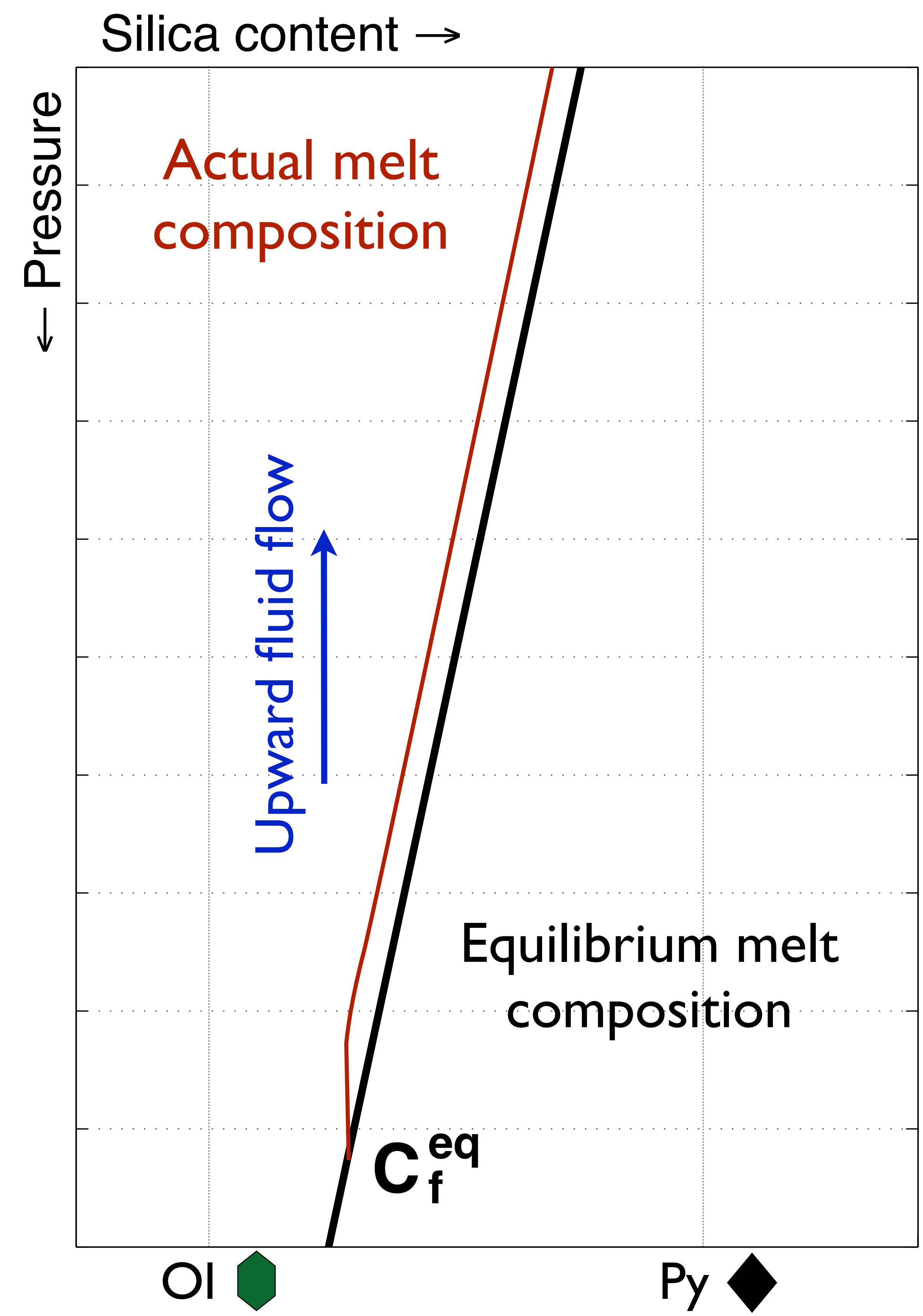

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Mantle flow

Magmatic flow

Aharonov et al, GRL 1995; Spiegelman, *JGR* '01

Reactive magmatic flow

- Ignore energy conservation.
- Ignore large-scale matrix shear.
- Disequilibrium melting
- Explicit melting rate (linear kinetics):

$$\Gamma = RA(\phi, c^s)\left[c^f_{eq} - c^f\right]$$

Aharonov et al, GRL 1995; Spiegelman, *JGR* '01

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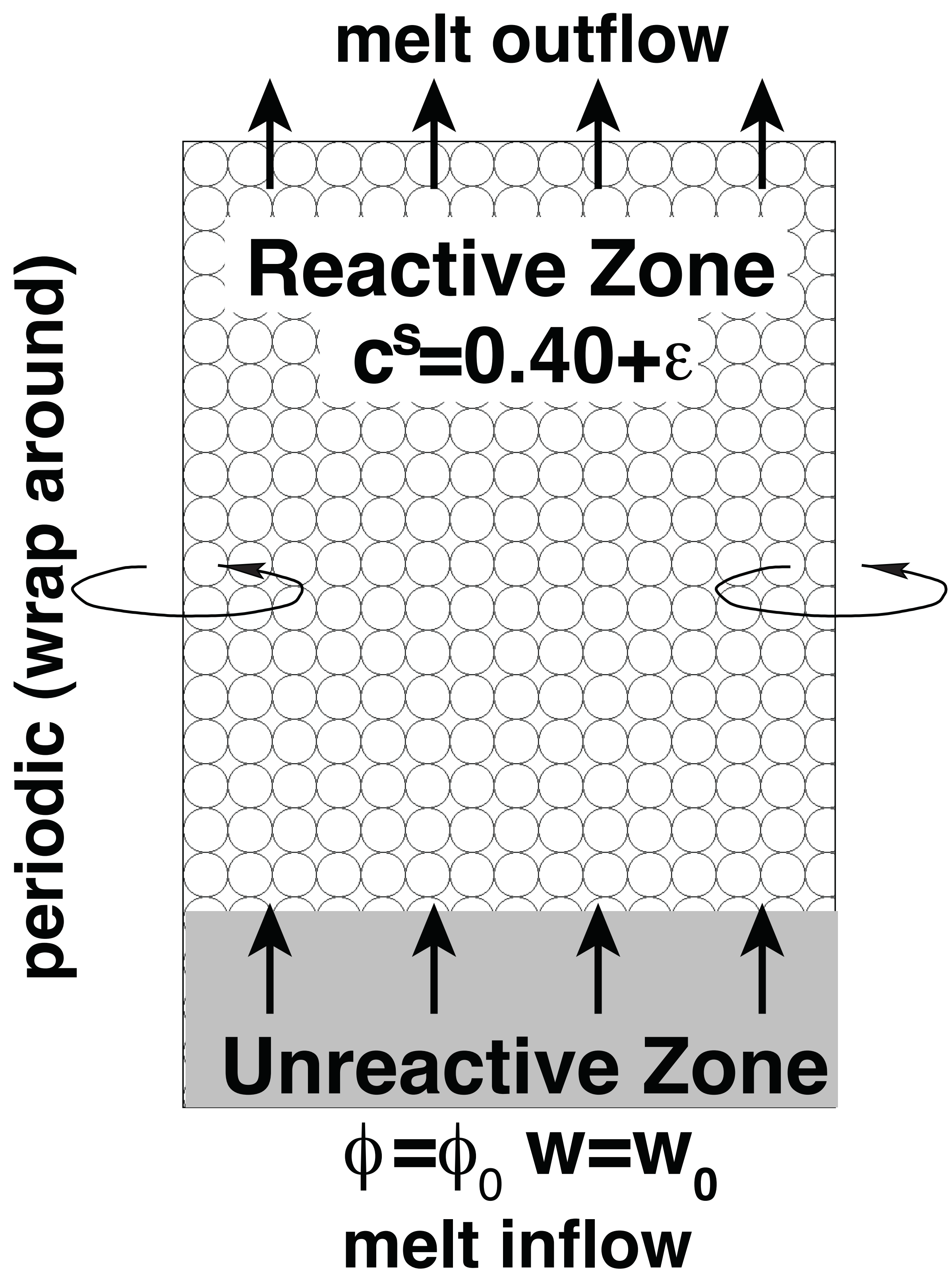
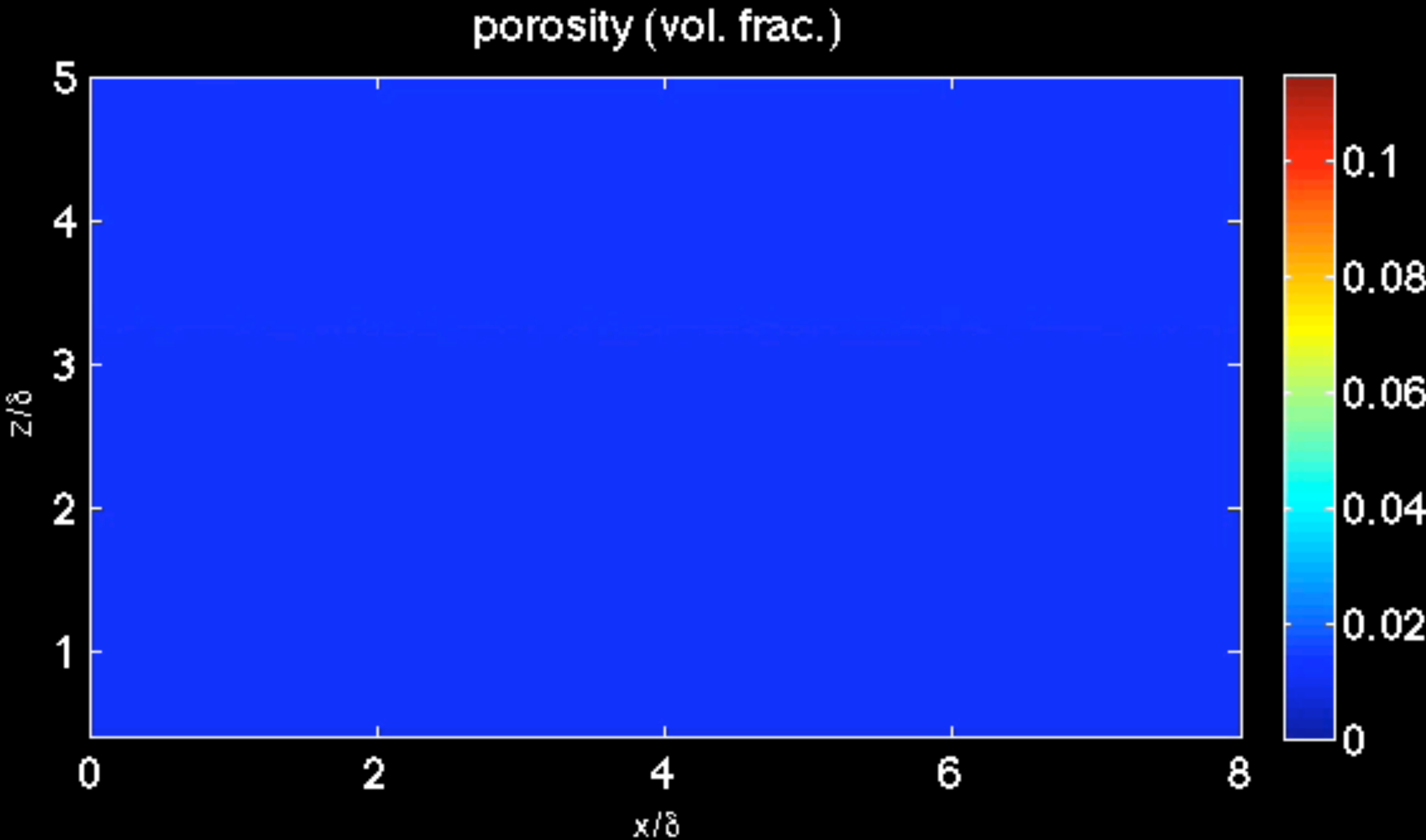


Aharonov et al, GRL 1995; Spiegelman, *JGR* '01

2.3

Reactive infiltration instability



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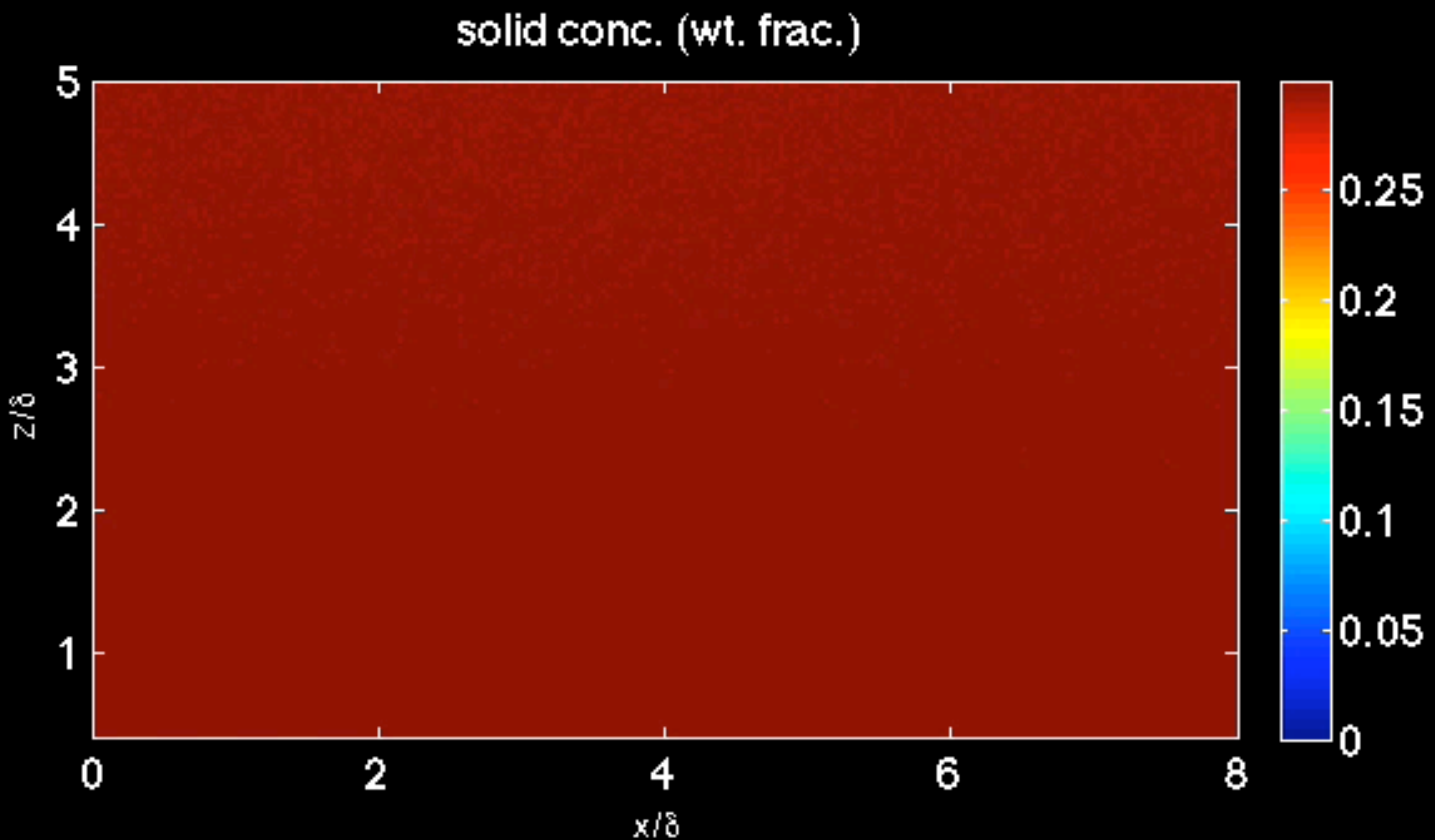

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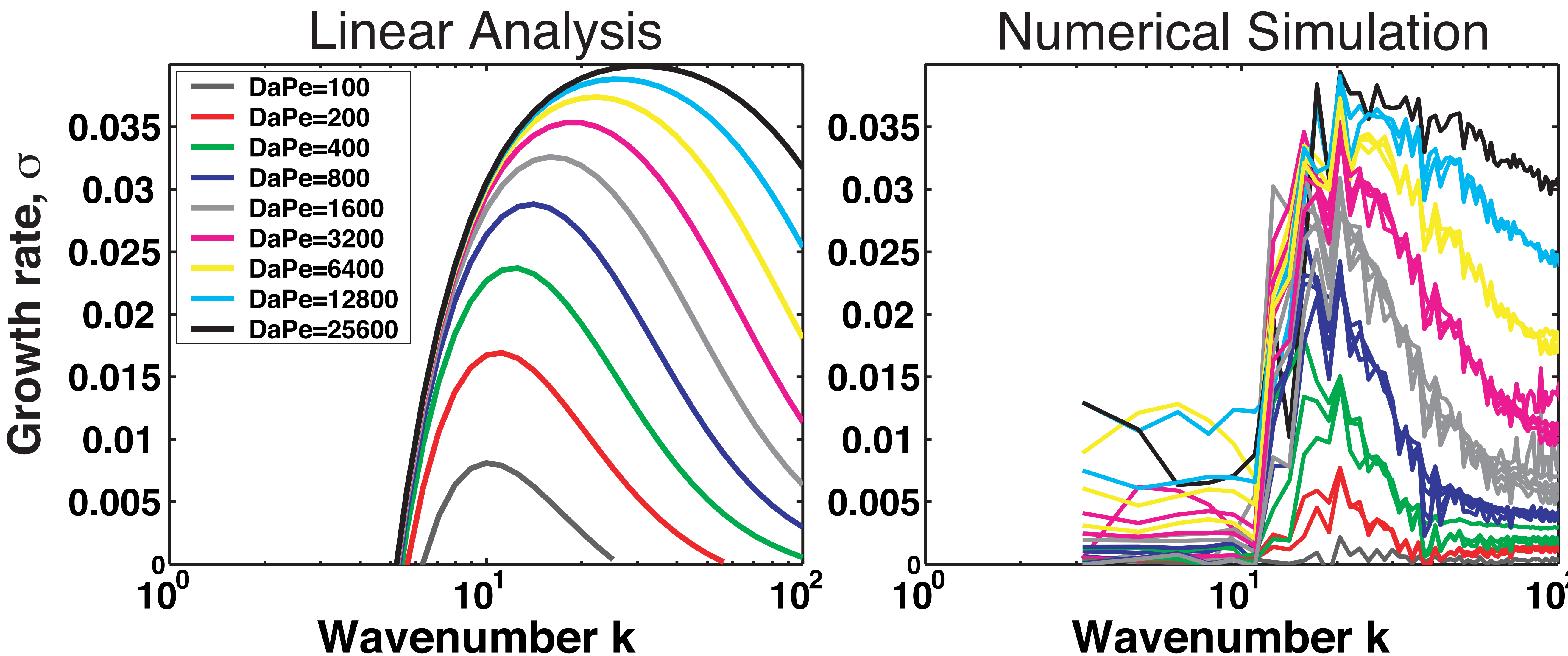
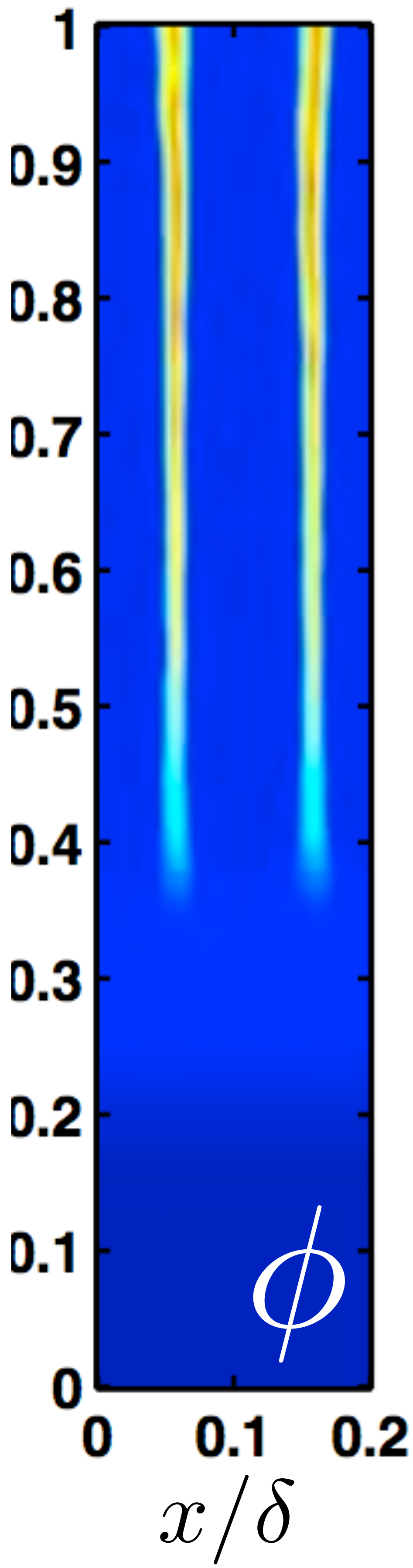
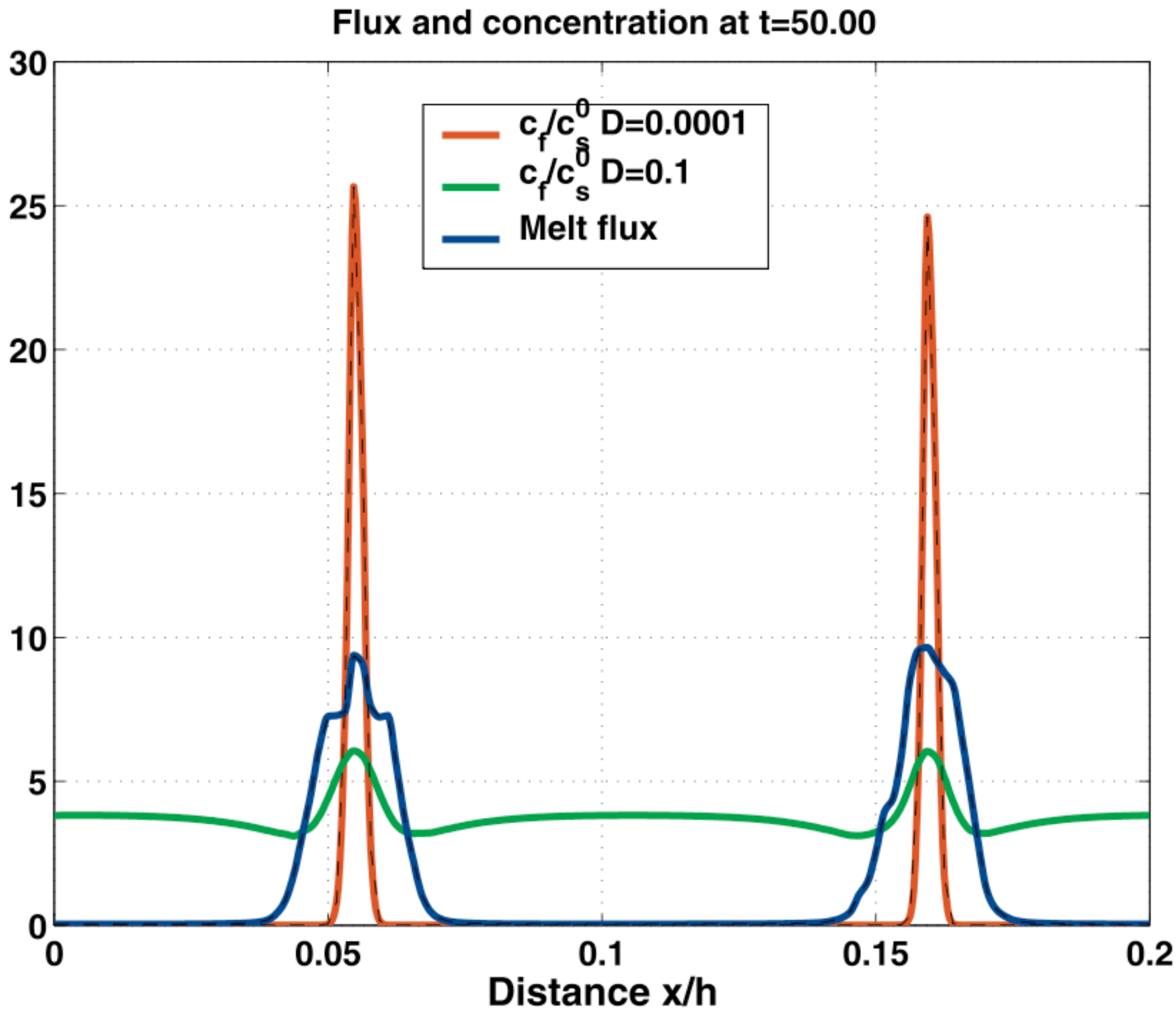


Chemical consequences





Spiegelman & Kelemen, *G-cubed* '03

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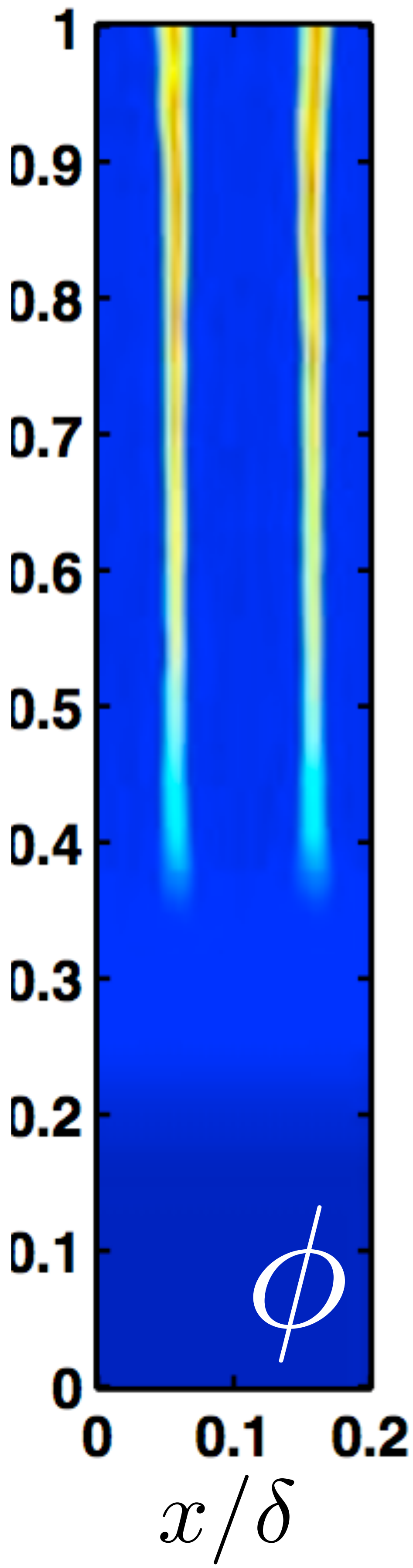
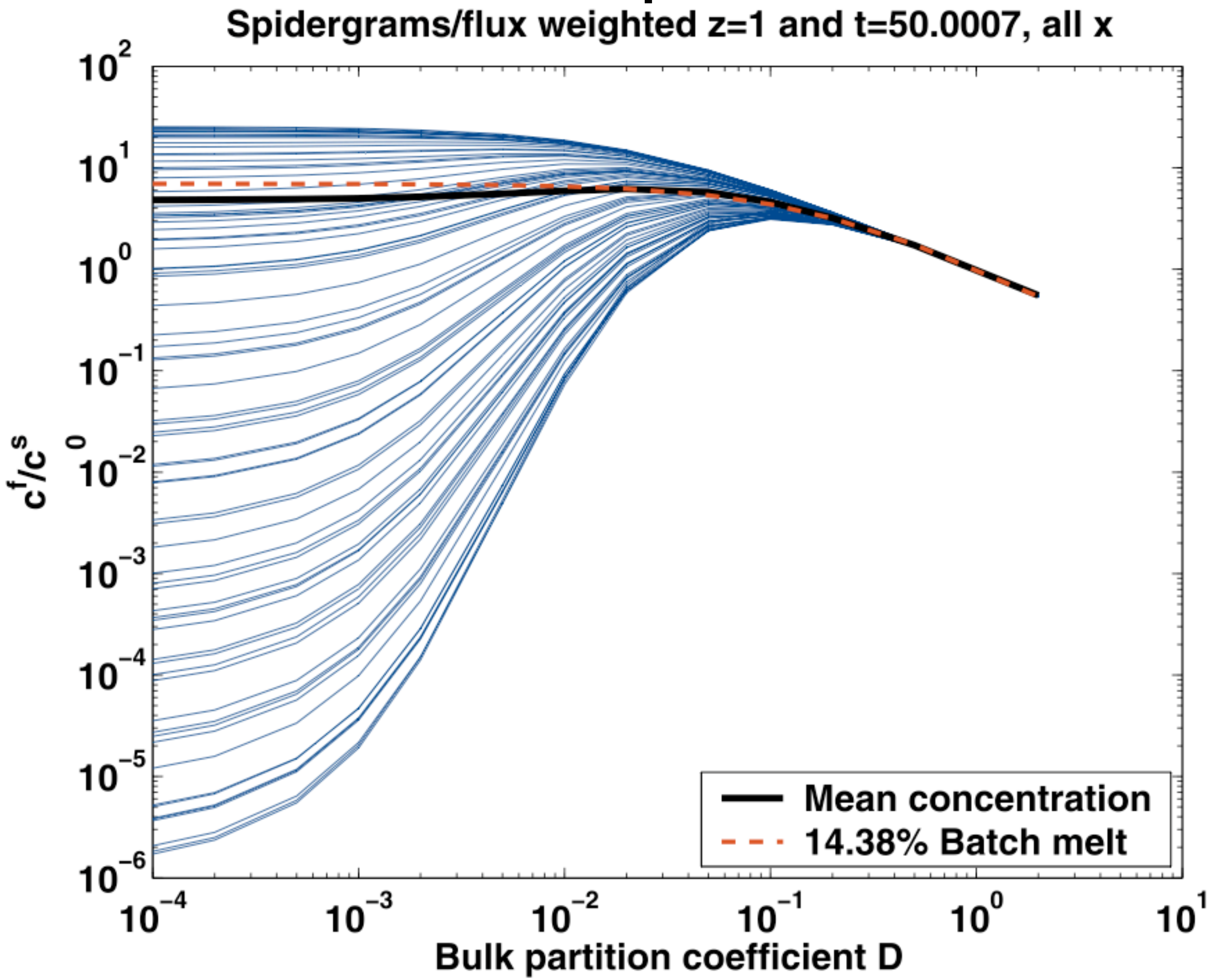


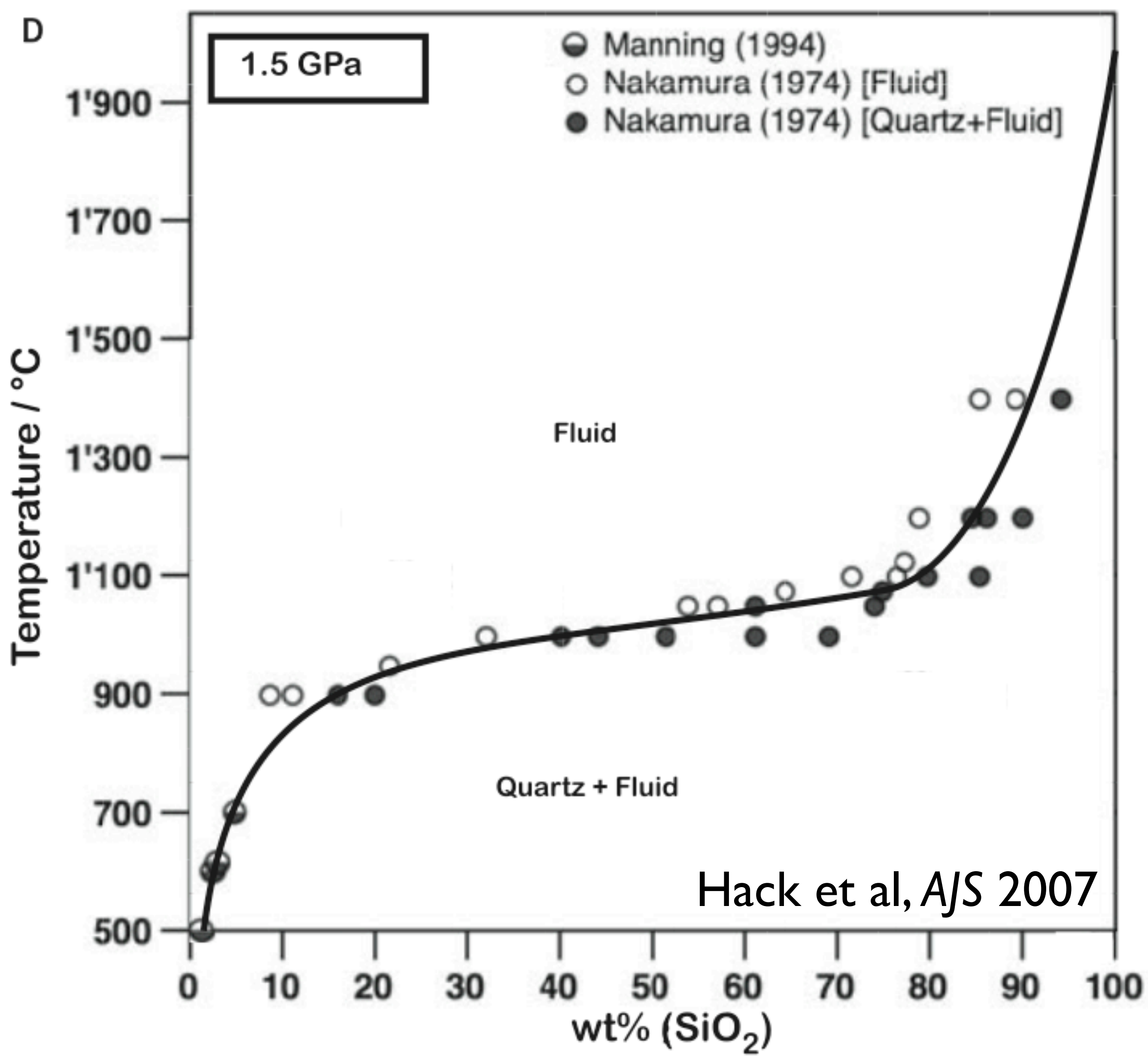


Spiegelman & Kelemen, *G-cubed* '03

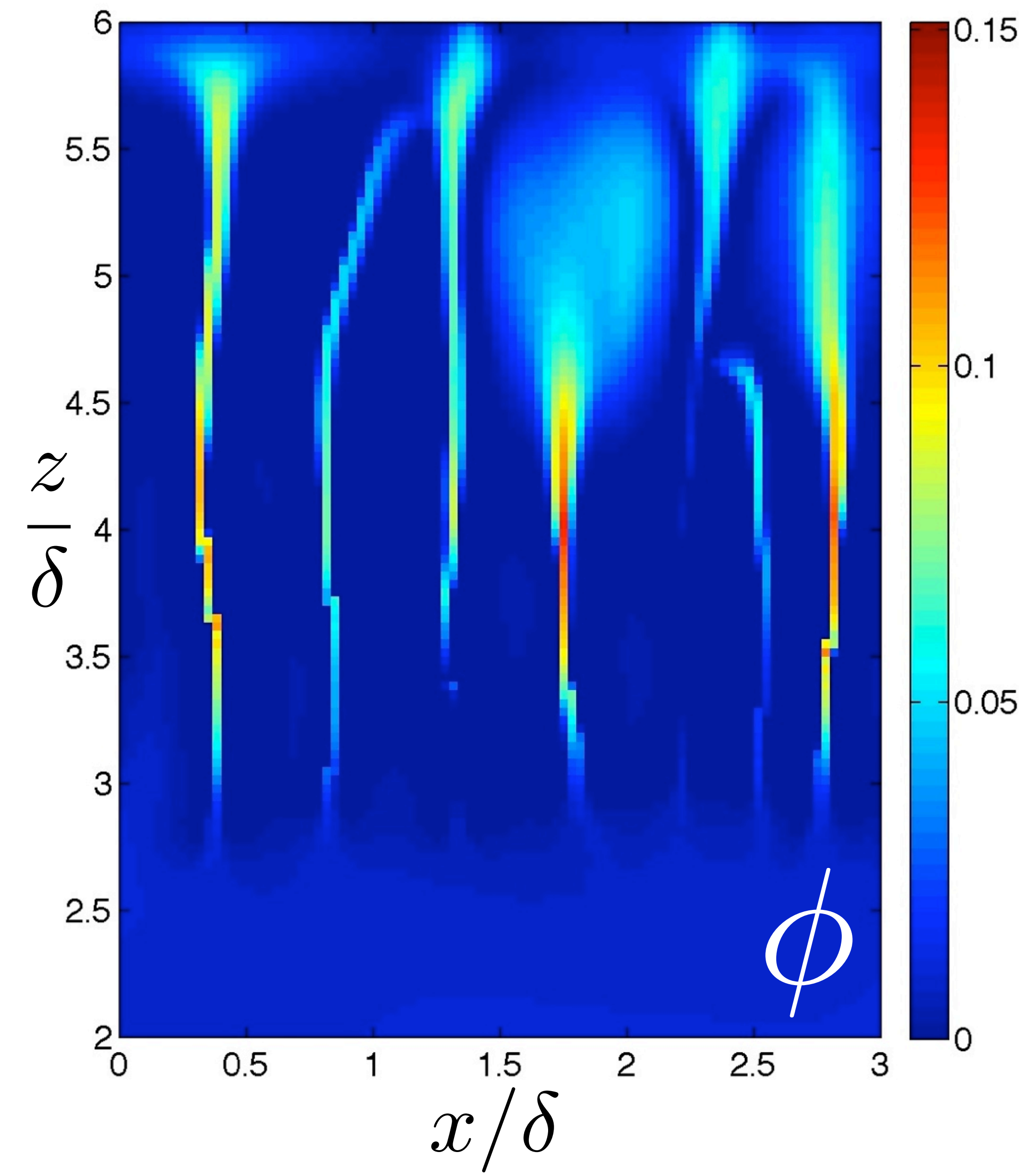
2.3

Hydrous reactive melting

- Reactive melting in a subduction zone above the slab?
- T-dependent rock solubility.
- Solve conservation of energy for T.



Hack et al, *AJS* 2007



Katz et al, *in prep.* 2009

Thermochemistry - Summary

- Reactive flow produces channelized melt flux via the Reactive Infiltration Instability.
- Predictions for trace elements & U-series in MORB.
- Consequences of channelized flow in arcs?
- Embedded in large-scale mantle flow. Interaction with shear instability? [SW]

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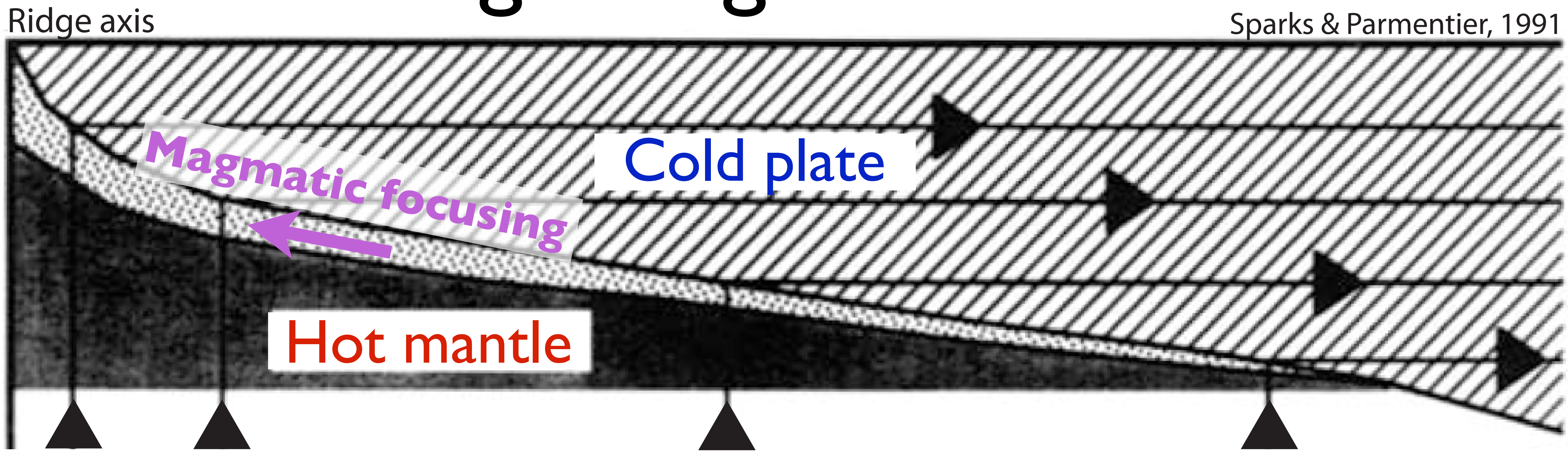
How to couple melting and magmatic flow with tectonic-scale mantle dynamics?

Putting it together: tectonic scale



Melt focusing by sublithospheric flow

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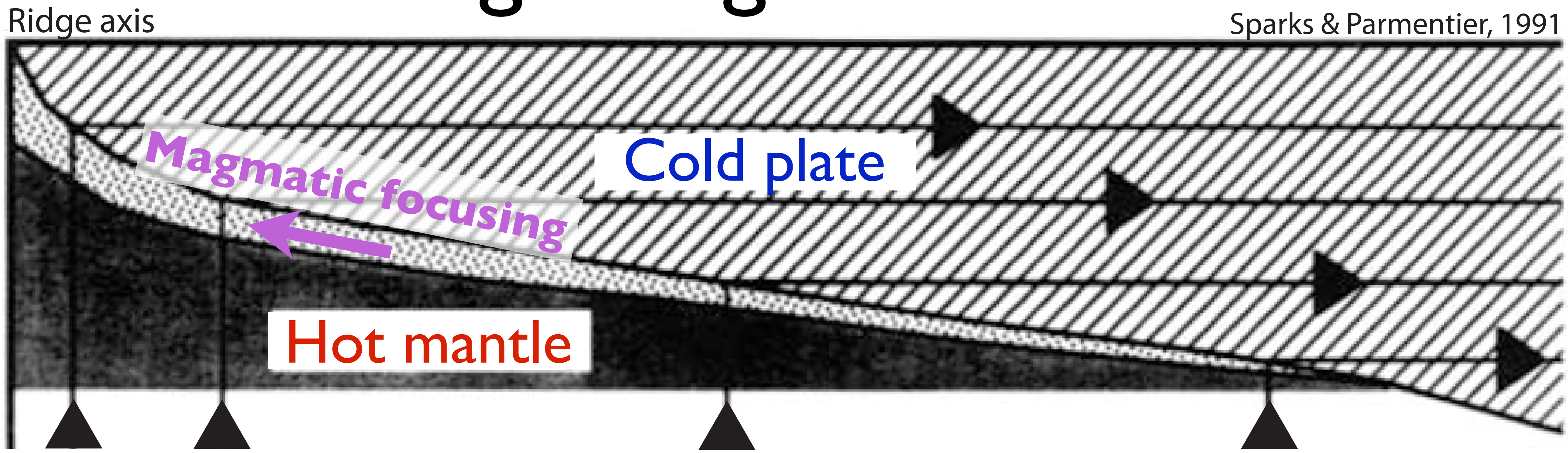
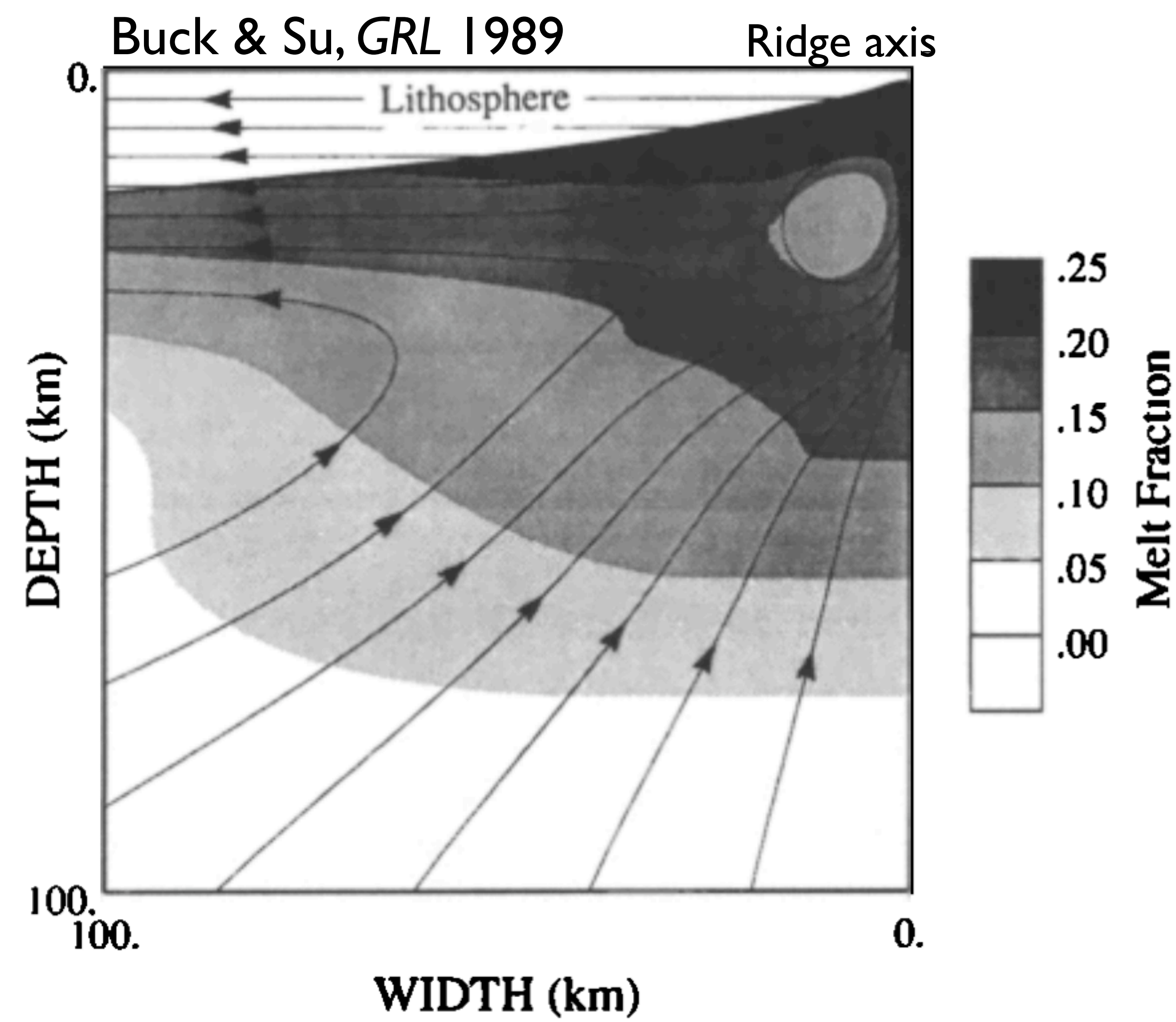


Melt focusing by sublithospheric flow

Porosity-driven mantle convection



Putting it together: problems & solns

- Expect zero porosity within the domain. Bulk viscosity is singular.
  Use pressure decomposition to reformulate.

- Expect melting and freezing. Treat these in a thermodynamically consistent way.
  Use Enthalpy Method to model thermochem.

- Melting is usually hydrous.
  Extend Enthalpy Method to 3-component system?

Putting it together: zero porosity

compaction viscosity: $\xi = \zeta - 2\eta/3$

compaction viscosity for small porosity: $\xi \mathrel{\tilde{\propto}} \dfrac{1}{\phi}$ singularity!

Pressure decomposition: $$\underbrace{P_f}_{\text{Fluid pressure}} = \underbrace{\rho g z}_{\text{Lithostatic}} + \underbrace{\mathcal{P}}_{\text{Compaction}} + \underbrace{P}_{\text{“Dynamic”}}$$

Katz *et al.* *PEPI* 2007

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$$\boxed{\begin{gathered}
\boldsymbol{\nabla}\cdot\mathbf{v}_m = \mathcal{P}/\xi \\
\boldsymbol{\nabla} P = \boldsymbol{\nabla}\cdot\left[\eta(\boldsymbol{\nabla}\mathbf{v}_m + \boldsymbol{\nabla}\mathbf{v}_m^T)\right] + \phi\Delta\rho\mathbf{g} \\
-\boldsymbol{\nabla}\cdot\frac{K}{\mu}\boldsymbol{\nabla}\mathcal{P} + \frac{\mathcal{P}}{\xi} = \boldsymbol{\nabla}\cdot\frac{K}{\mu}\left[\boldsymbol{\nabla} P + \Delta\rho\mathbf{g}\right] + \Gamma\frac{\Delta\rho}{\rho}
\end{gathered}}$$

Katz *et al.* *PEPI* 2007

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compaction viscosity for small porosity: $\xi \tilde{\propto} \dfrac{1}{\phi}$ singularity!

Pressure decomposition: $P_f = \rho g z + \mathcal{P} + P$

Fluid pressure

Lithostatic

Compaction

"Dynamic"

$$\nabla \cdot \mathbf{v}_m = \mathcal{P}/\xi$$

$$\nabla P = \nabla \cdot [\eta(\nabla \mathbf{v}_m + \nabla \mathbf{v}_m^T)] + \phi \Delta \rho \mathbf{g}$$

$$-\nabla \cdot \frac{K}{\mu} \nabla \mathcal{P} + \frac{\mathcal{P}}{\xi} = \nabla \cdot \frac{K}{\mu} [\nabla P + \Delta \rho \mathbf{g}] + \Gamma \frac{\Delta \rho}{\rho}$$

Katz *et al. PEPI* 2007

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Katz *et al. PEPI* 2007

Putting it together: Enthalpy method

Assume: $d\mathcal{H} = c_P dT + \rho^{-1}(1-\alpha T)dP$ thermodynamic eq

$C = \phi C_f + (1-\phi)C_m$ two components

Define: $T = \mathcal{T}\mathrm{e}^{\alpha g z/c_P}$ potential temperature

Cons. of energy

$$\underbrace{\frac{\partial\mathcal{H}}{\partial t}}_{\text{bulk enthalpy}} + \underbrace{\rho c_P \mathrm{e}^{\frac{\alpha g z}{c_P}}\boldsymbol{\nabla}\cdot\overline{\mathbf{v}}\mathcal{T}}_{\text{advection of sensible heat}} = \underbrace{\rho L\boldsymbol{\nabla}\cdot(1-\phi)\mathbf{v}_m}_{\text{advection of latent heat}} + \underbrace{k\mathrm{e}^{\frac{\alpha g z}{c_P}}\nabla^2\mathcal{T}}_{\text{diffusion}}$$

Katz, *JPet* 2008

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Cons. of energy

bulk enthalpy, advection of sensible heat, advection of latent heat, diffusion

$$\frac{\partial \mathcal{H}}{\partial t} + \rho c_P \mathrm{e}^{\frac{\alpha g z}{c_P}} \boldsymbol{\nabla}\cdot\overline{\mathbf{v}}\mathcal{T} = \rho L \boldsymbol{\nabla}\cdot(1-\phi)\mathbf{v}_m + k\mathrm{e}^{\frac{\alpha g z}{c_P}}\nabla^2\mathcal{T}$$

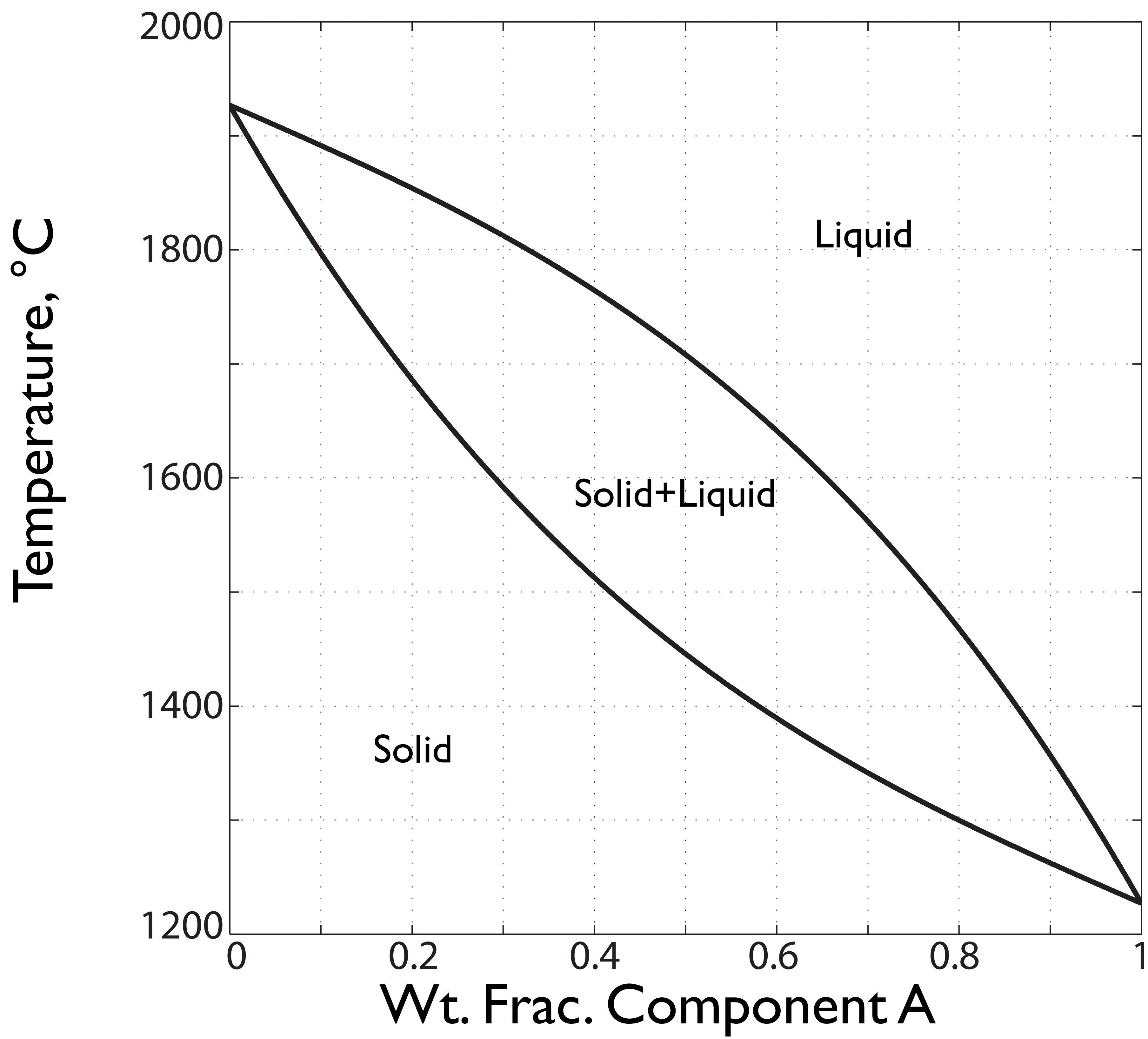
Cons. of species mass

bulk composition, advection, diffusion

$$\frac{\partial C}{\partial t} + \underbrace{\boldsymbol{\nabla}\cdot\phi\mathbf{v}_f C_f + \boldsymbol{\nabla}\cdot(1-\phi)\mathbf{v}_m C_m}_{\text{advection}} = \mathcal{D}\boldsymbol{\nabla}\cdot\phi\boldsymbol{\nabla}C_f$$

Katz, *JPet* 2008

3.1 Putting it together: Enthalpy method



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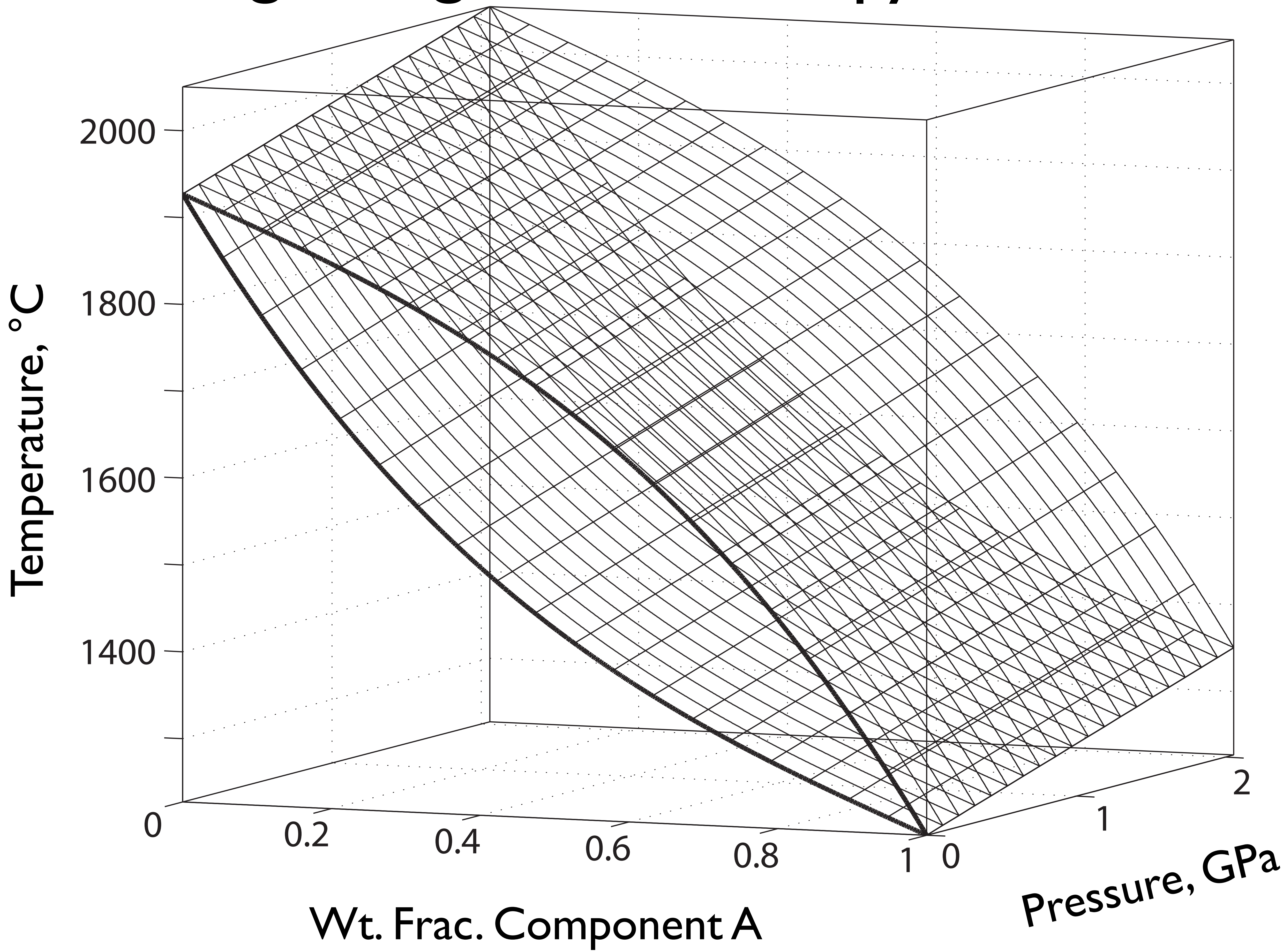


Melt focusing at mid-ocean ridges

$$\eta = \eta_0 \exp\left(\frac{E^*}{RT}\right)\exp(-\alpha\phi)$$

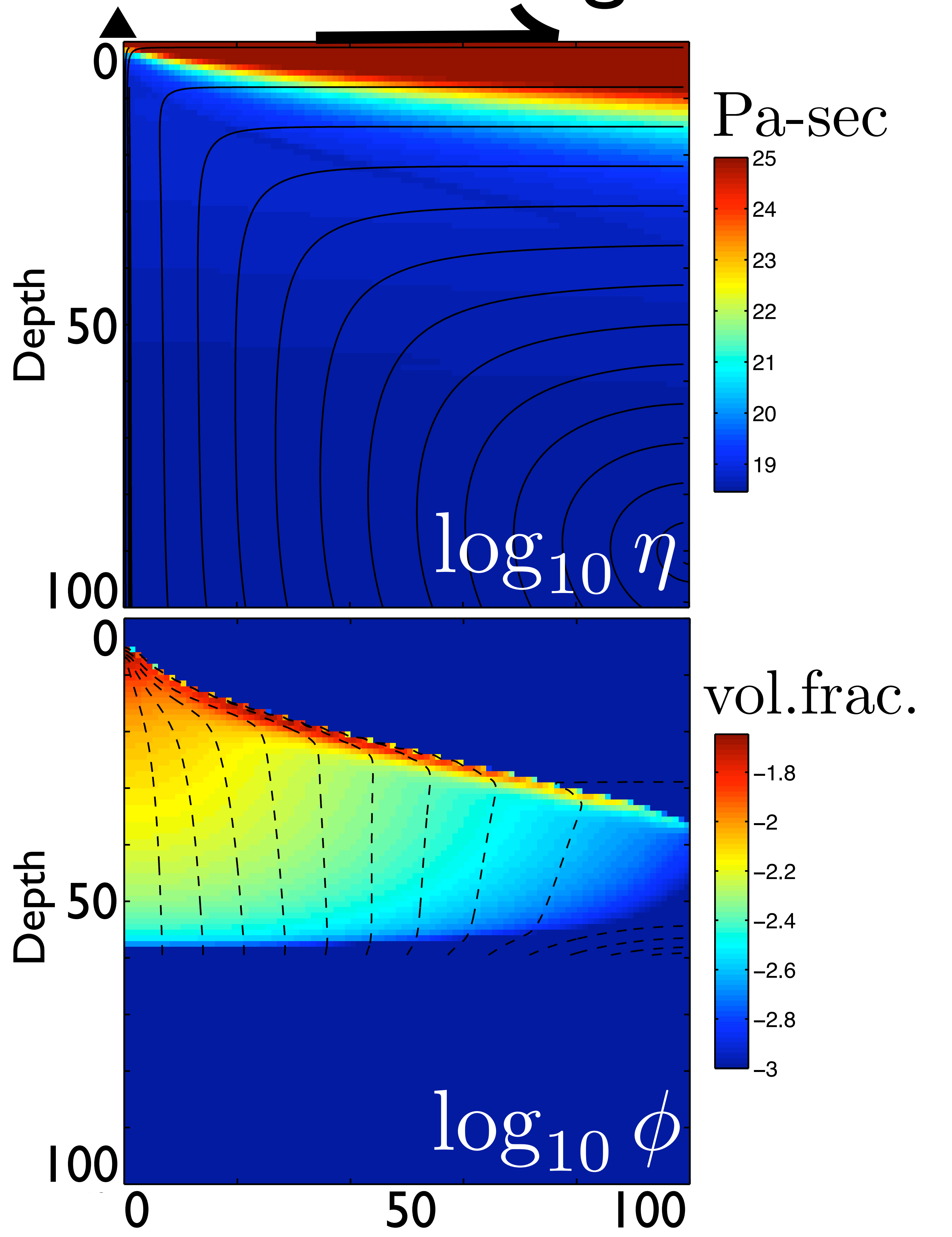
$$\zeta = \zeta_0 \exp\left(\frac{E^*}{RT}\right)\phi^{-1}$$

$$\mathbf{v}_m(x, z=0) = (U_0, 0)$$

Ridge axis boundary:
reflection*

Bottom/right boundary:
inflow/outflow

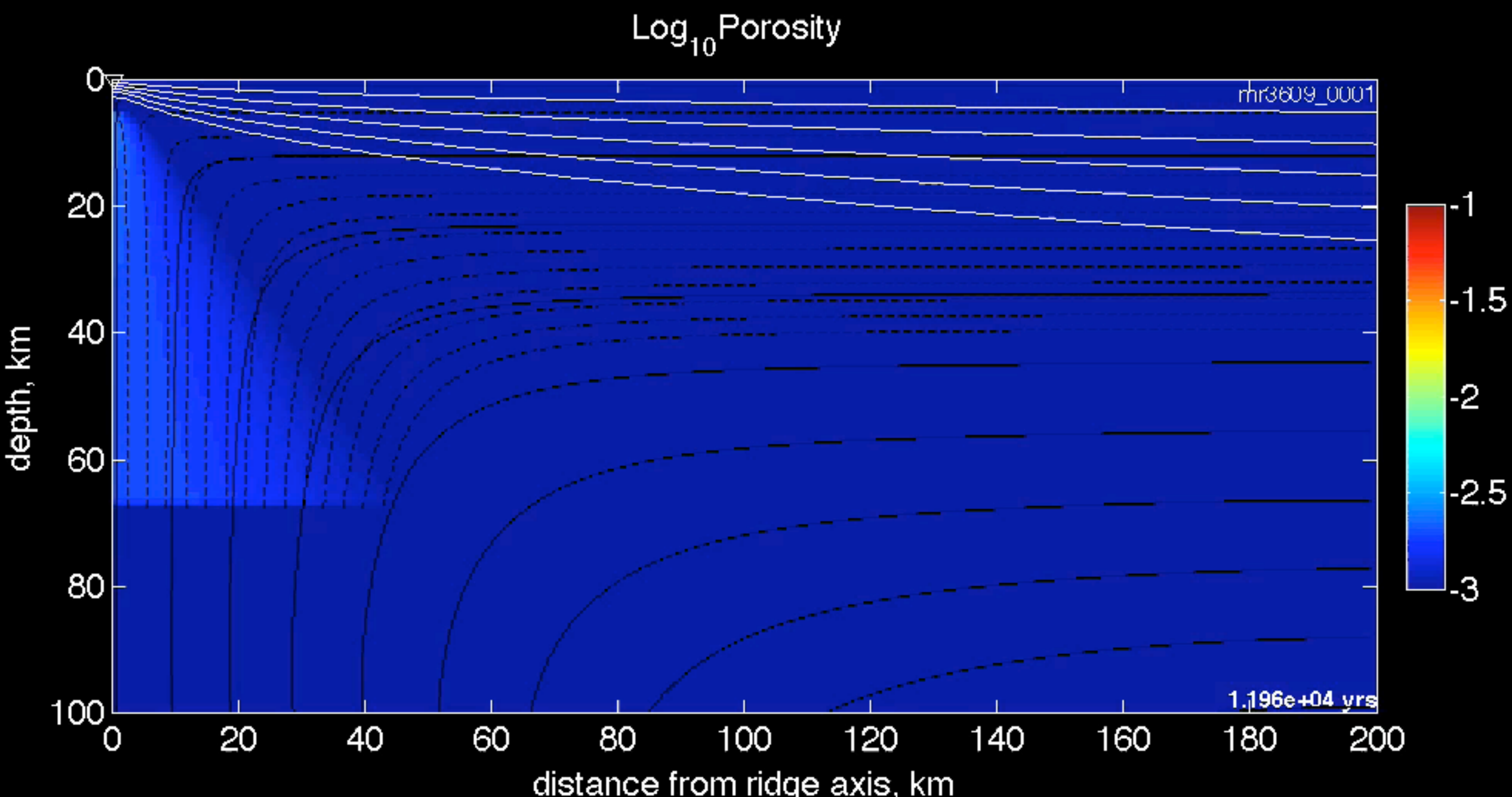
Melt outflow:
dike under axis to base of the lithosphere.



3.2

Melt focusing at mid-ocean ridges

$U_0 = 4 \text{ cm/yr}, \ K_0 = 10^{-7} \text{ m}^2, \ \eta_0 = 1e19, \ \zeta_0 = 5e19 \text{ Pa-s, etc}$



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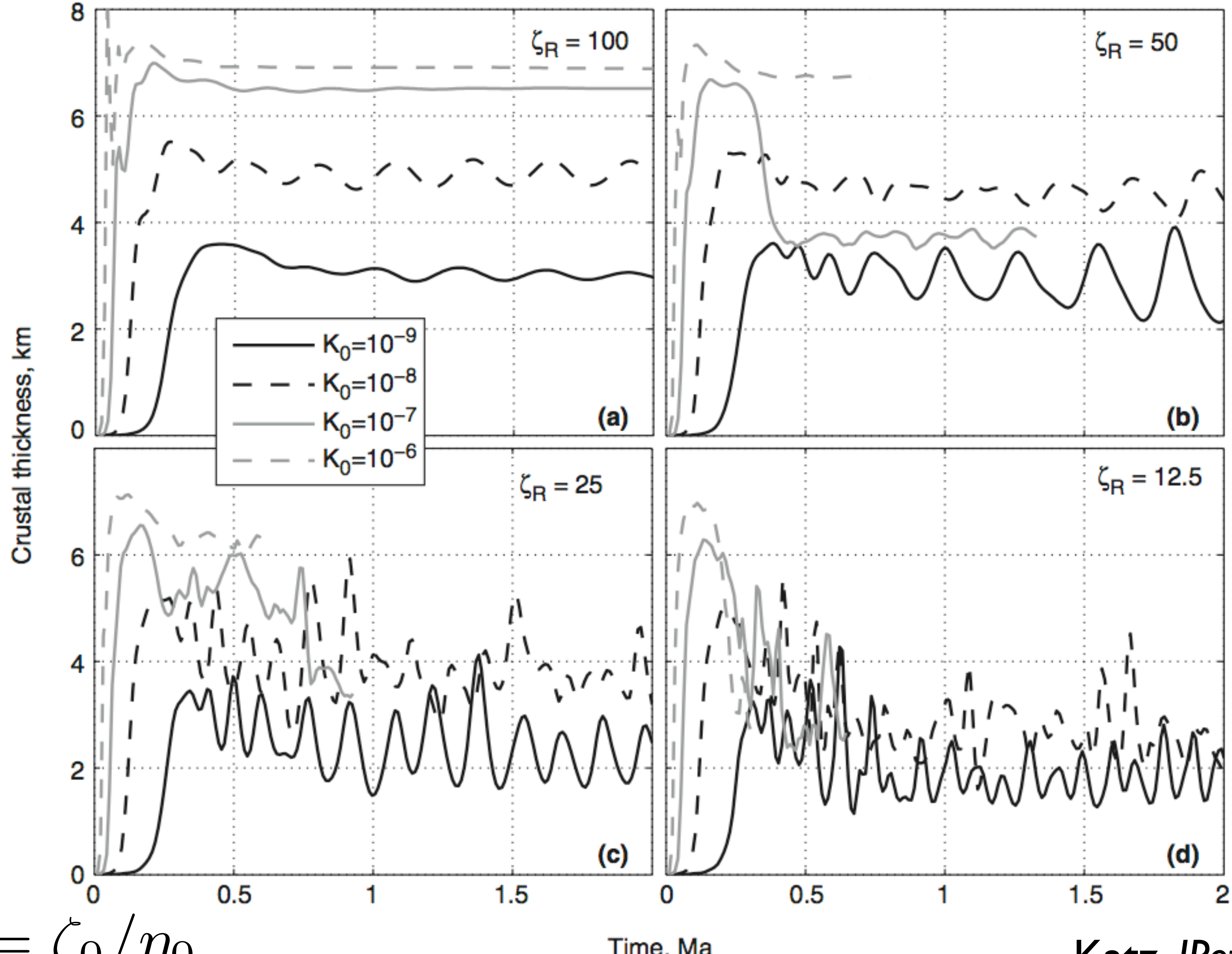
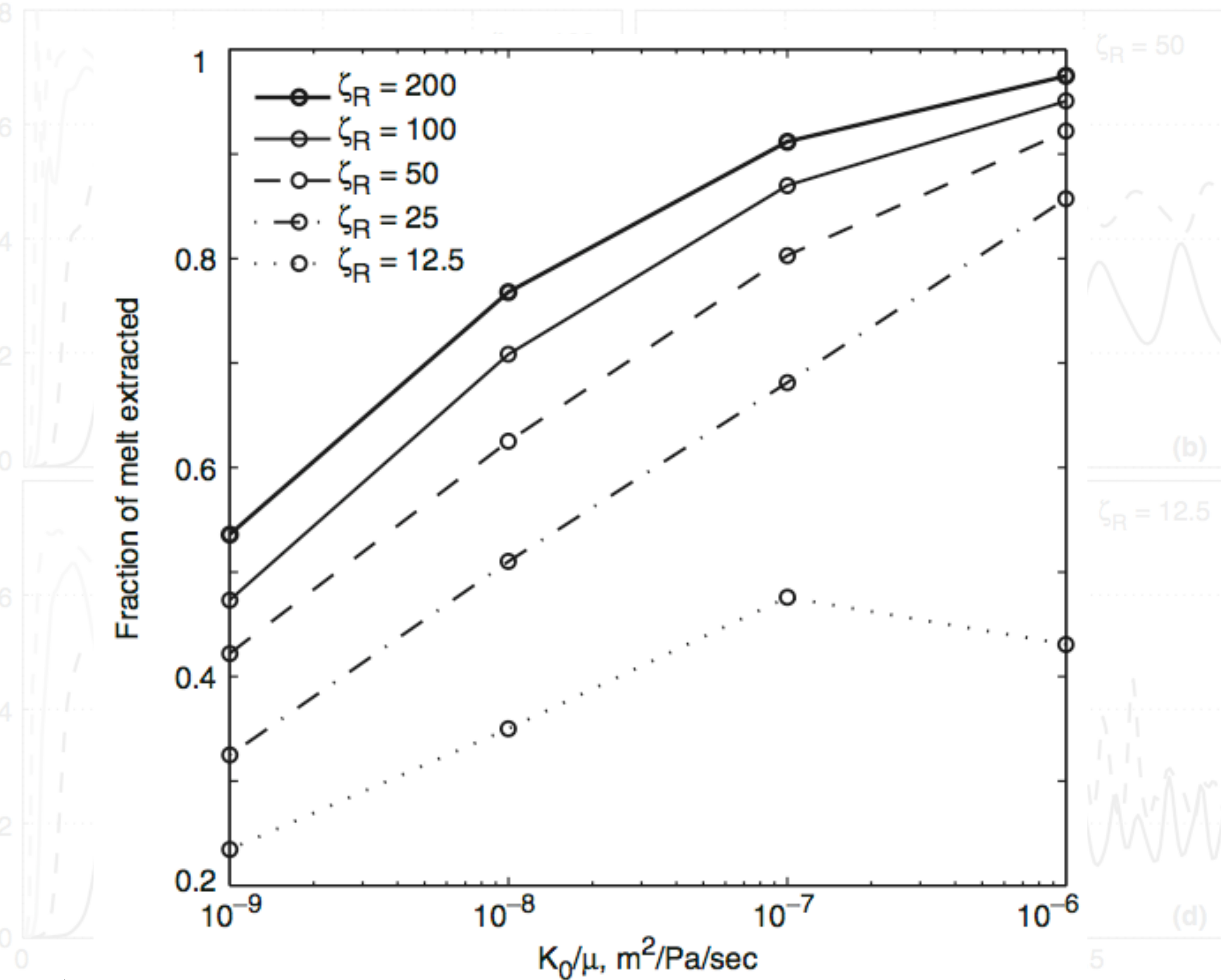


$$\zeta_R = \zeta_0 / \eta_0$$

Katz, *JPet* 2008

Melt focusing at mid-ocean ridges



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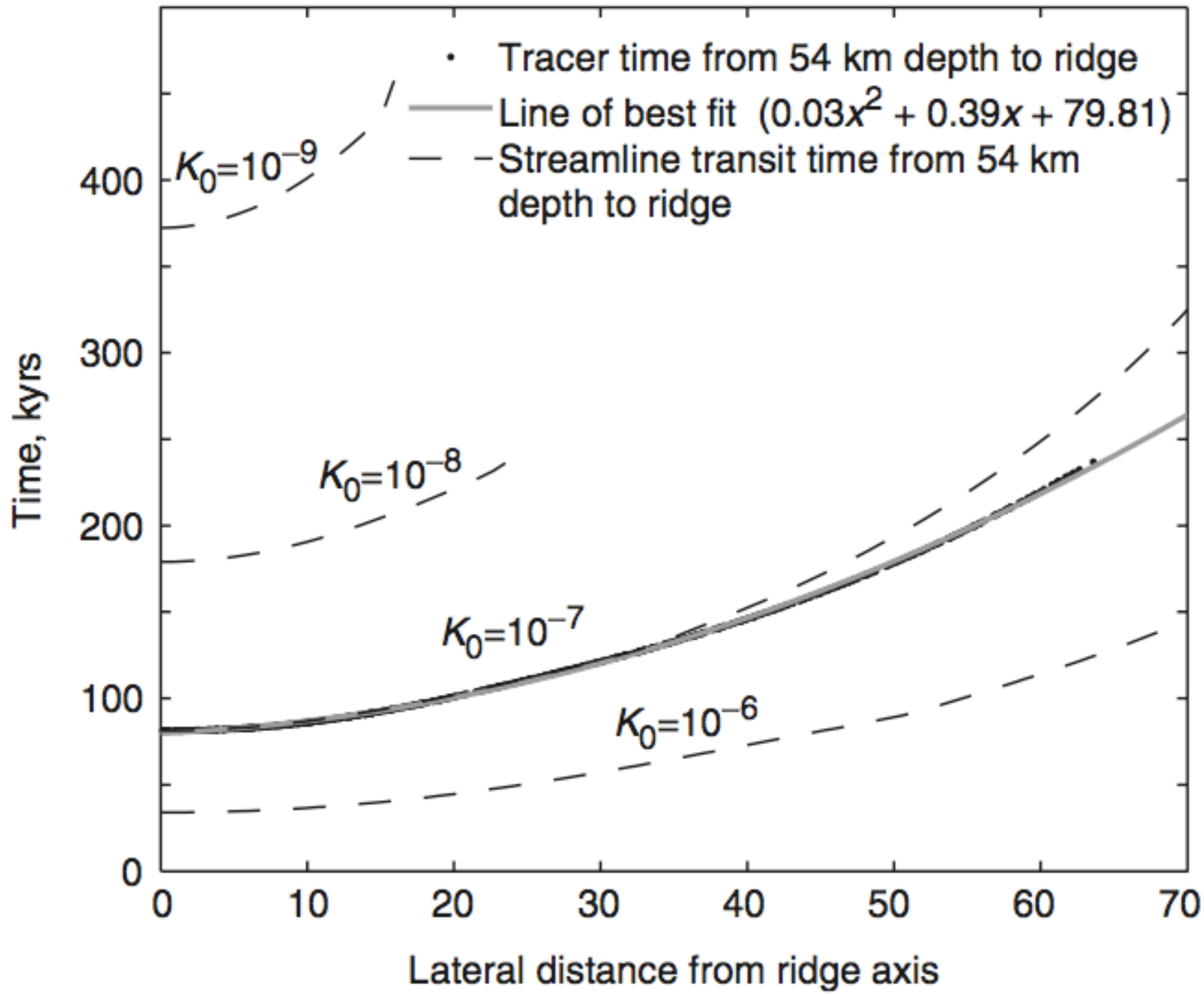
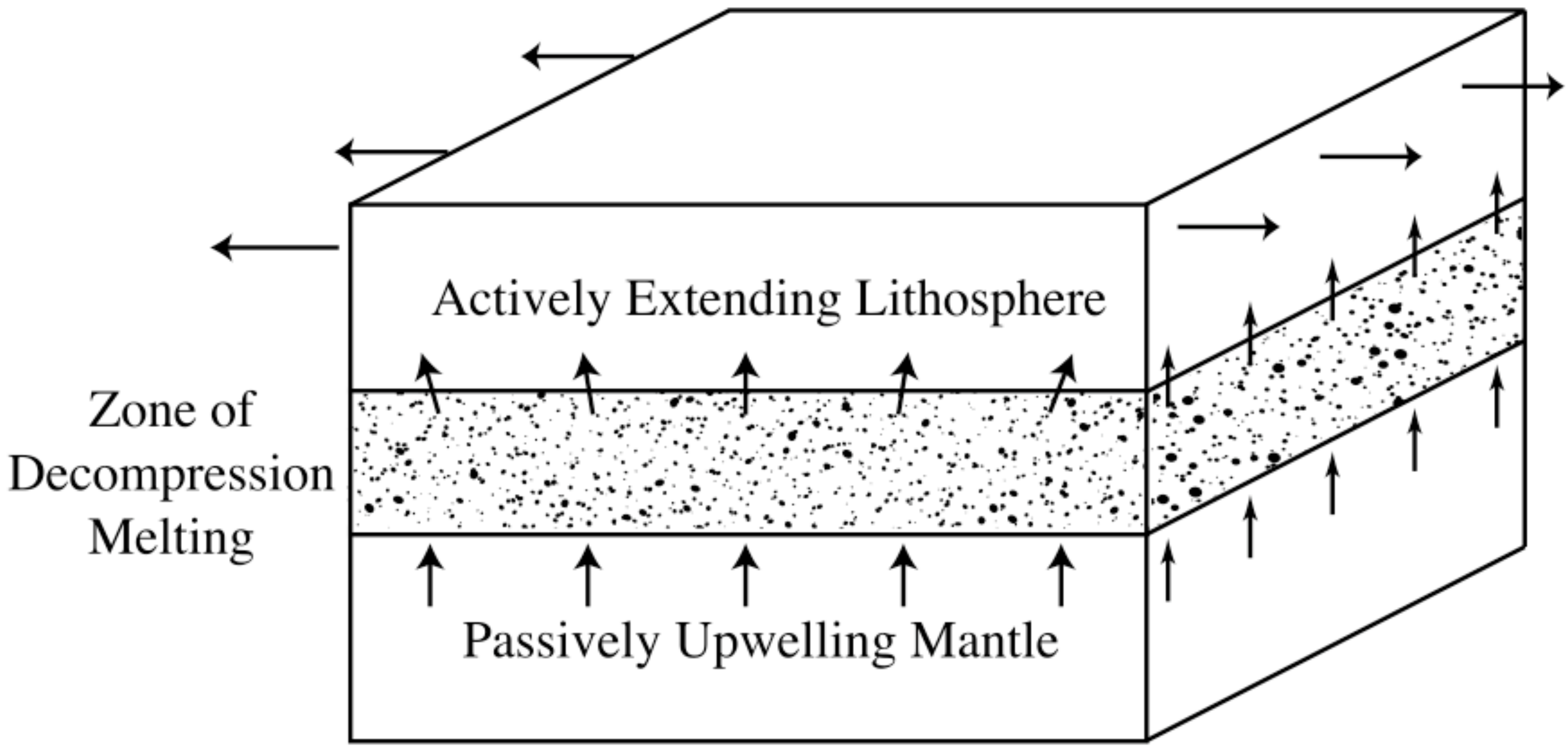


Katz, *JPet* 2008

3.2

Local convection due to porosity



Hernlund et al, *JGR* 2008

3.2

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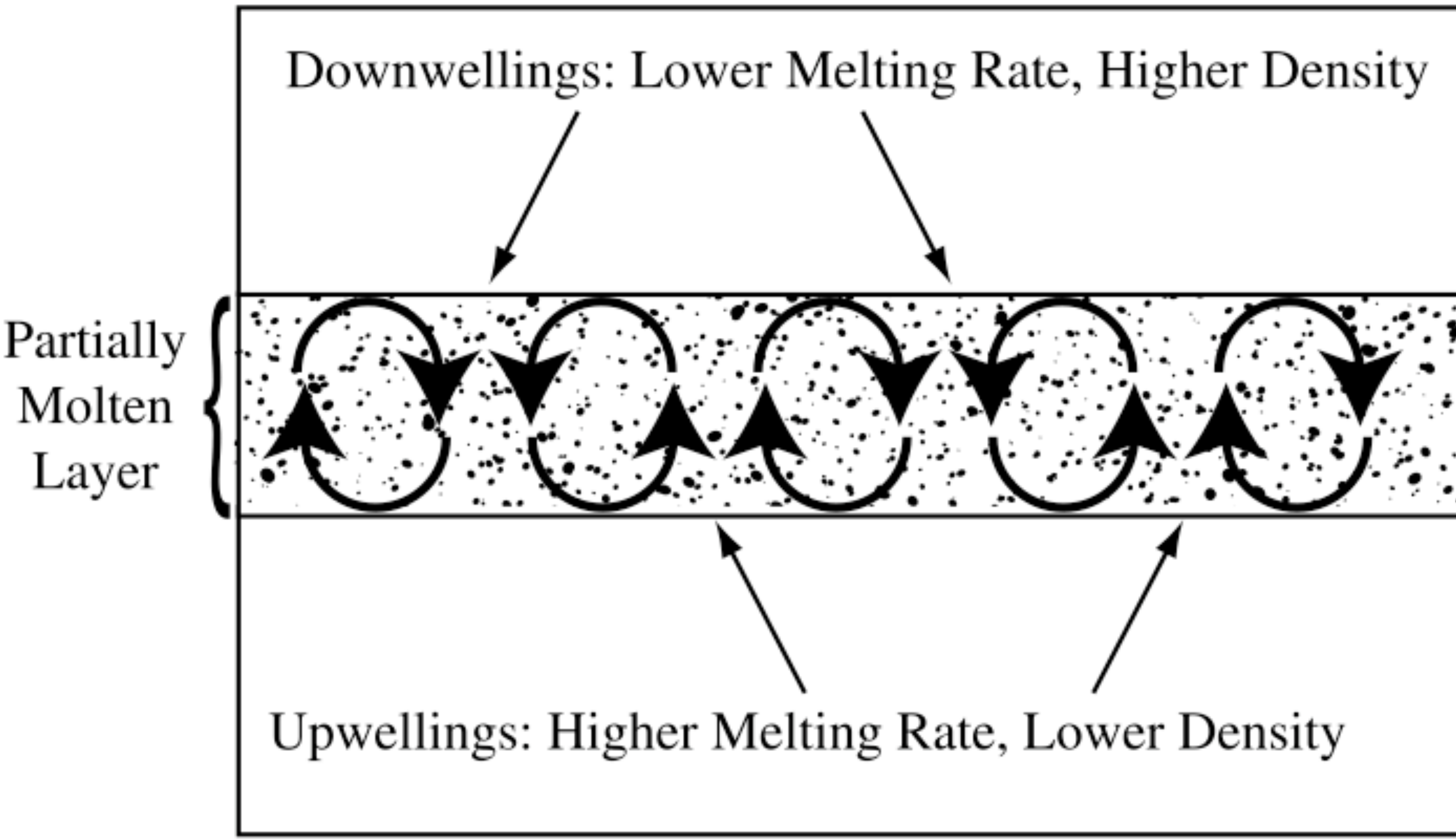

Hernlund et al, *JGR* 2008

Putting it together - summary

- Tectonic scale models generate insight into coupled magma/mantle processes.
- Investigate large-scale processes such as local convection and magmatic focusing.
- Need to incorporate more physics (e.g. reactive channelization) and chemistry (e.g. trace elements, U-series).
- Mantle heterogeneity, hydrous melting, subduction, plumes, 3D...

Talk summary

- Early (exciting) days for magma dynamics.
- Interesting problems in fundamental fluid mechanics, geodynamics, thermodynamics.
- Haven’t addressed: anisotropy, capillarity, melting with volatile elements, etc.
- Model testing via geochemistry & seismology.
- Challenging computations, much room for improvement.


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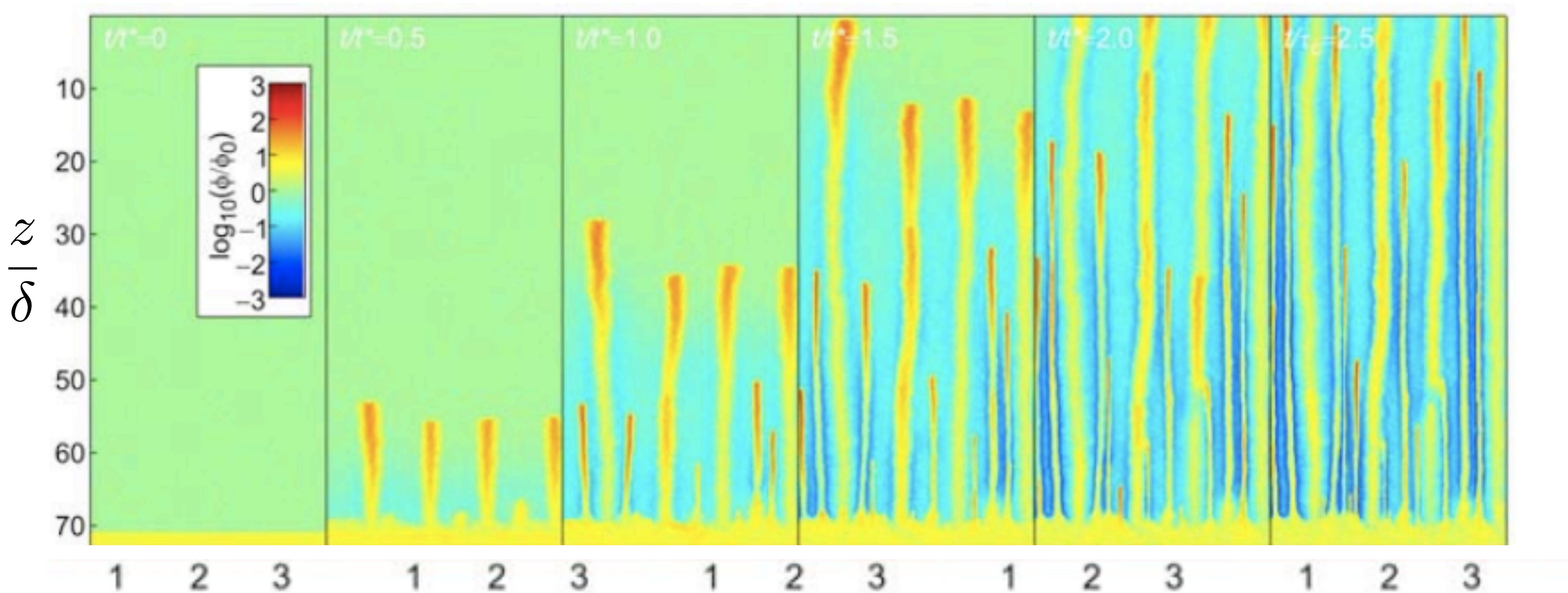
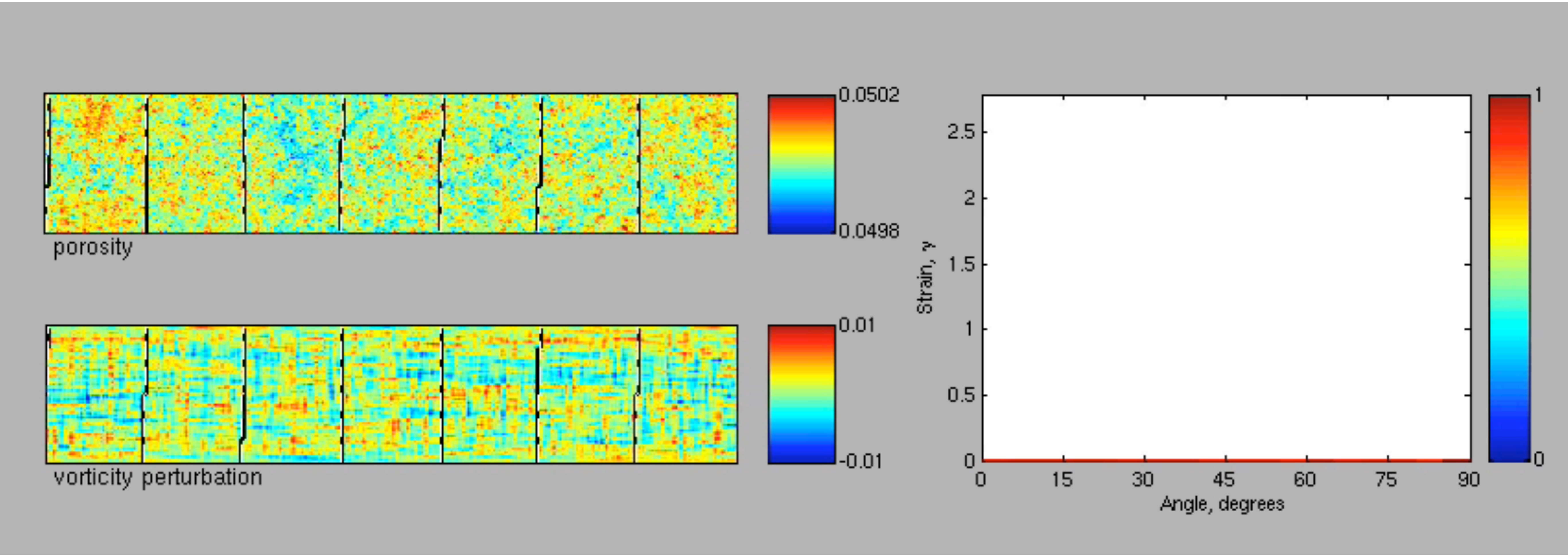
Supplementary slides

Solitary waves



Connolly & Podlachikov, *JGR* 2007

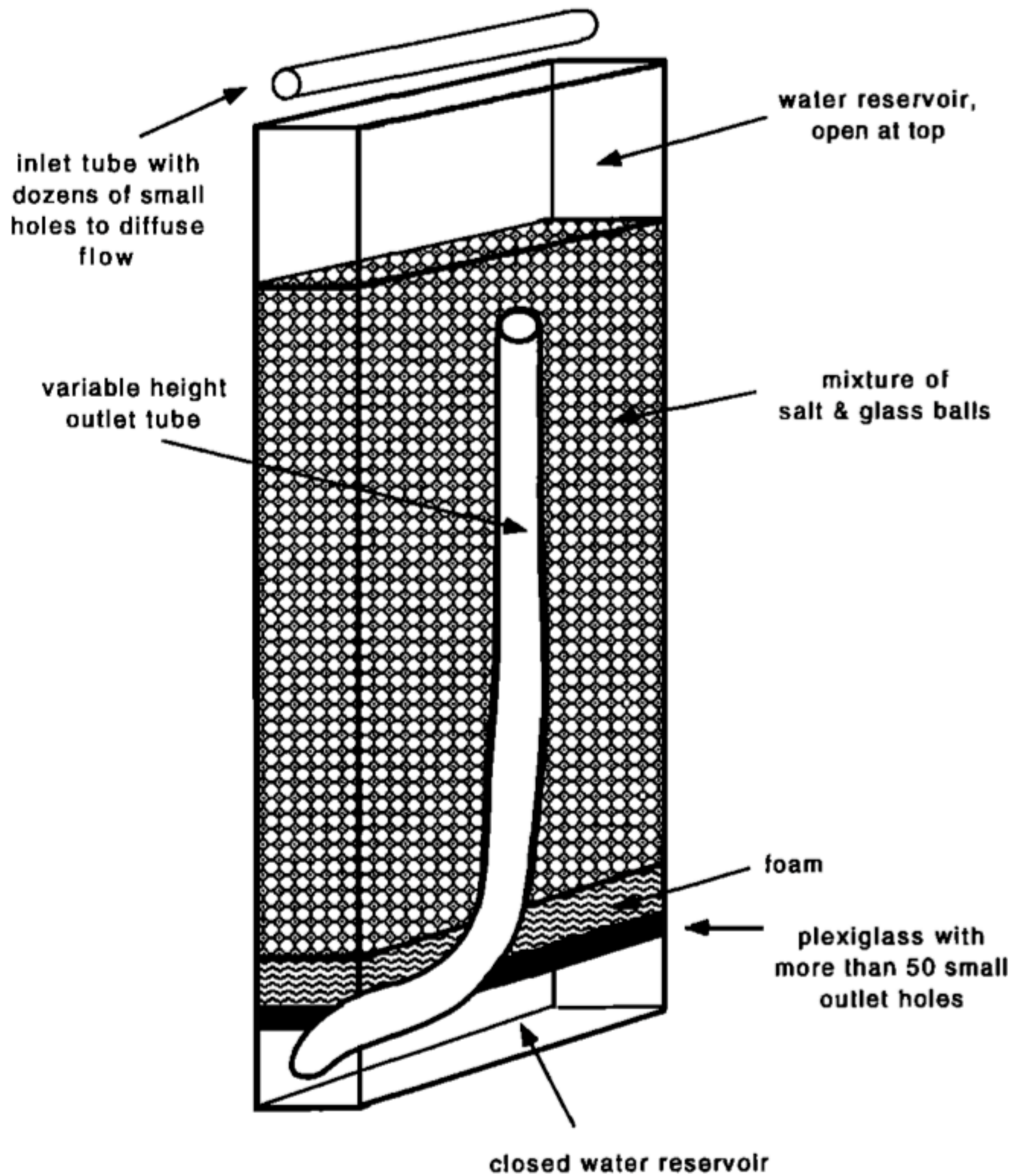
Movie of shear band formation



JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 100, NO. B1, PAGES 475–496, JANUARY 10, 1995

Experiments on flow focusing in soluble porous media, with applications to melt extraction from the mantle

Peter B. Kelemen,[1] J. A. Whitehead,[2] Einat Aharonov,[3] and Kelsey A. Jordahl[4]
Woods Hole Oceanographic Institution, Woods Hole, Massachusetts



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