# Crystal Settling and Growth caused by Ostwald Ripening in a Rotating Terrestrial Magma Ocean

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## Introduction

About 4.5 billion years ago the earth was covered by a heavily convecting and rotating global magma ocean approximately $1000$ km deep, caused by an impact of a mars-sized impactor during the earth's accretion. Some time after the formation of the earth's core, when metal and silicate have separated [1], the magma ocean begins to crystallize (Fig. 1). Small silicate crystals emerge and if the fluid is supersaturated with crystals, they grow by Ostwald Ripening. This process can be observed in solutions and emulsions and results in shrinking of small crystals and growing of large crystals on behalf of smaller ones. Due to the change in crystal radius the settling time of the crystals changes. One question which is important for example for the starting model of plate tectonics or the development of the mantle is whether fractional or equilibrium crystallization occurred in the magma ocean. Fractional crystallization would lead to a fully differentiated mantle after the solidification of the magma ocean whereas equilibrium crystallization would result in a well mixed mantle. These two scenarios can be distinguished by comparing the settling times of the crystals with the magma ocean crystallization time.

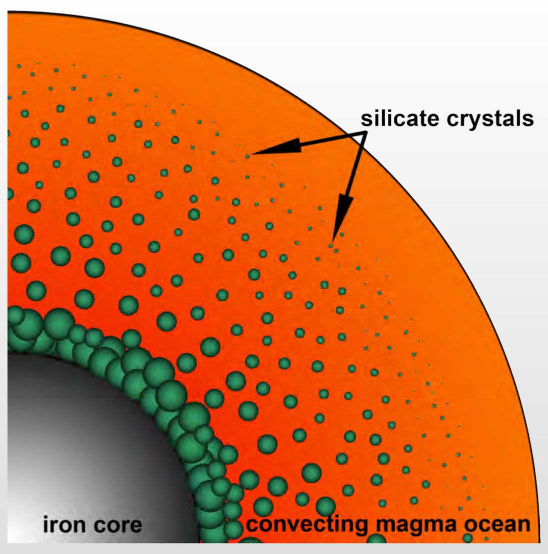


Fig. 1: Magma ocean in an early stage of crystallization.

## Numerical Model

To study the settling, shrinking and growing of crystals in the vigorously convecting and rotating magma ocean, we employed a $3D$ Cartesian model with finite Prandtl number. Due to the low viscosity and strong rotation of the early magma ocean the influence of rotation on the fluid flow cannot be neglected. Our numerical model is divided in two parts, the fluid model and the particle model.

**Fluid model:** The fluid code is based on a Finite Volume discretization and uses the non-dimensionalized Boussinesq equations below. It is possible to tilt the rotation axis to study the situation at the pole and at the equator (Fig. 2). At the pole the rotation axis is parallel, at the equator it is perpendicular to gravity. The Rossby number $Ro$ is used to properly balance the Coriolis force and the buoyant force, because the estimated parameter ($Ra = 10^{27} - 10^{29}$, $Ta = 20^{25} - 10^{27}$ (see below and [2],[4])) are not computational feasible.

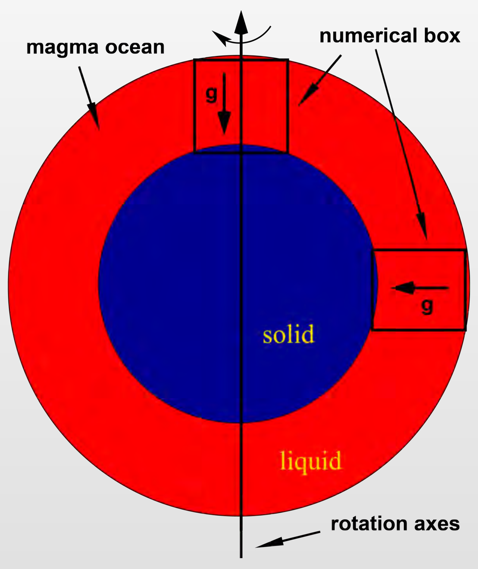


Fig. 2: A schematic view of the numerical model [1].

Equations:

- Continuity eq.: $\nabla \cdot \vec{v} = 0$
- Heat transport: $\frac{\partial T}{\partial t} + (\vec{v}\cdot\nabla)T = \nabla^2 T$
- Momentum eq.: $\frac{1}{Pr}\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right) = -\nabla p - \sqrt{Ta}\,(\vec{e_\omega}\times\vec{v}) + \nabla^2\vec{v} + Ra(T - BC)\vec{e_z}$

Parameter:

- Rayleigh number: $Ra = \frac{\alpha g \Delta T d^3}{\kappa \nu_0}$, $Ra = 10^8$
- Prandtl number: $Pr = \frac{\nu_0}{\kappa}$, $Pr = 1$
- Taylor number: $Ta = \frac{4\Omega^2 d^4}{\nu_0^2}$, $0 \le Ta \le 10^{10}$
- Buoyancy number: $B = \frac{\rho_p - \rho_{fl}}{\rho_{fl}\alpha\Delta T}$, $B \approx 2.5$
- Rossby number: $Ro = \sqrt{\frac{Ra}{PrTa}}$, $0.1 \le Ro \le \infty$

**Particle model:** The silicate crystals are modelled with a discrete element model. They influence the fluid flow and are able to grow, shrink, vanish and form. In addition they interact with each other through a collision algorithm and gravitational, Coriolis and drag forces caused by the fluid act on them. To compare different simulations, we use the Stokes' settling time $t_s = \frac{9}{2}\frac{1}{a^2 RaB}$. This is the time in which a particle falls from the top to the bottom of the numerical box through a motionless fluid.

## 1) Crystal settling

At first we study the crystal settling at the pole and equator depending on different rotation rates without incorporating Ostwald Ripening.

**Pole:** At the Pole without rotation ($Ro = \infty$) and with weak rotation ($Ro = 1$) a large fraction of crystals is kept in suspension (Fig. 3). With strong rotation ($Ro = 0.1$) the silicate crystals settle and form a thick crystal layer at the bottom of the magma ocean which can be explained with the Taylor-Proudman Theorem.

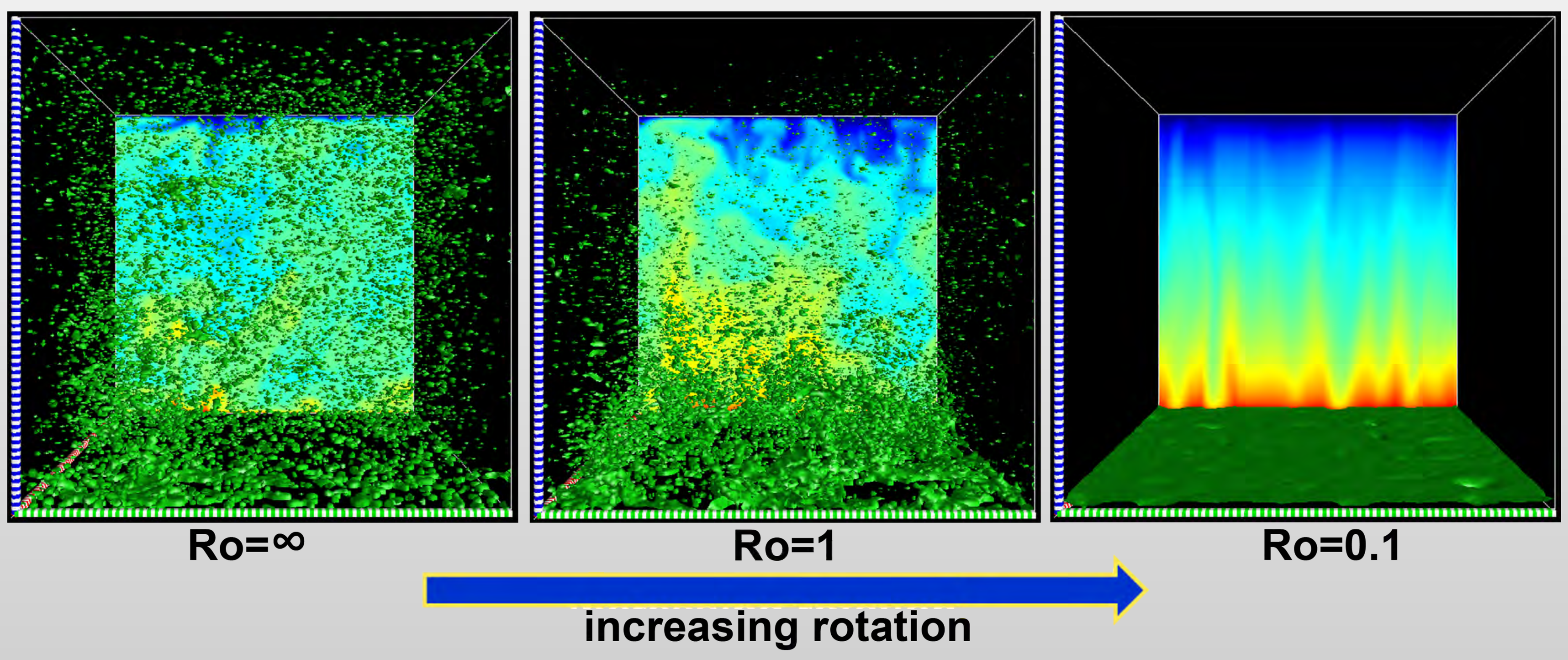


Fig. 3: Settling of silicate crystals at the **pole** without rotation (left), weak rotation (middle) and high rotation (right). In the background the temperature field.

**Equator:** In contrast we find four regimes (Fig. 4) here [1]. Without rotation the crystal's behaviour is comparable to that at the pole. At weak rotation ($Ro = 1$) a high fraction of silicate crystals settle at the bottom of the magma ocean, at stronger rotation ($Ro = 0.14$) the silicates form a thick crystal layer in the bottom $1/3$ of the box. With strong rotation ($Ro = 0.1$) all crystals are suspended and we observe a ribbon like structure in the middle that develops due to strong shear flows in y-direction.

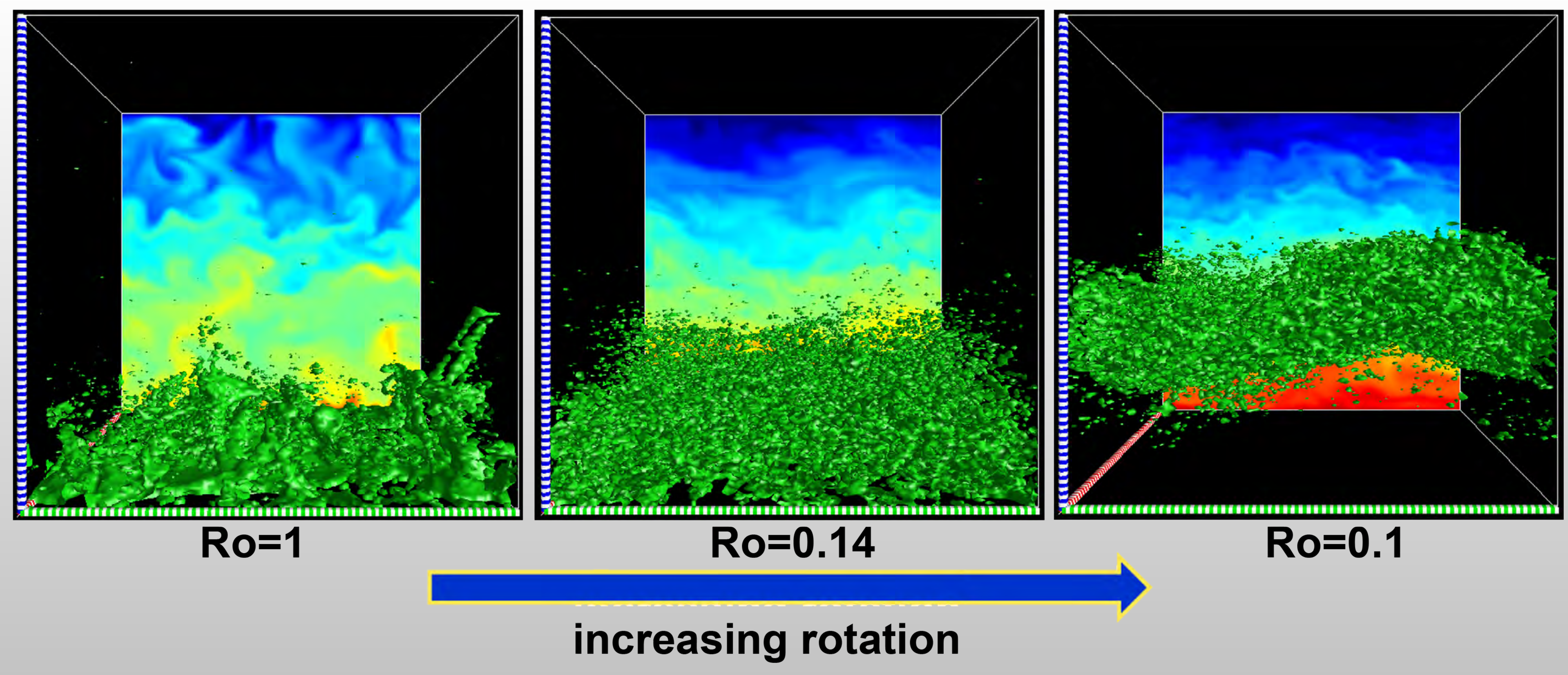


Fig. 4: Settling of silicate crystals at the **equator** for weak (left), moderate (middle) and high rotation (right).

## 2) Ostwald Ripening

With a second model we investigate growing and shrinking of crystals by Ostwald Ripening and include formation and melting of particles (Fig. 1). With Ostwald Ripening smaller crystals tend to dissolve and larger crystals grow on behalf on smaller ones (Fig. 6) if a given saturation is reached. An explanation for this process is that larger crystals are energetically more stable than smaller ones. For the magma ocean Ostwald Ripening means that the silicate crystals form if the temperature and pressure are high enough at a certain depth. In a first study we divided the box after works of [2] and [3] in three zones (Fig. 5). The silicate crystals form in a formation zone and fall through the fluid. In the growing zone we assume that the fluid is supersaturated. If two crystals with different radius collide here, the larger crystal grows and the smaller crystal shrinks. In the zone above the formation region the crystals melt.

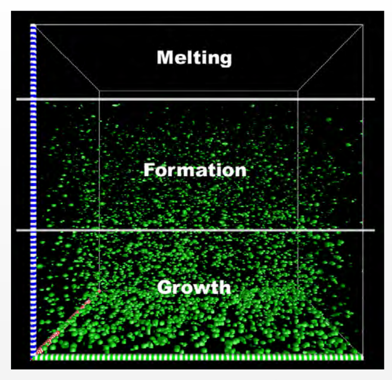


Fig. 5: Zones for Ostwald Ripening model.

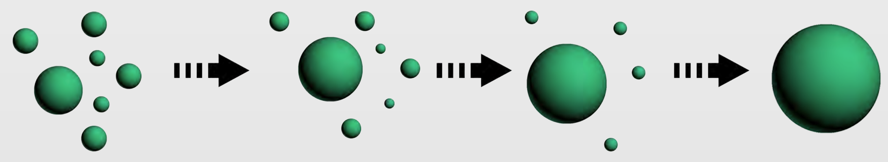

Fig. 6: Principle of Ostwald Ripening: Smaller crystals dissolve, larger crystals grow on behalf of small crystals.

Very first results show that if we include Ostwald Ripening, in general we observe the same three regimes as without Ostwald Ripening.

**Pole:** However, the evolution of the averaged crystal radius depends on the rotational strength. Without rotation ($Ro = \infty$) the averaged crystal radius is nearly constant with time (Fig. 7), with increasing rotation the radius and the growth speed increase. The crystal radius and growth speed is maximum for the highest rotation ($Ro = 0.1$).

**Equator:** At the equator, although the calculations for these cases have not reached a statistically steady state yet, we can observe that without rotation ($Ro = \infty$) the averaged crystal radius is constant with time as it is at the pole. With increasing rotation the crystals grow faster and also get larger compared to the pole (Fig. 8). The fastest growth of the pole and equator can be observed at the equator for high rotation ($Ro = 0.1$).

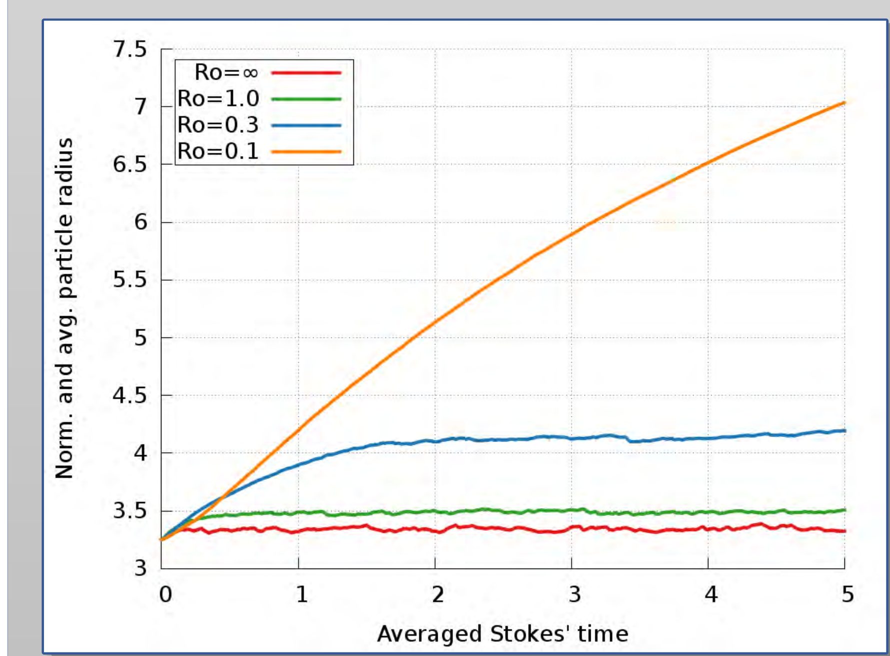


Fig. 7: Evolution of the crystal radius at the **pole** for different rotation rates.

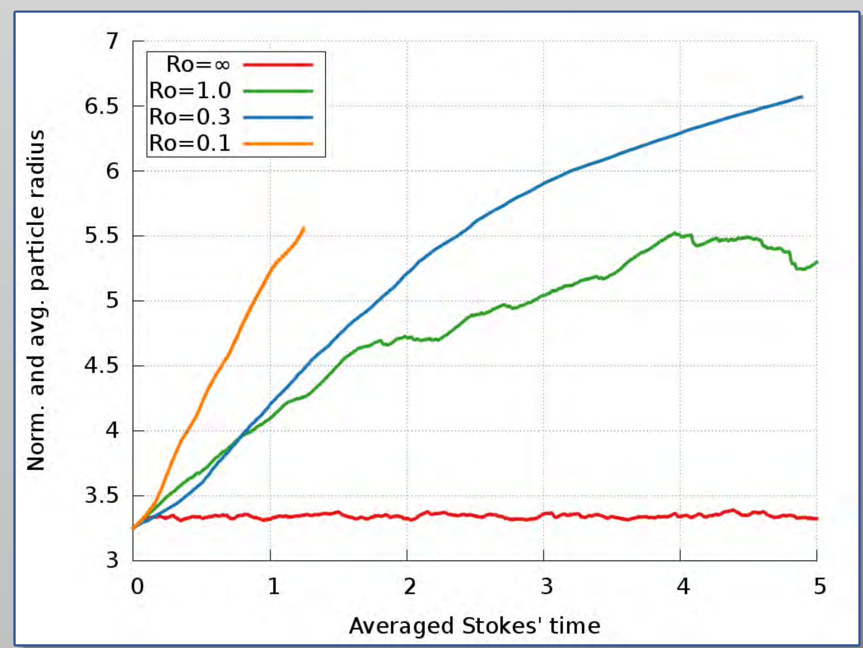


Fig. 8: Evolution of the crystal radius at the **equator** for different rotation rates.

## Conclusion

1) Crystal settling: depends on rotational strength and orientation of rotation axis.

- Pole: weak rotation: high fraction of crystals in suspension; strong rotation: fast and total crystal settling
- Equator: weak rotation: crystal settling; moderate rotation: thick crystal layer, strong rotation: ribbon-like structure in the middle of the magma ocean

2) Ostwald Ripening: growth and avg. crystal radius and therefore sedimentation time depends on rotational strength and orientation of rotation axis.

- Pole: weak - moderate rotation: crystal radius small and slow growth $\rightarrow$ large sedimentation time; strong rotation: crystals grow larger at moderate speed
- Equator: moderate - strong rotation: crystals grow faster and get larger than at the pole $\rightarrow$ small sedimentation time

$\longrightarrow$ asymmetrical crystallization of magma ocean possible.

$\longrightarrow$ Extreme case: well mixed mantle at the pole, fully differentiated at the equator.

## References

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